Threshold ECDSA from ECDSA assumptions: *the multiparty case*

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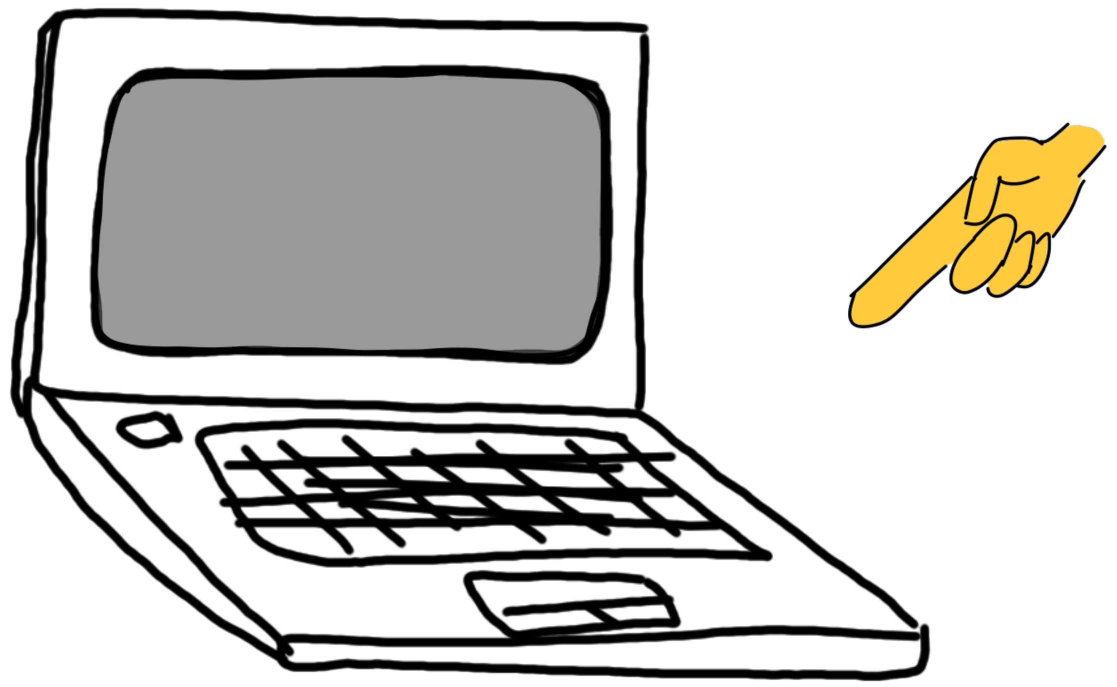
Traditional Signature

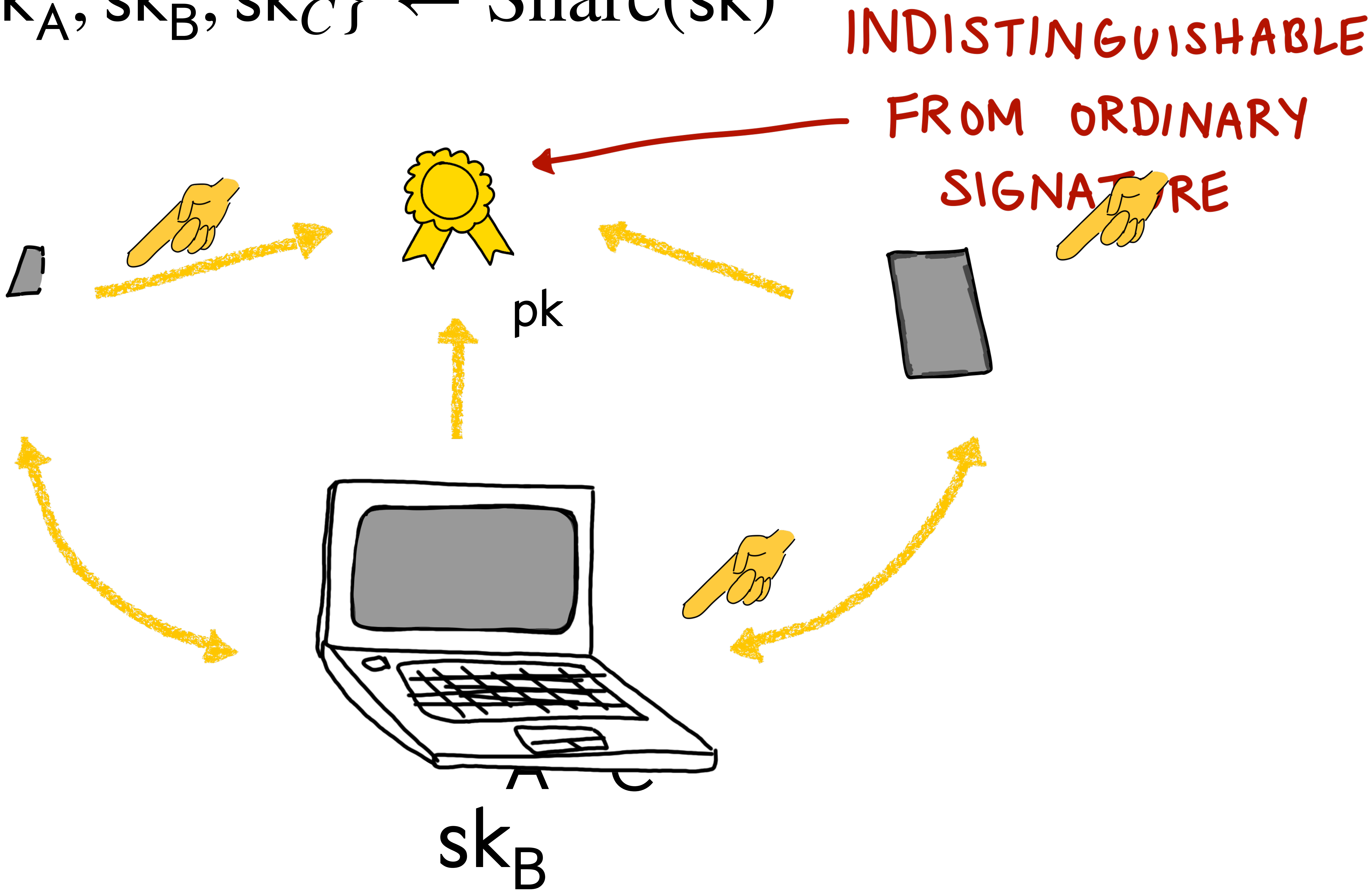
pk



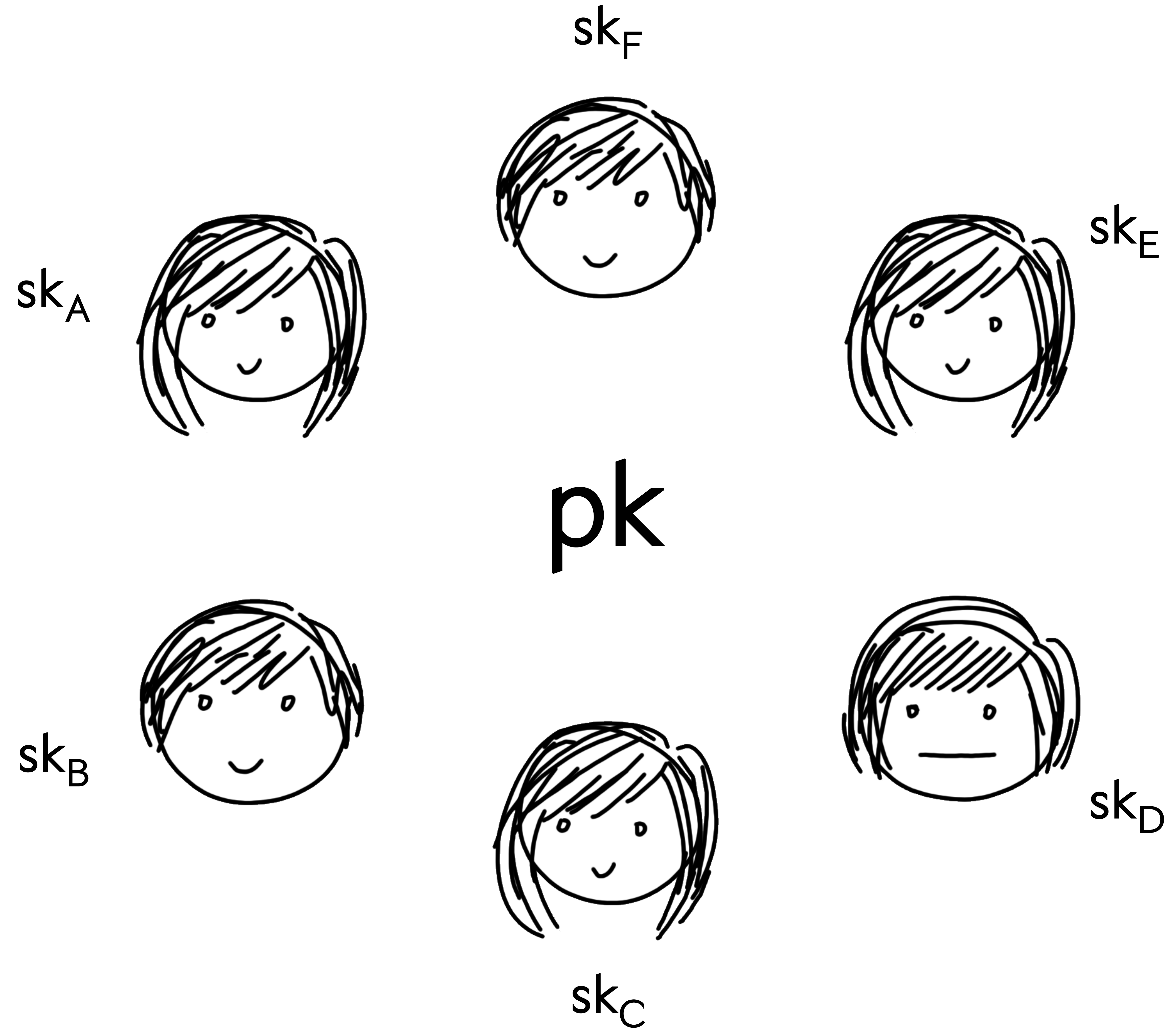
sk

Threshold Signature

$$\{sk_A, sk_B, sk_C\} \leftarrow \text{Share}(sk)$$



3-of-n Signature Scheme



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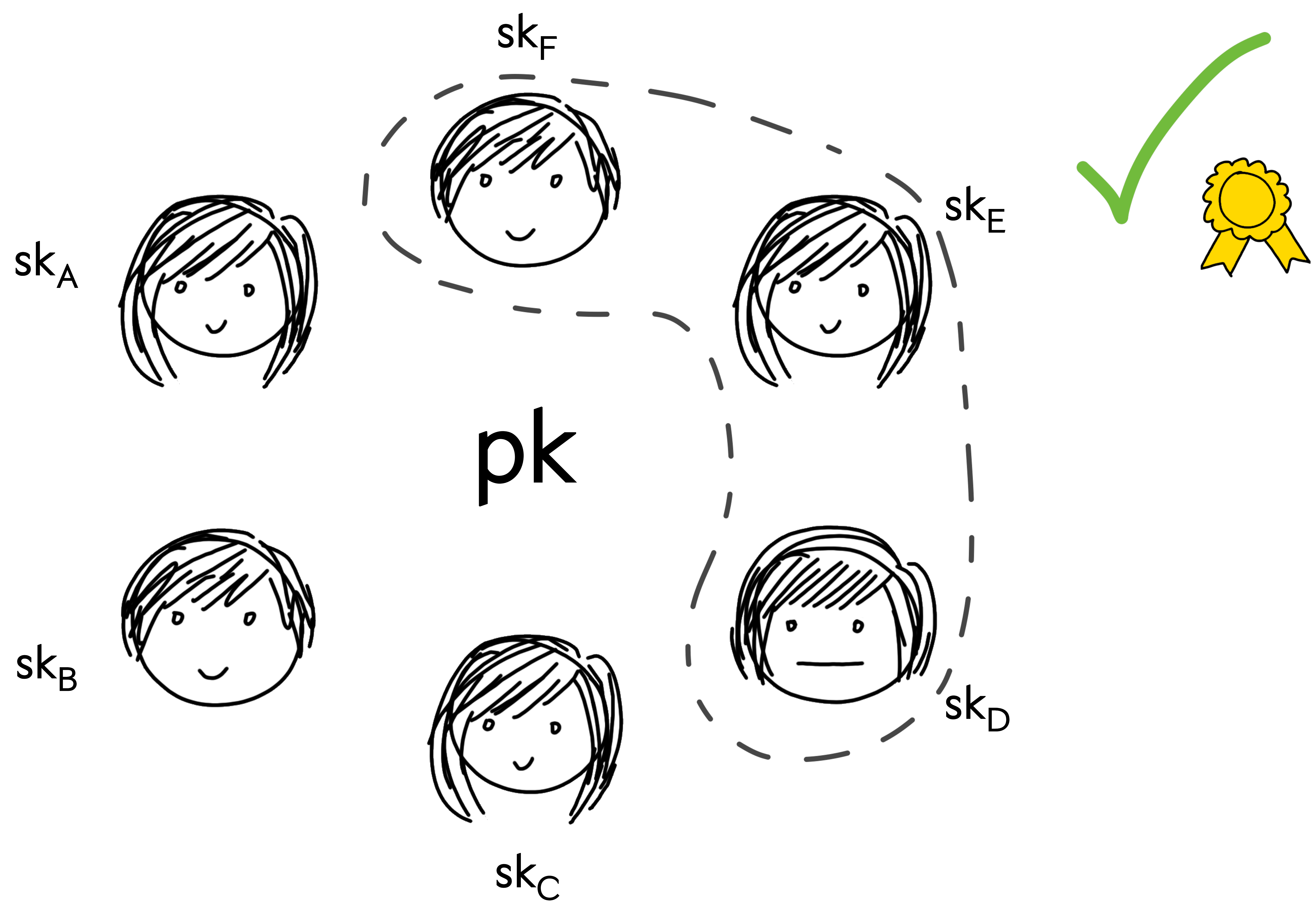

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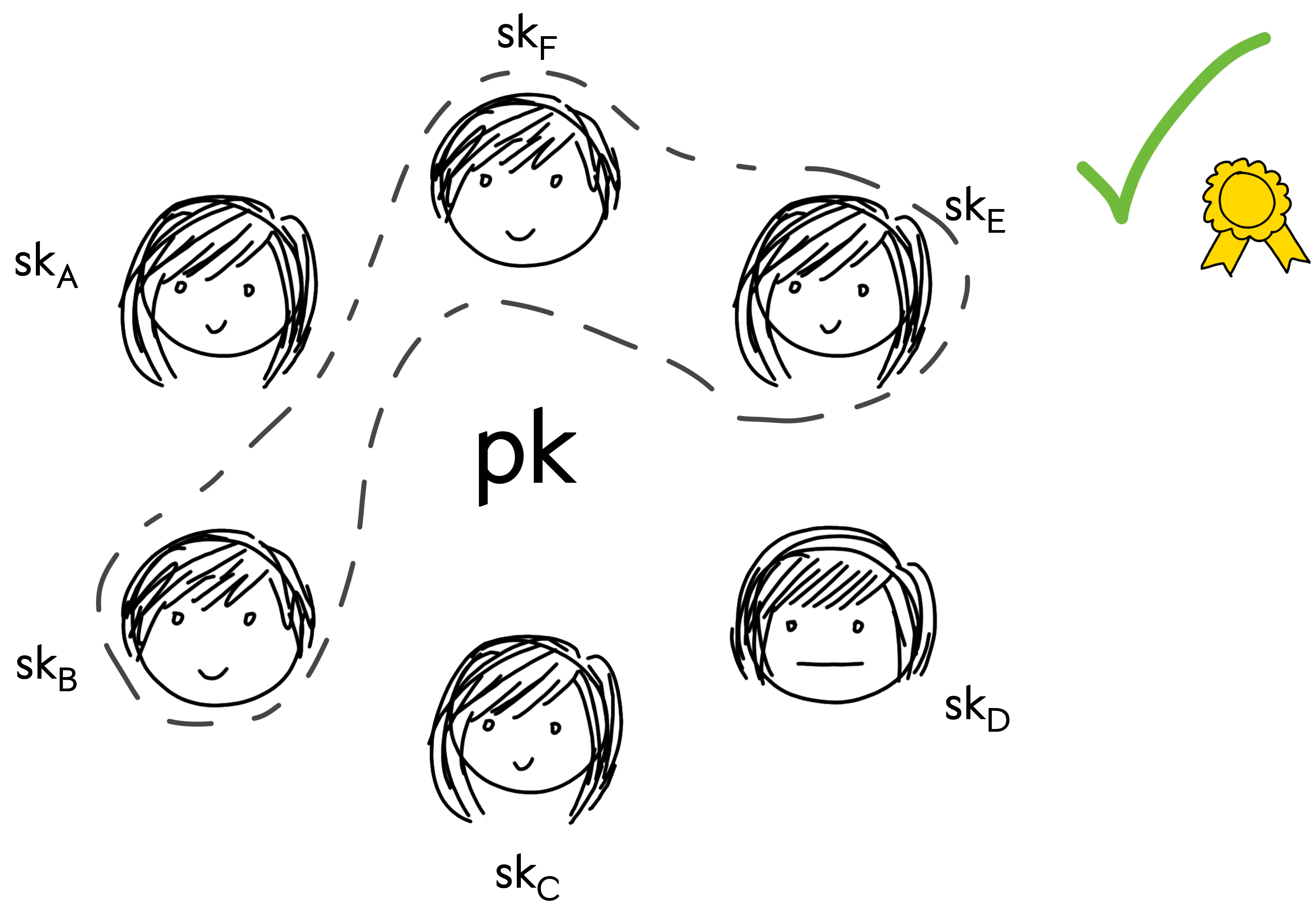

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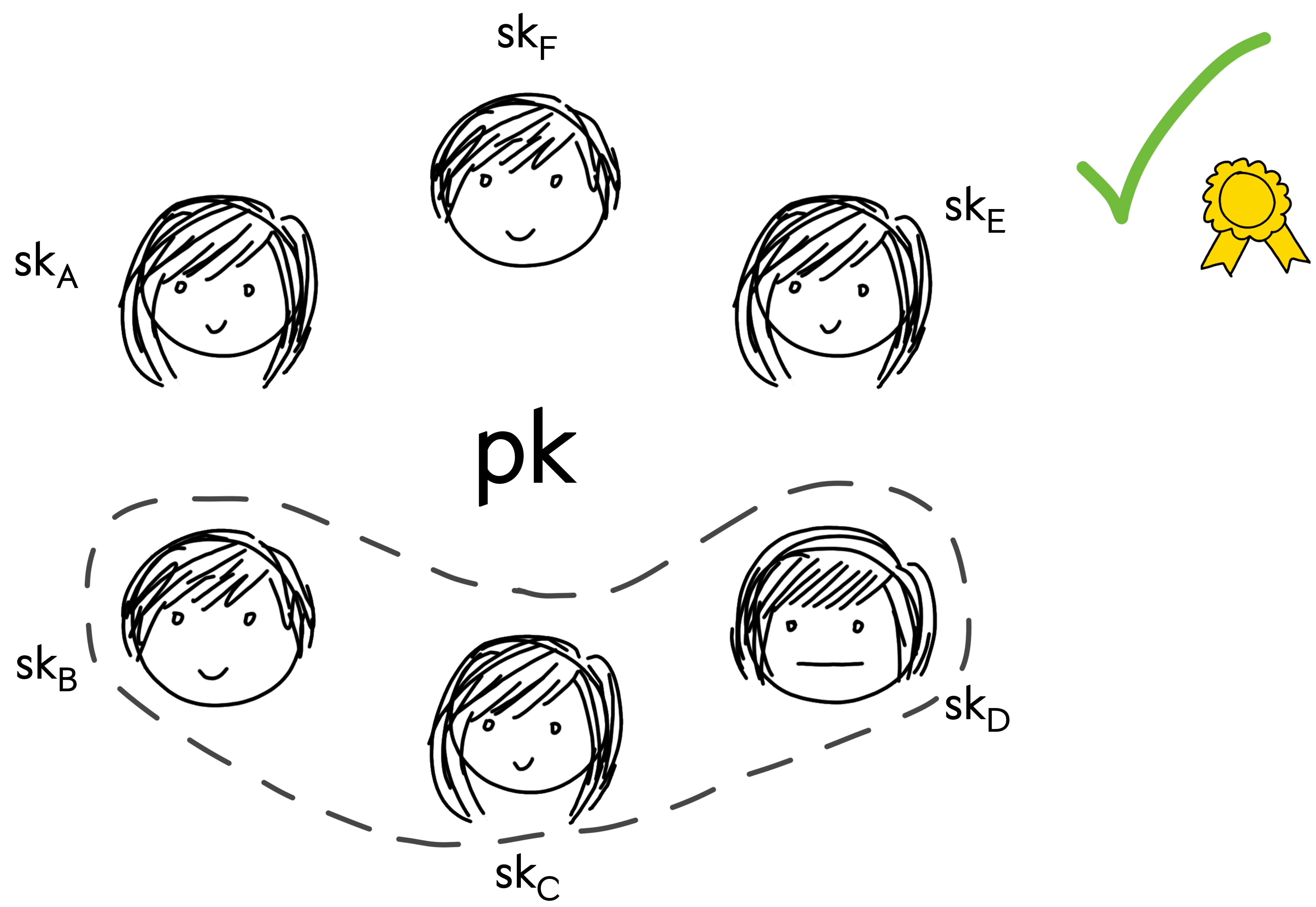

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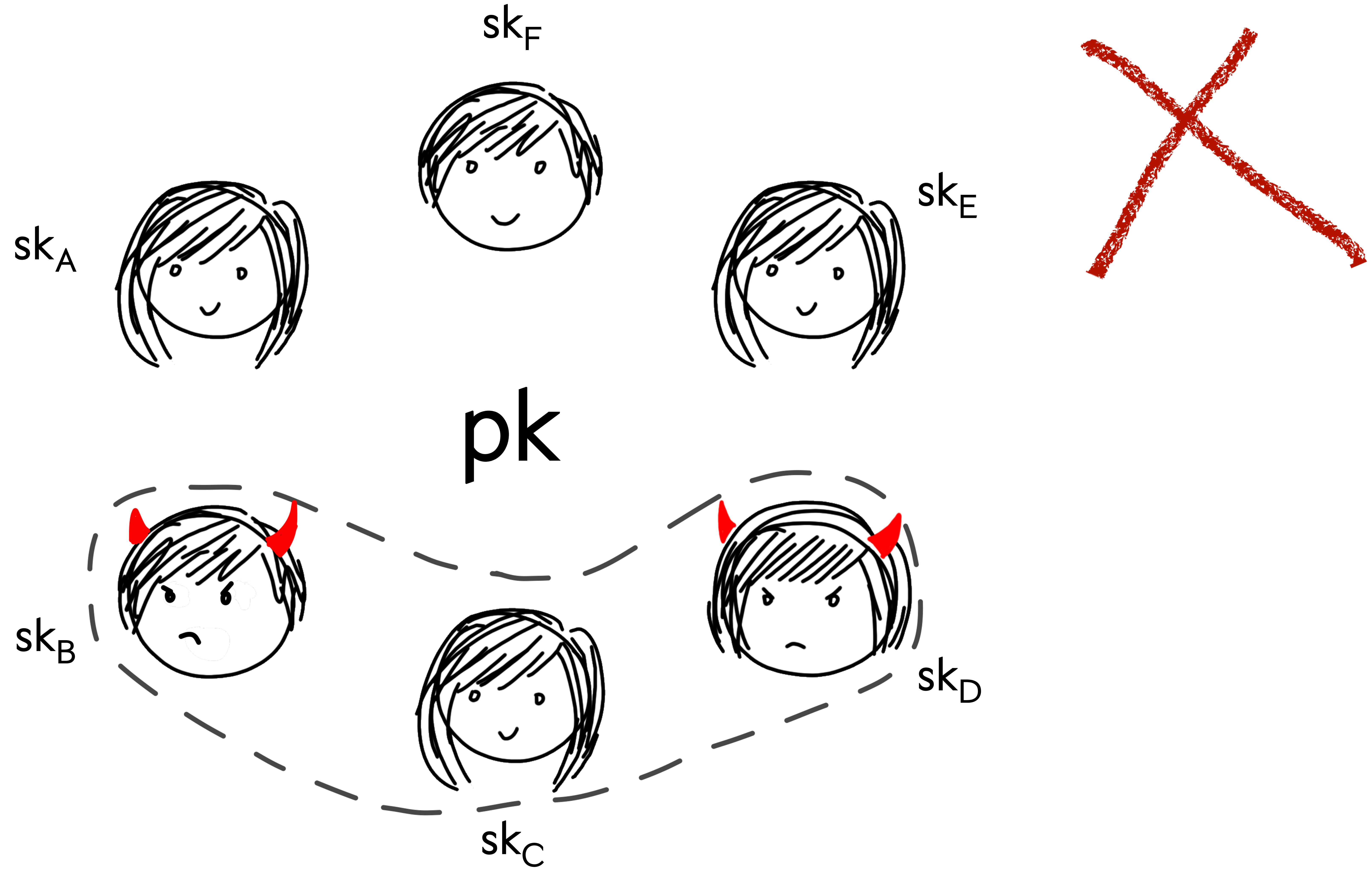

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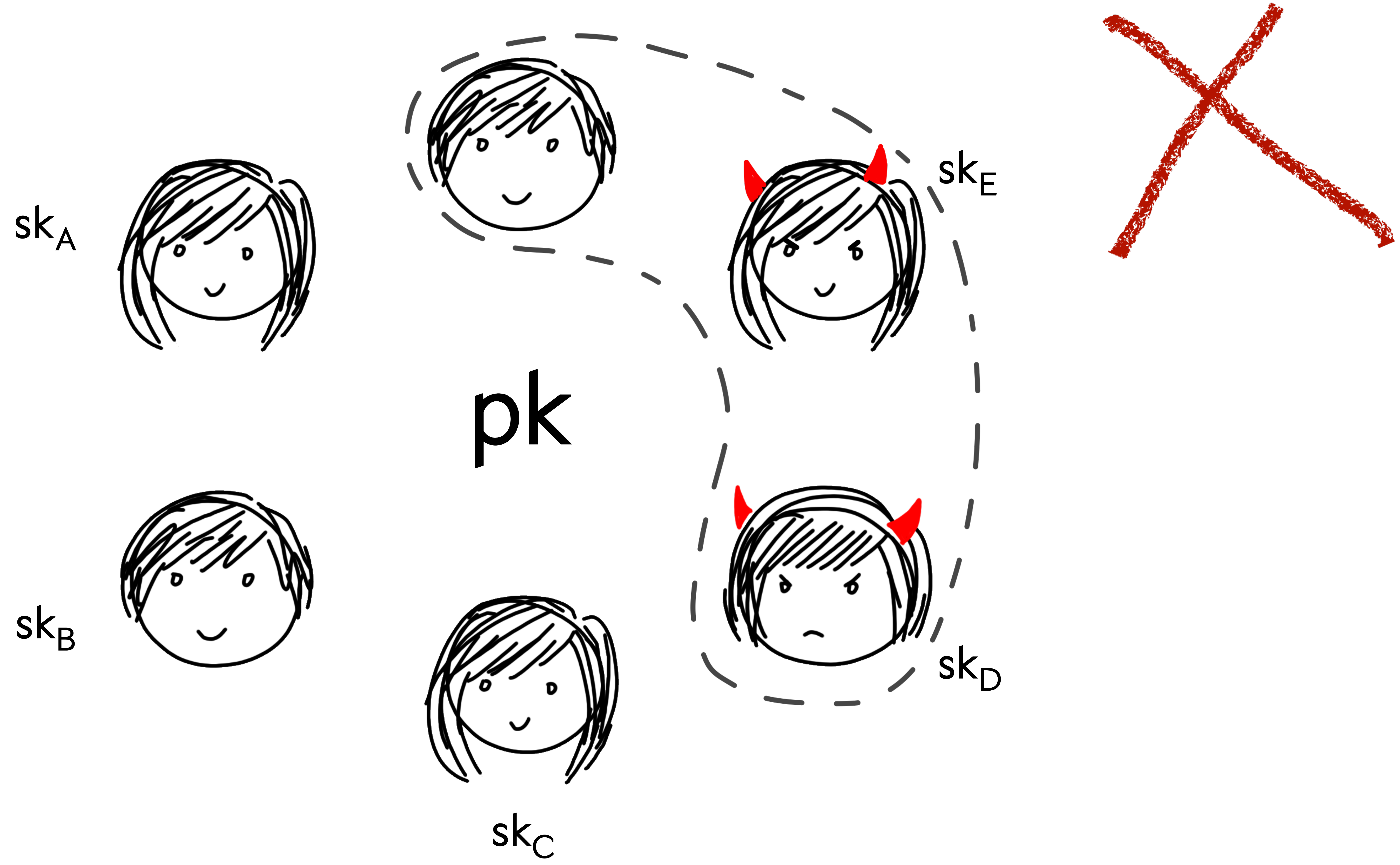

Full Threshold

- Scheme can be instantiated with any $t <= n$
- Adversary corrupts up to t-1 parties

ECDSA

- **E**lliptic **C**urve **D**igital **S**ignature **A**lgorithm
- Devised by David Kravitz, standardized by NIST
- Widespread adoption across the internet

TLS

Notation

Elliptic curve parameters	G	q
Secret values	sk	k
Public values	pk	R

ECDSA Recap

x-coordinate of ***R***

$$R = k \cdot G$$

$$sign(m, \mathsf{sk}, k) = \frac{H(m) + \mathsf{sk} \cdot r_x}{k}$$

Non-linearity makes ‘thresholdization’ difficult

Threshold ECDSA

- Limited schemes based on Paillier encryption: [MacKenzie Reiter 04], [Gennaro Goldfeder Narayanan 16], [Lindell 17]
- Practical key generation and efficient signing (full threshold):
  - [Gennaro Goldfeder 18]: Paillier-based
  - [Lindell Nof Ranellucci 18]: El-Gamal based
- **Our work last year** [DKLs18]: 2-of-n ECDSA under native assumptions
- **This work**: Full-Threshold ECDSA under native assumptions

Our Approach

- 2-party multipliers: **Oblivious Transfer** in **ECDSA curve**
  - **Pros**:
    - With **OT Extension** (no extra assumptions) just a **few milliseconds**
    - **Native assumptions** (**CDH** in the same curve)
  - **Con:** Higher bandwidth (**100s of KB/party**)

Our Approach

- OT-MUL secure up to choice of inputs

- **Light consistency check (unique to our protocol)**:
  - Verify shares in the exponent before reveal
  - Costs **5 exponentiations+curve points**/party
  - Subverting checks implies solving **CDH** in the same curve

Tradeoffs

- Our work avoids **expensive zero-knowledge proofs** and **assumptions foreign to ECDSA** itself, required by other works in the area
- Using OT-MUL is very light on computation, but more demanding of bandwidth than alternative approaches; we argue this is not an issue for most applications
- Our wall clock times (even WAN) are an **order of magnitude** better than the next best concurrent work

Our Model

- **Universal Composability** [Canetti ’01] (static adv., local RO)
- **Functionality (trusted third party *emulated* by protocol)**:
  - Store secret key
  - Compute ECDSA signature when enough parties ask
- **Assumption**: CDH is hard in the ECDSA curve
- **Network**: Synchronous, broadcast
- Security with abort

Our Approach

- **Setup:** MUL setup, VSS for [sk]
- **Signing:**

1. Get candidate shares [k], [$1/k$], and $R=k\cdot G$
2. Compute [sk/k] = MUL([$1/k$], [sk])
3. Check relations in exponent
4. Reconstruct sig = [$1/k$]$\cdot H(m)$+[sk/k]

Setup

- **Fully distributed**
- **MUL setup**: Pairwise among parties (128 OTs)
- **Key generation**: (Pedersen-style)
  - Every party Shamir-shares a random secret
  - Secret key is sum of parties’ contributions
  - Verify in the exponent that parties’ shares are on the same polynomial

Our Approach

- **Setup**: MUL setup, VSS for [sk]
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  1. Get candidate shares [k], [$1/k$], and $R=k\cdot G$
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  3. Check relations in exponent
  4. Reconstruct *sig* = [$1/k$]$\cdot H(m)$+[sk/k]

Obtaining Candidate Shares

- **Building Block**: Two party MUL with full security [DKLs18]
- **One approach** (implemented):
  - Each party starts with multiplicative shares of *k* and *1*/*k*
  - Multiplicative to additive shares: log(*t*)+c rounds
- **Alternative**: [Bar-Ilan&Beaver '89] approach yields constant round protocol (work in progress)

Our Approach

- **Setup**: MUL setup, VSS for [sk]
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  1. Get candidate shares [k], [$1/k$], and $R=k \cdot G$
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  3. Check relations in exponent
  4. Reconstruct sig = [$1/k$]·$H(m)$+[sk/k]

Our Approach

- **Setup**: MUL setup, VSS for [sk]
- **Signing**:
  1. Get candidate shares [k], [$1/k$], and $R=k\cdot G$
  2. Compute [sk/k] = MUL([$1/k$], [sk]) => Standard GMW
  3. Check relations in exponent
  4. Reconstruct sig = [$1/k$]$\cdot H(m)$+[sk/k]

Our Approach

- **Setup**: MUL setup, VSS for [sk]
- **Signing**:

1. Get candidate shares [k], [$1/k$], and $R=k\cdot G$
2. Compute [sk/k] = MUL([$1/k$], [sk])
3. Check relations in exponent
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Major challenges from 2 to Multi-party

2-party check does not obviously generalize [LNR18]

Can't use Diffie-Hellman Exchange for *R*

Check in Exponent

- There are **three** relations that have to be verified

$$[k] \qquad \left[\frac{1}{k}\right] \qquad \left[\frac{\mathsf{sk}}{k}\right]$$

Check in Exponent

$$[k] \quad \left[\frac{1}{k}\right] \quad \left[\frac{\mathsf{sk}}{k}\right]$$

- **Technique**: Each equation is verified in the exponent, using ‘auxiliary’ information that’s already available
- **Cost**: 5 exponentiations, 5 group elements per party independent of party count, and no ZK proofs

Check in Exponent

- **Task:** verify relationship between [*k*] and [1/*k*]

- **Idea**: verify $\left[\frac{1}{k}\right][k] = 1$ by verifying $\left[\frac{1}{k}\right][k] \cdot G = G$

Check in Exponent

Attempt at a solution:

Public R

Broadcast $\Gamma_i = \left[\frac{1}{k}\right]_i \cdot R$

Verify $\sum_{i \in [n]} \Gamma_i = G$

Check in Exponent

Attempt at a solution:

Adversary's contribution

Honest Party's contribution

Public $$R = k_A k_h \cdot G$$

Broadcast $$\Gamma_i = \left[\frac{1}{k_A}\frac{1}{k_h}\right]_i \cdot R$$

Verify $$\sum_{i \in [n]} \Gamma_i = G$$

Check in Exponent

Attempt at a solution:

Adversary's contribution

Honest Party's contribution

Public $$R = k_A k_h \cdot G$$

Broadcast $$\Gamma_i = \left[\left(\frac{1}{k_\mathrm{A}} + \epsilon\right)\frac{1}{k_h}\right]_i \cdot R$$

Verify $$\sum_{i \in [n]} \Gamma_i = G + \underline{\epsilon k_A \cdot G}$$

Easy for Adv. to offset

Idea: Randomize Target

- Currently we expect $\sum \Gamma_i$ to hit a fixed target *G*
- **Idea**: randomize the multiplication so target is unpredictable
- Compute $\left[\frac{\phi}{k}\right]$ instead of $\left[\frac{1}{k}\right]$
- Reveal ϕ only after *every* other value is committed

Check in Exponent

Attempt at a solution:

Adversary's contribution

Honest Party's contribution

Public

$$R = k_A k_h \cdot G$$

Broadcast

$$\Gamma_i = \left[\frac{1}{k_A}\frac{1}{k_h}\right]_i \cdot R$$

Check in Exponent

Attempt at a solution:

Adversary's contribution

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Public $R = k_A k_h \cdot G$

Broadcast $\Gamma_i = \left[\frac{\phi_A}{k_A} \frac{\phi_h}{k_h} \right]_i \cdot R$

Verify $\sum_{i \in [n]} \Gamma_i = \phi_A \phi_h \cdot G$

Check in Exponent

Attempt at a solution:

Adversary's contribution

Honest Party's contribution

Public $$R = k_A k_h \cdot G$$

Broadcast $$\Gamma_i = \left[\frac{\phi_A}{k_A} \frac{\phi_h}{k_h}\right]_i \cdot R$$

Verify $$\sum_{i \in [n]} \Gamma_i = \Phi$$

Check in Exponent

Attempt at a solution:

Adversary's contribution

Honest Party's contribution

Public $$R = k_A k_h \cdot G$$

Broadcast $$\Gamma_i = \left[\left(\frac{\phi_A}{k_A} + \epsilon \right) \frac{\phi_h}{k_h} \right]_i \cdot R$$

Verify $$\sum_{i \in [n]} \Gamma_i = \Phi + \epsilon \underline{\phi_h} k_A \cdot G$$

Completely unpredictable

Check in Exponent

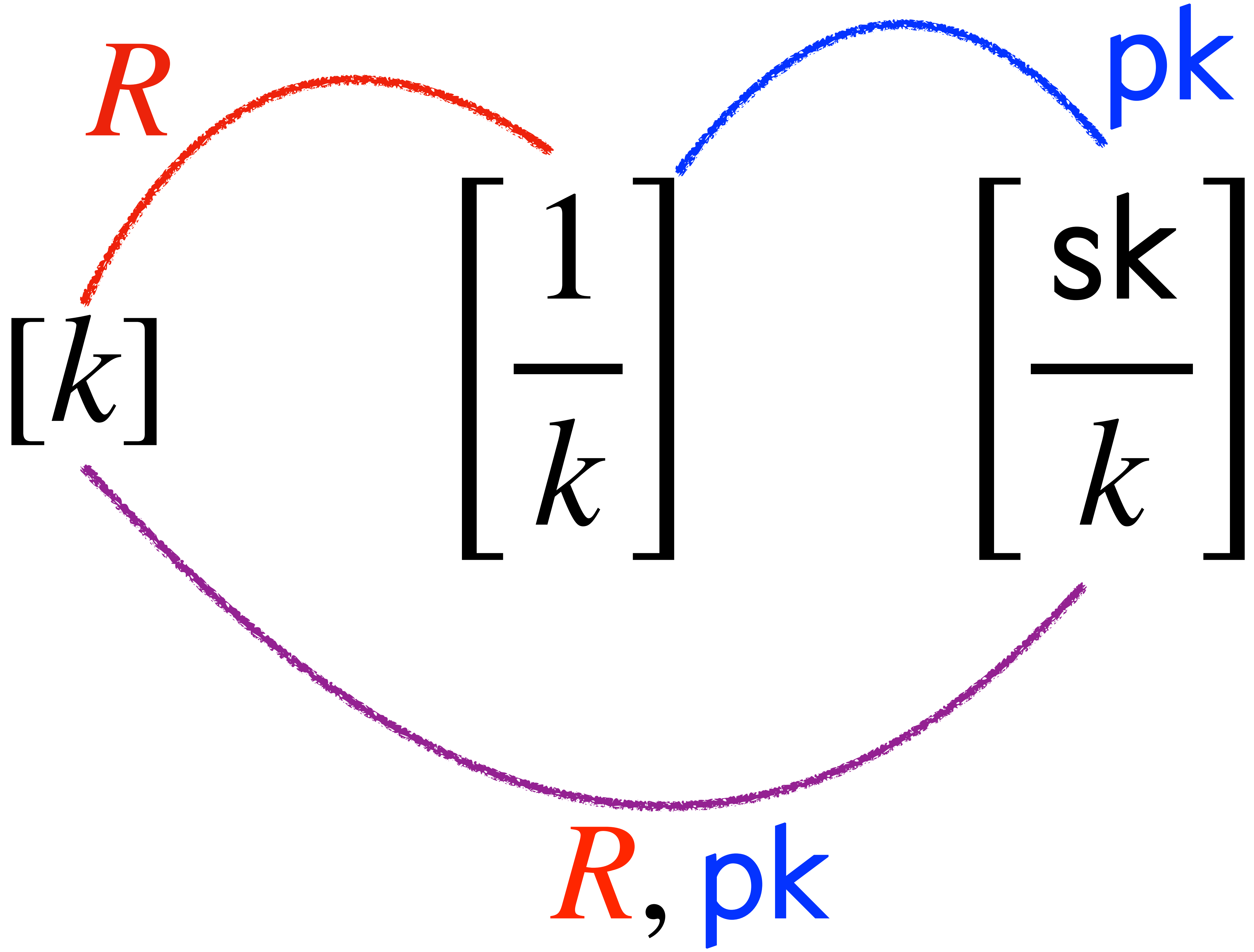
There are **three** relations that have to be verified

Each costs, per party:

-2 exponentiations

-2 field elements

Two broadcast rounds



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Broadcast linear combination of shares

Dominant Costs

	Rounds	Public Key	Bandwidth
Setup	5	520*n*	21*n* KB
Signing	log(*t*)+6	5	<100*t* KB

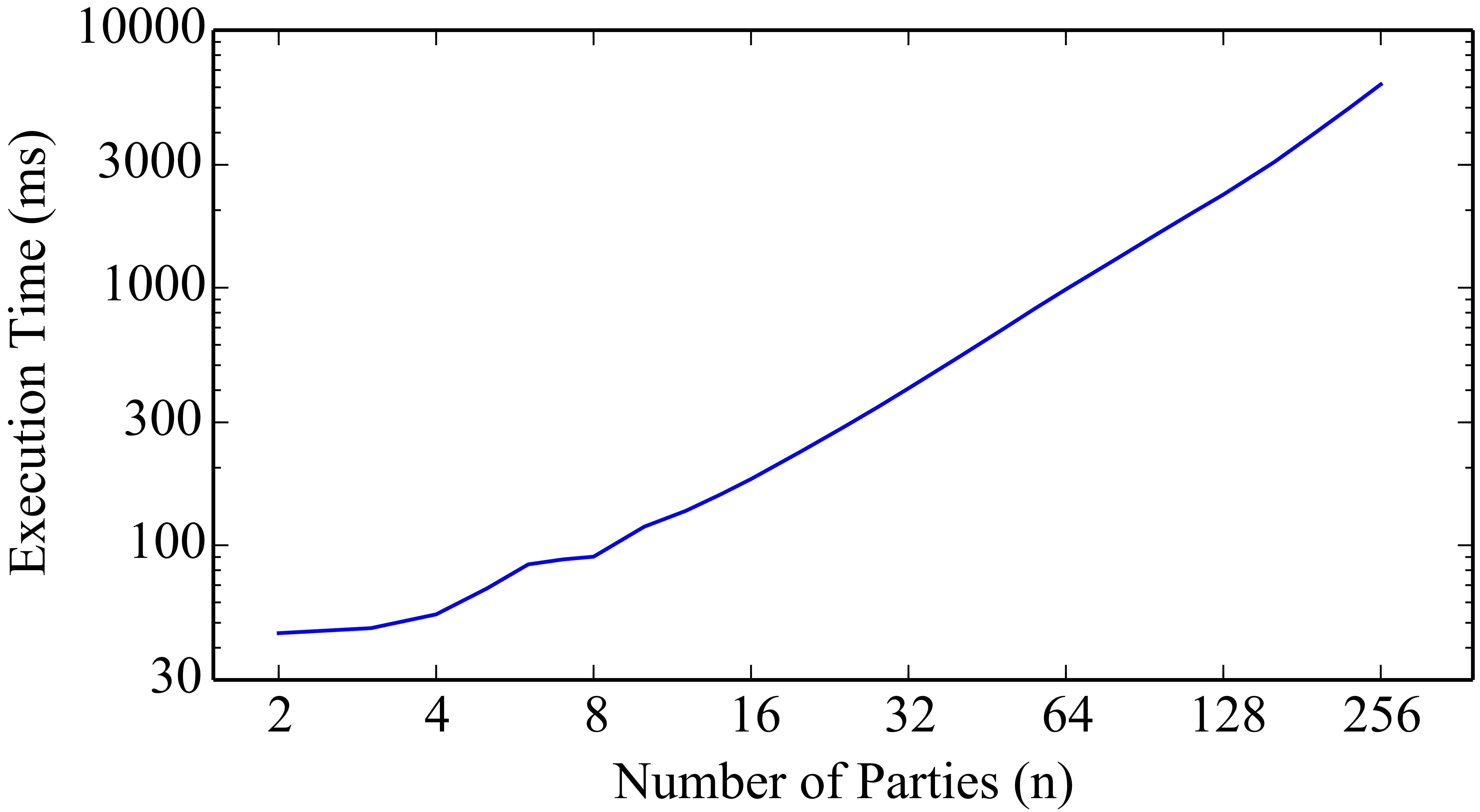
Journal version (in progress): **8 round signing**

(à la [Bar-Ilan Beaver 89])

Benchmarks

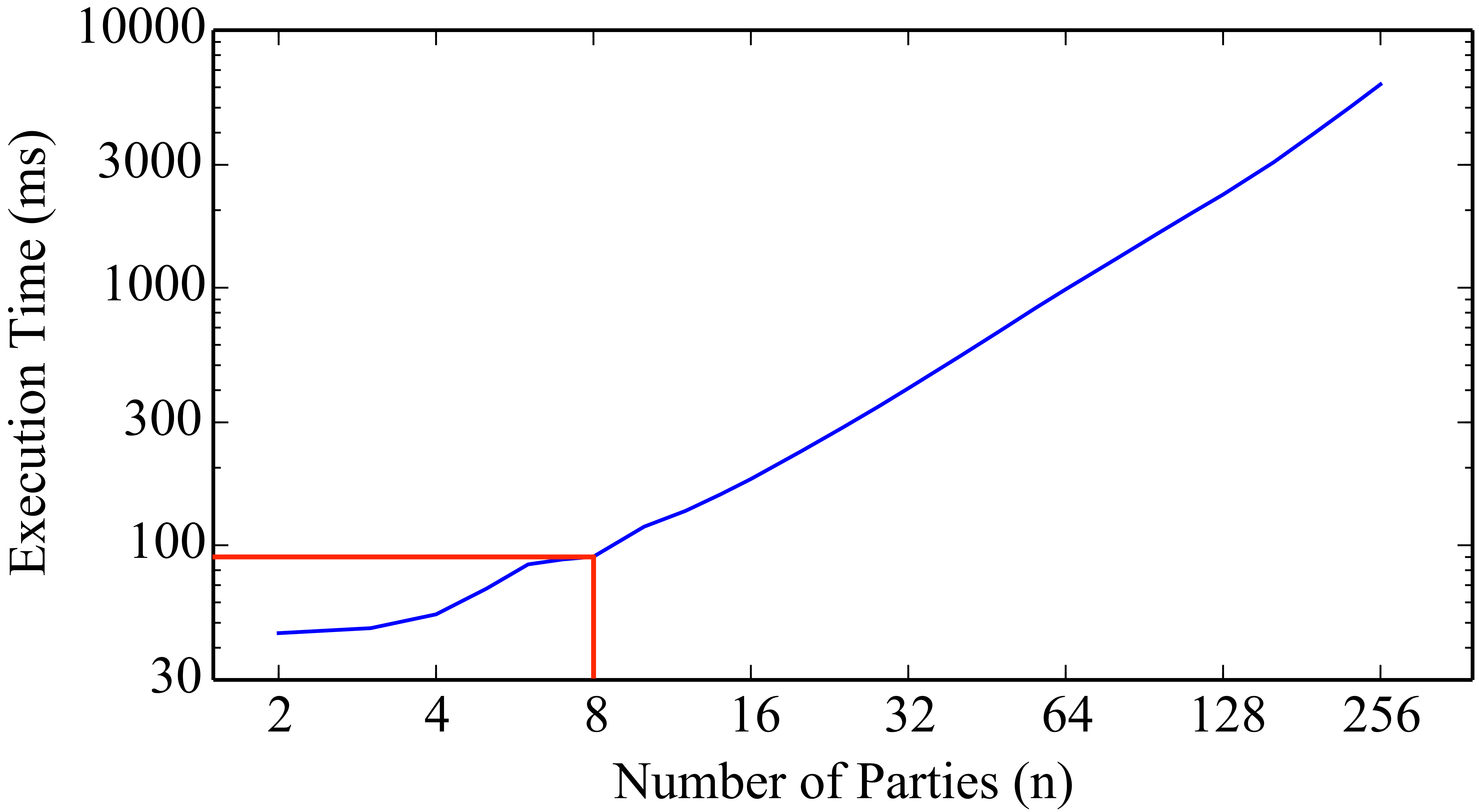
- Implementation in **Rust**
- Ran benchmarks on Google Cloud
- One node per party
- **LAN** and **WAN** tests (up to **16 zones**)
- **Low Power Friendliness**: Raspberry Pi (~93ms for 3-of-3)

LAN Setup



Broadcast PoK (DLog), **Pairwise**: 128 OTs

LAN Setup



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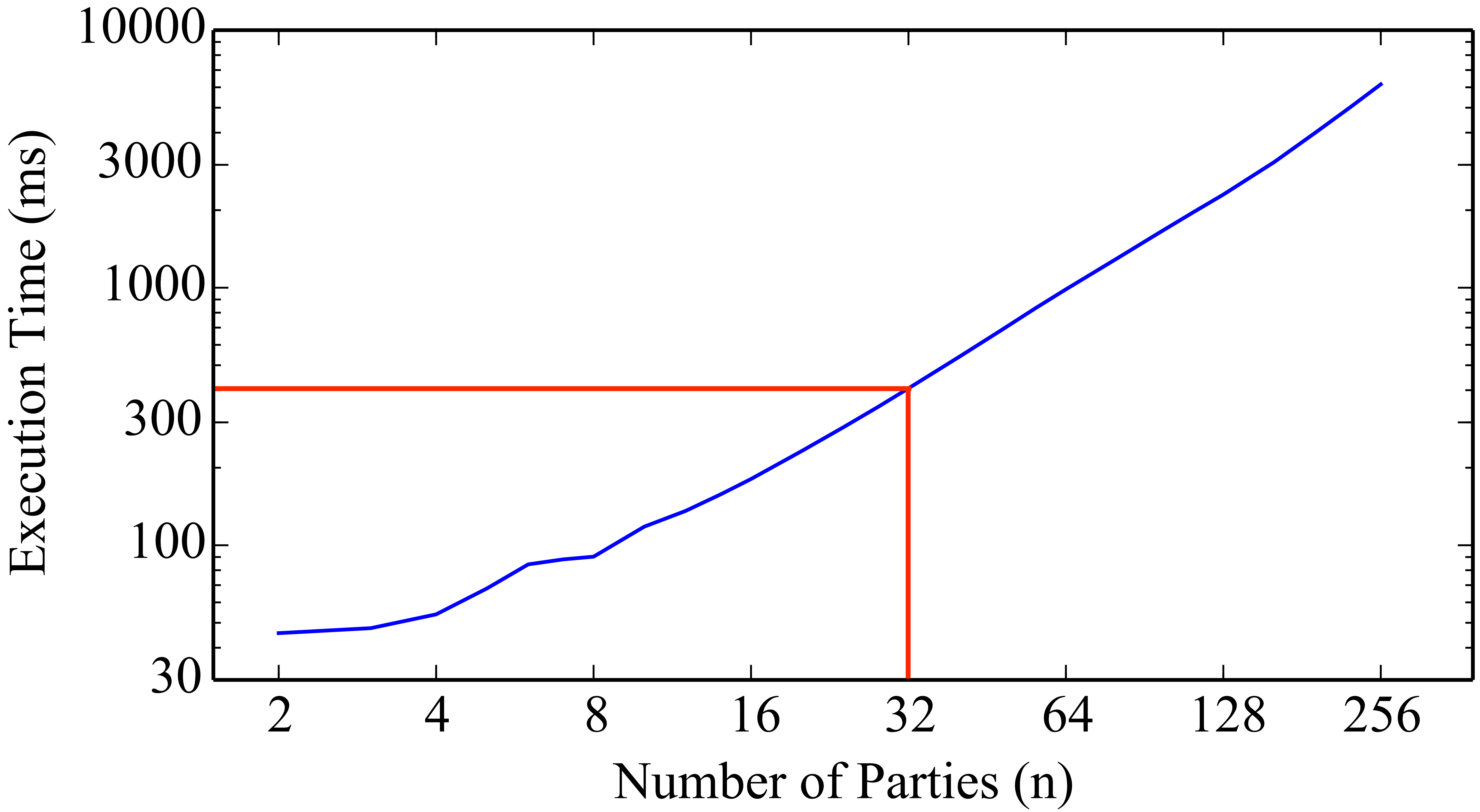

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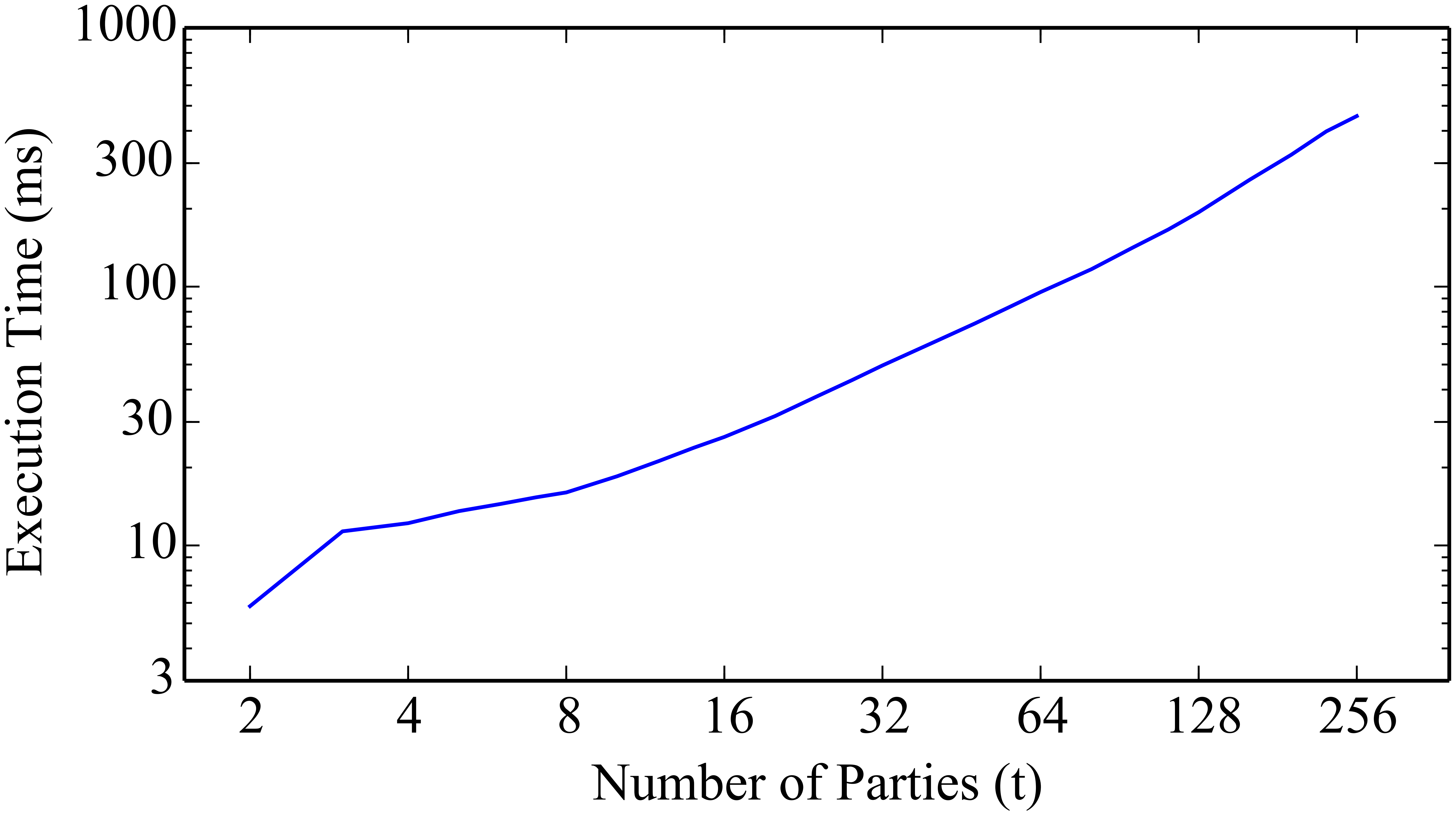
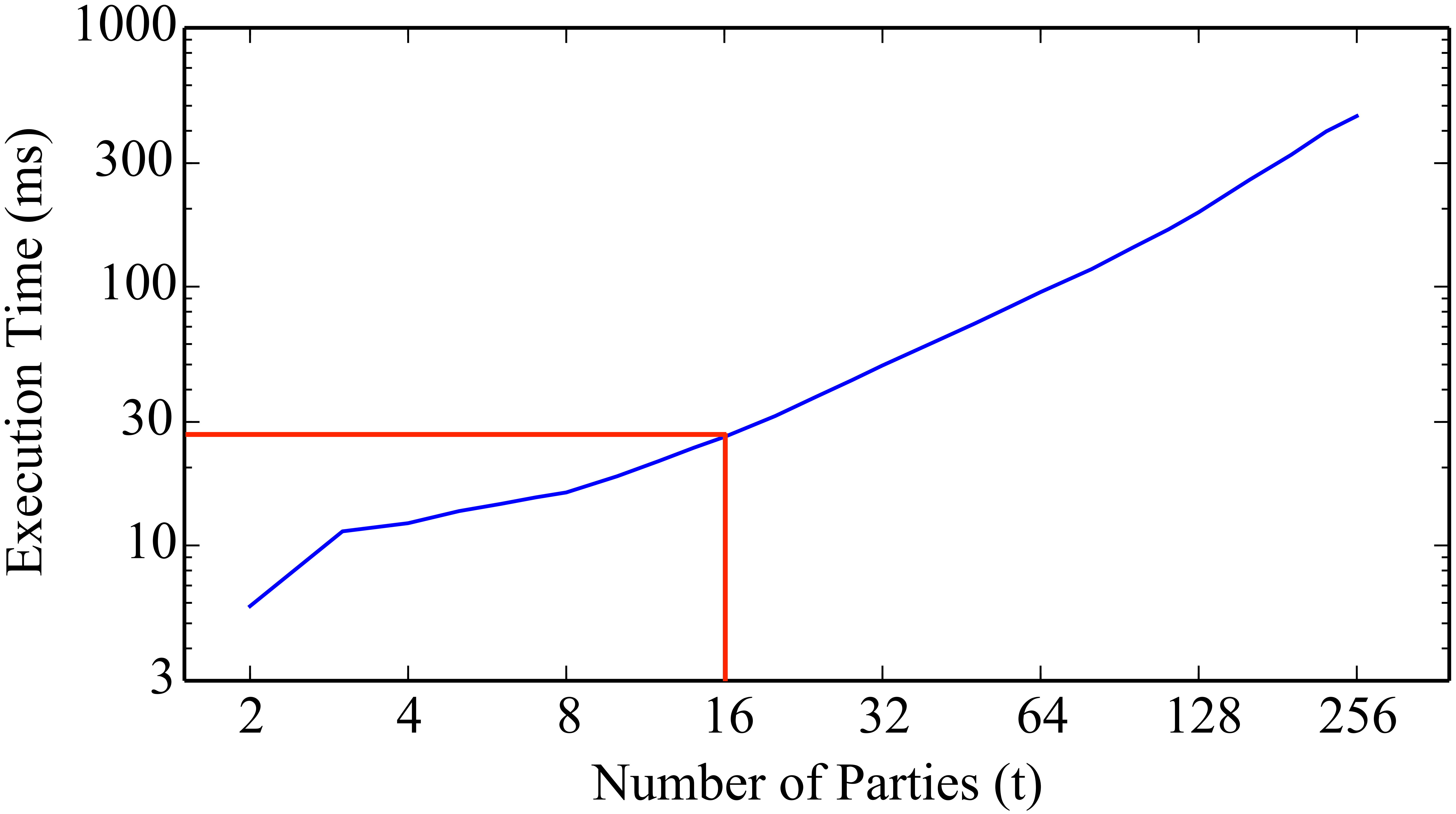
LAN Signing



1000
300
100
30
10
3
Execution Time (ms)
2
4
8
16
32
64
128
256
Number of Parties (t)

LAN Signing



1000
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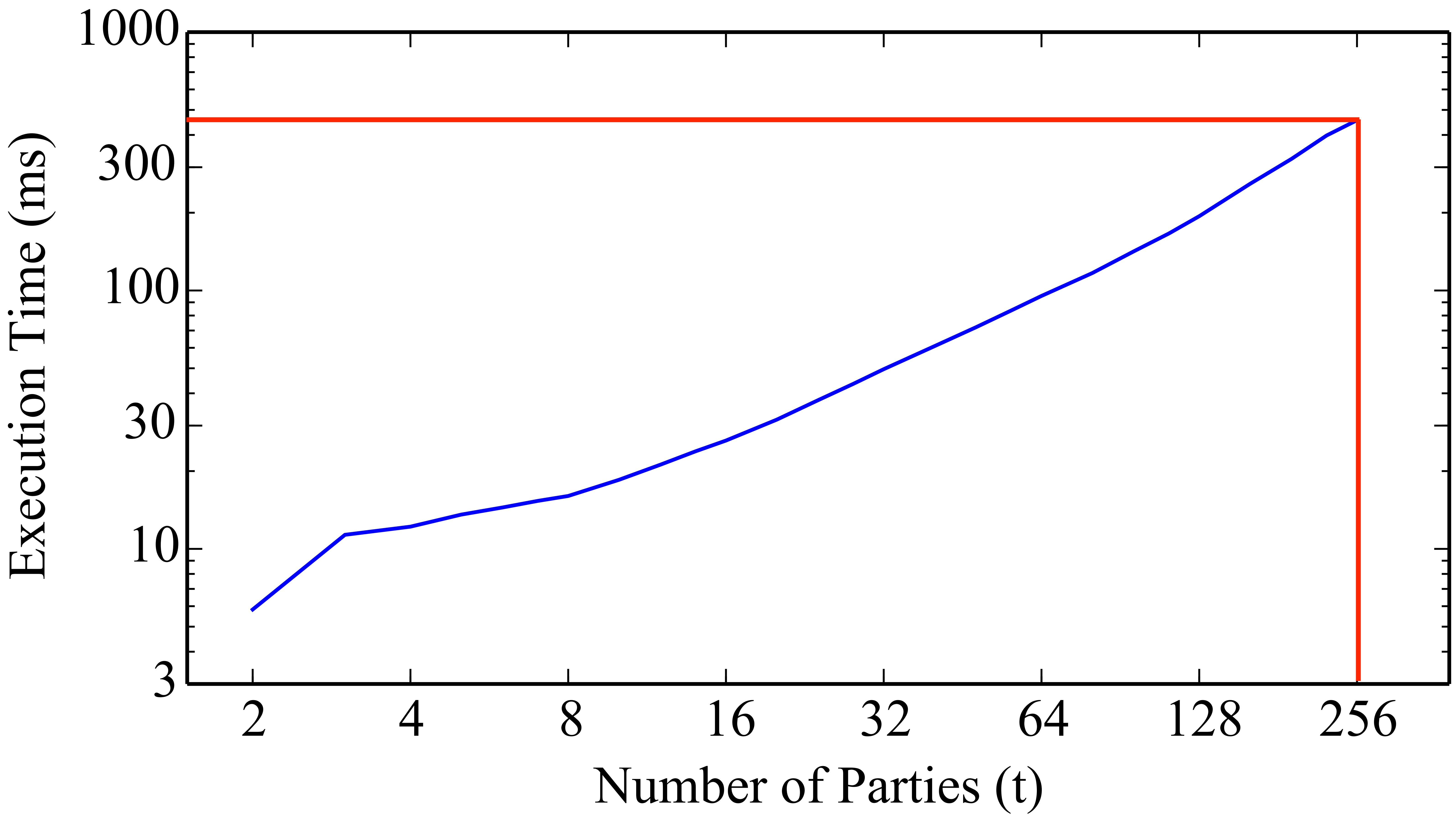
1000
300
100
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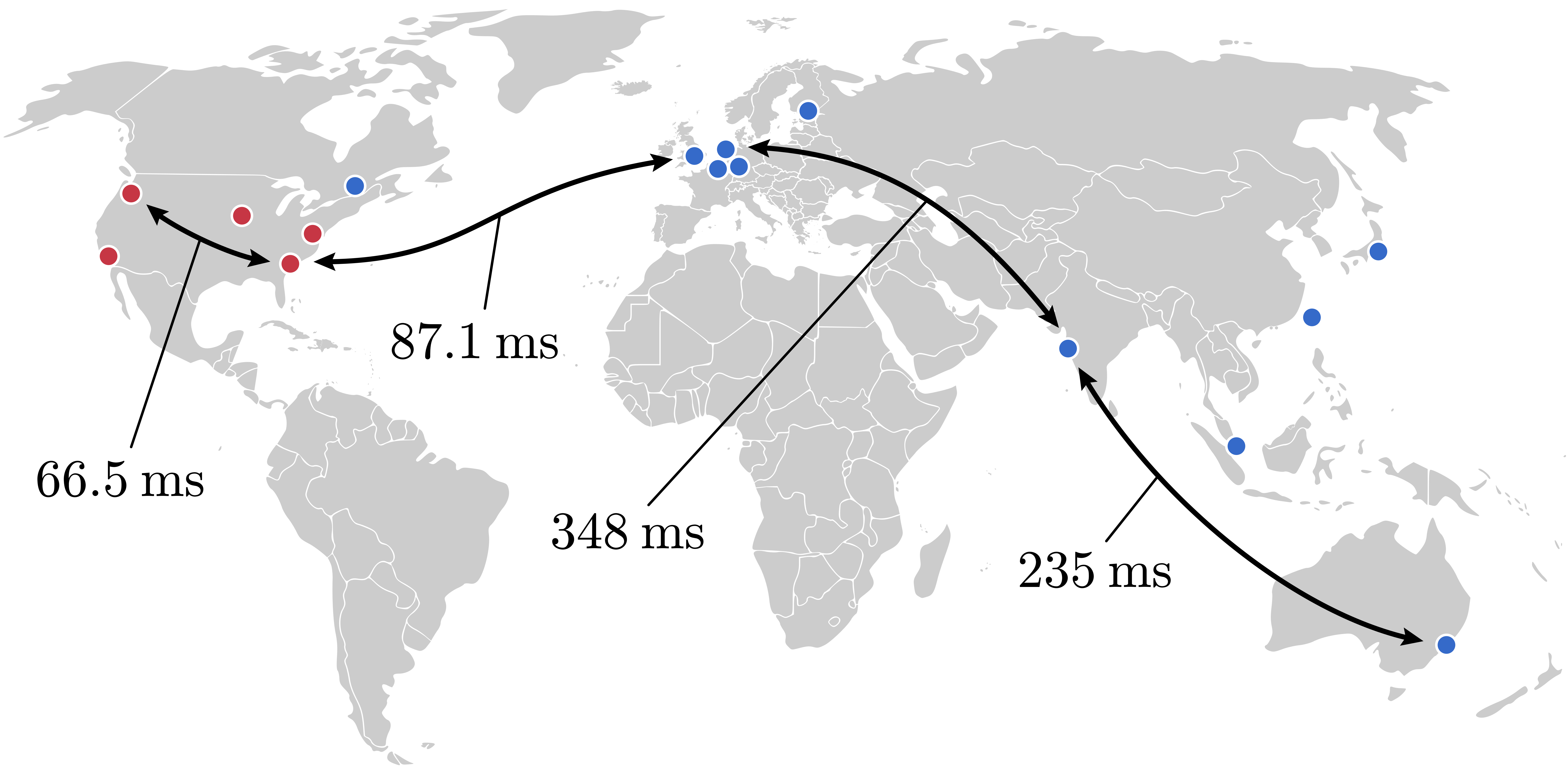
WAN Nodes



WAN Benchmarks

All time values in milliseconds

Parties/Zones	Signing Rounds	Signing Time	Setup Time
5/1	9	13.6	67.9
5/5	9	288	328
16/1	10	26.3	181
16/16	10	3045	1676
40/1	12	60.8	539
40/5	12	592	743
128/1	13	193.2	2300
128/16	13	4118	3424

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Comparison

All time figures in milliseconds

Protocol	Signing		Setup	
	$t = 2$	$t = 20$	$n = 2$	$n = 20$
This Work	**9.5**	**31.6**	**45.6**	**232**
GG18	77	509	–	–
LNR18	304	5194	$\sim$11000	$\sim$28000

Note: Our figures are wall-clock times; includes network costs

Comparison

All time figures in milliseconds

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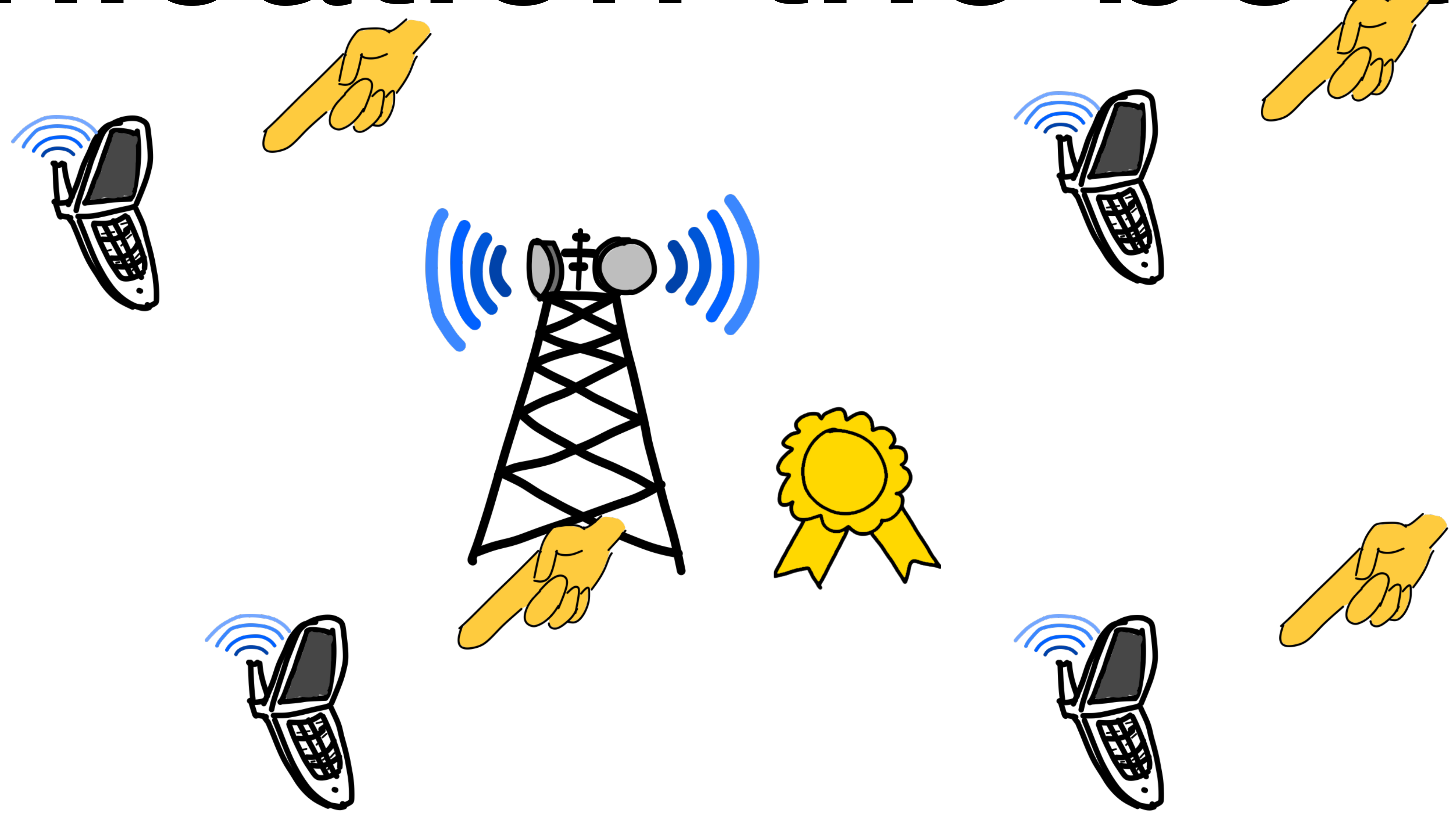
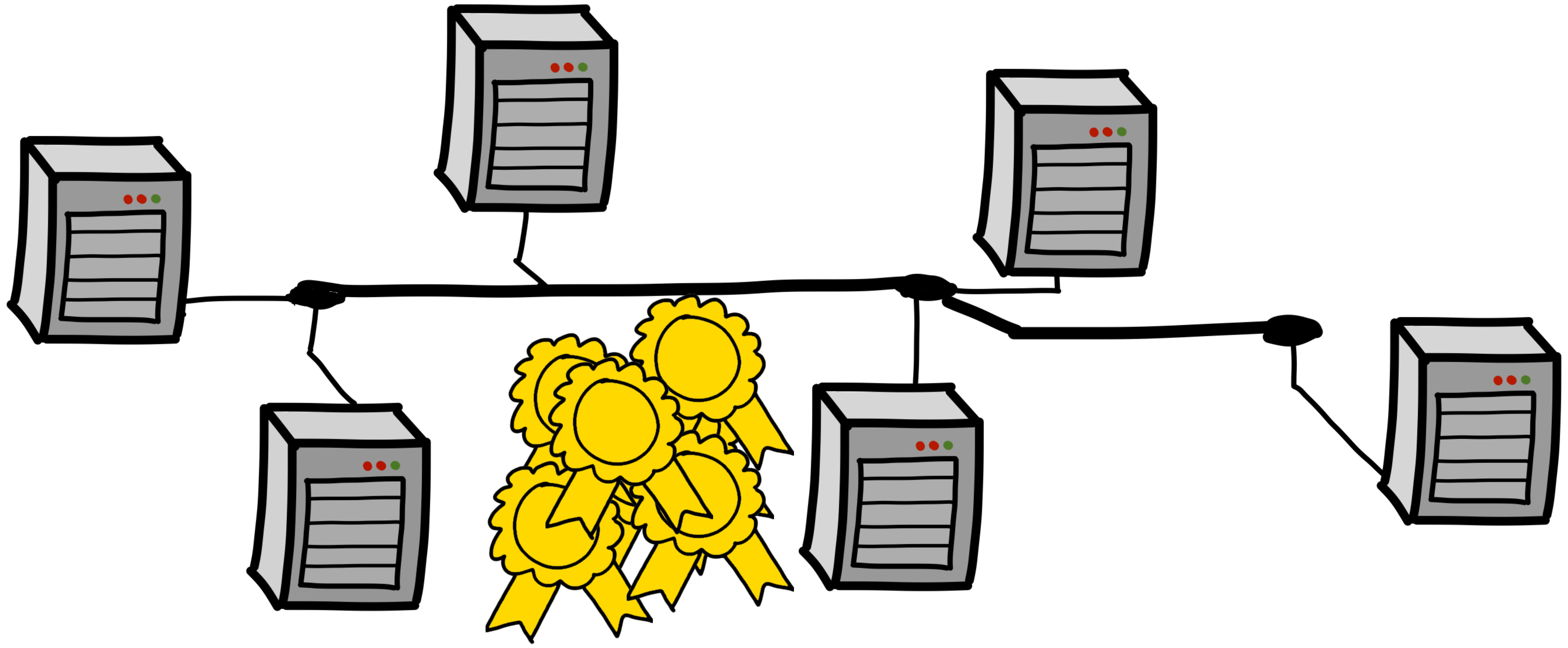
Is communication the bottleneck?



- **Mobile applications (human-initiated)**:
  - eg. t=4, <4Mb transmitted per party
  - Well within LTE envelope for responsivity

Is communication the bottleneck?



- **Large-scale automated distributed signing:**
  - Threshold 2: 3.8ms/sig <= ~263 sig/second
  - Threshold 20: 31.6ms/sig <= ~31 sig/second
- Both settings need <500Mb bandwidth

Conclusion

- **Efficient full-threshold ECDSA with fully distributed keygen**
- **Paradigm**: ‘produce candidate shares, verify by exponent check’ costs **5 exponentiations** (+ many hashes) to sign, **no ZK** online
- **Instantiation**: Cryptographic assumptions native to ECDSA itself (**CDH** in the same curve)
- Lightweight computation **but communication well within practical range (<100*t* KB/party)**
- **Wall-clock times**: Practical in realistic scenarios

Thank you!

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