On the Security of Two-Round Multi-Signatures

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Multi-signatures

$(pk_1,sk_1) \leftarrow Kg$ $(pk_2,sk_2) \leftarrow Kg$ $(pk_3,sk_3) \leftarrow Kg$

$Sign((pk_1,pk_2,pk_3), sk_1, m) \leftrightarrow Sign((pk_1,pk_2,pk_3), sk_2, m) \leftrightarrow Sign((pk_1,pk_2,pk_3), sk_3, m)$

$\rightarrow \sigma$ $\rightarrow \sigma$ $\rightarrow \sigma$

$$Verify((pk_1,pk_2,pk_3), m, \sigma) = 1$$

Every signer must agree to sign m

Goal: short signature (preferably ≈ single signature,

efficiently verifiable definitely << N signatures)

Multi-signatures

$(pk_1, sk_1) \leftarrow Kg$ $(pk_2, sk_2) \leftarrow Kg$ $(pk_3, sk_3) \leftarrow Kg$

$\text{Sign}((pk_1, pk_2, pk_3), sk_1, m) \leftrightarrow \text{Sign}((pk_1, pk_2, pk_3), sk_2, m) \leftrightarrow \text{Sign}((pk_1, pk_2, pk_3), sk_3, m)$

$\rightarrow \sigma$ $\rightarrow \sigma$ $\rightarrow \sigma$

Key aggregation: $apk \leftarrow \text{KAgg}(pk_1, pk_2, pk_3)$

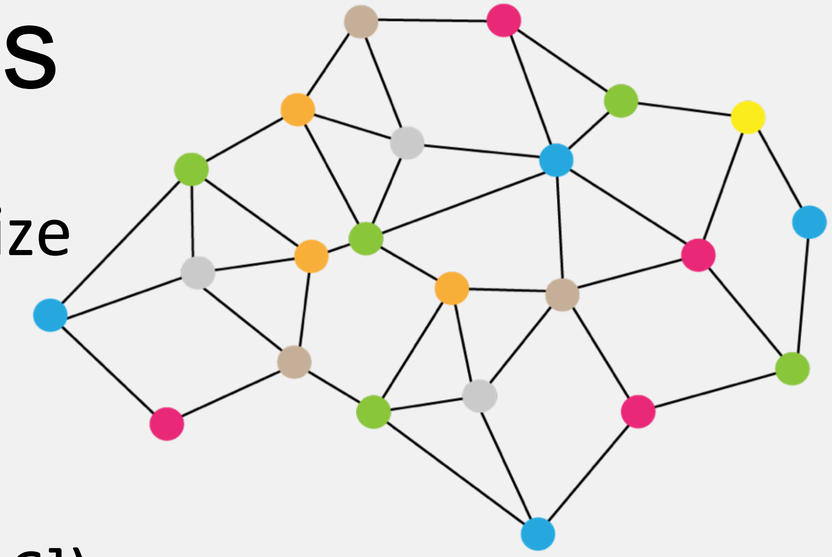
$\text{Verify}(apk, m, \sigma) = 1$

Every signer must agree to sign m

Goal: short signature (preferably ≈ single signature,

efficiently verifiable definitely << N signatures)

Applications of multi-signatures



- Improve Bitcoin throughput / reduce blockchain size
  - ”multisig” transactions as small as other transactions
  - Reduce size of multi-input multi-output transactions
- Collective signing by co-thorities (e.g., CoSi [STV+16])
- Distributed random beacons (e.g., RandHound [SJK+17])
- Block certification in proof-of-stake / permissioned blockchains
  - e.g., Dfinity, OmniLedger, Ziliqa, Harmony, Algorand, …

Existing multi-signatures

Schnorr signatures

$pk = g^{sk}$

$r \leftarrow_R Z_q$

$t \leftarrow g^r$

$c \leftarrow H(t,m)$

$s \leftarrow r + c \cdot sk \bmod q$

$\sigma \leftarrow (c, s)$

Verification:

$c = H(g^s \cdot pk^{-c}, m)$

Efficient & Provably secure

- under discrete-log assumption
- in the random-oracle model: model hash function as ideal random function

"Plain" Schnorr multi-signatures

$pk_1 = g^{sk1}$

$r_1 \leftarrow_R Z_q$
$t_1 \leftarrow g^{r1}$

$t \leftarrow t_1 \cdot t_2 \cdot t_3$
$c \leftarrow H(t,m)$
$s_1 \leftarrow r_1 + c \cdot sk_1 \bmod q$

$s \leftarrow s_1 + s_2 + s_3 \bmod q$
$\sigma \leftarrow (c, s)$

$\leftrightarrow$

$pk_2 = g^{sk2}$

$r_2 \leftarrow_R Z_q$
$t_2 \leftarrow g^{r2}$

$t \leftarrow t_1 \cdot t_2 \cdot t_3$
$c \leftarrow H(t,m)$
$s_2 \leftarrow r_2 + c \cdot sk_2 \bmod q$

$s \leftarrow s_1 + s_2 + s_3 \bmod q$
$\sigma \leftarrow (c, s)$

$\leftrightarrow$

$pk_3 = g^{sk3}$

$r_3 \leftarrow_R Z_q$
$t_3 \leftarrow g^{r3}$

$t \leftarrow t_1 \cdot t_2 \cdot t_3$
$c \leftarrow H(t,m)$
$s_3 \leftarrow r_3 + c \cdot sk_3 \bmod q$

$s \leftarrow s_1 + s_2 + s_3 \bmod q$
$\sigma \leftarrow (c, s)$

$apk \leftarrow pk_1 \cdot pk_2 \cdot pk_3$

Check $c = H(g^s \cdot apk^{-c}, m)$

Problem 1: Rogue-key attacks

$pk_1 = g^{sk1}$

$pk_2 = g^{sk2} / pk_1$

$apk = pk_1 \cdot pk_2 = g^{sk2}$

can compute signatures under apk by himself!

Known remedies:

- Per-signer challenges [BN06]
- Proofs of possession added to pk [RY07,BCJ08]
- MuSig key aggregation: $apk \leftarrow \Pi\, pk_i^{H(pki,\, \{pk1,\ldots,pkN\})}$ [MPSW18]

Problem 2: Signature simulation

pk_1 pk_2

$c, s_1 \leftarrow_R Z_q$

$t_1 \leftarrow g^{s1}\, pk_1^{-c}$ $\longrightarrow t_1$

$t \leftarrow t_1 \cdot t_2$ $\longleftarrow t_2$

$c \leftarrow H(t,m)$

Standard Schnorr proof technique does not work

(cannot program random oracle,
because adversary knows t before simulator does)

Multi-signatures from discrete logarithms

Scheme	Rounds	Rogue keys	Signature simulation
BN [BN06]	3	per-signer challenges	preliminary round $H(t_i)$
~~BCJ-1 [BCJ08]~~	~~2~~	~~per-signer challenges~~	~~homomorphic equivocable (HE) commitments~~
~~BCJ-2 [BCJ08]~~	~~2~~	~~proofs of possession~~	
~~MWLD [MWLD10]~~	~~2~~	~~per-signer challenges~~	~~witness-indinstinguishable keys~~
~~CoSi [STV+16]~~	~~2~~	~~proofs of possession~~	~~(no security proof)~~
~~MuSig-1 [MPSW18a]~~	~~2~~	~~MuSig key aggregation~~	~~DL oracle in one-more DL assumption~~
mBCJ [this work]	2	proofs of possession	per-message HE commitments
BDN-DL, MuSig-2 [BDN18, MPSW19]	3	MuSig key aggregation	preliminary round $H(t_i)$
BDN-DLpop [BDN18]	3	proofs of possession	preliminary round $H(t_i)$
BLS [Bol03,RY07]	1	proofs of possession	pairings
BDN-P [BDN18]	1	MuSig key aggregation	pairings

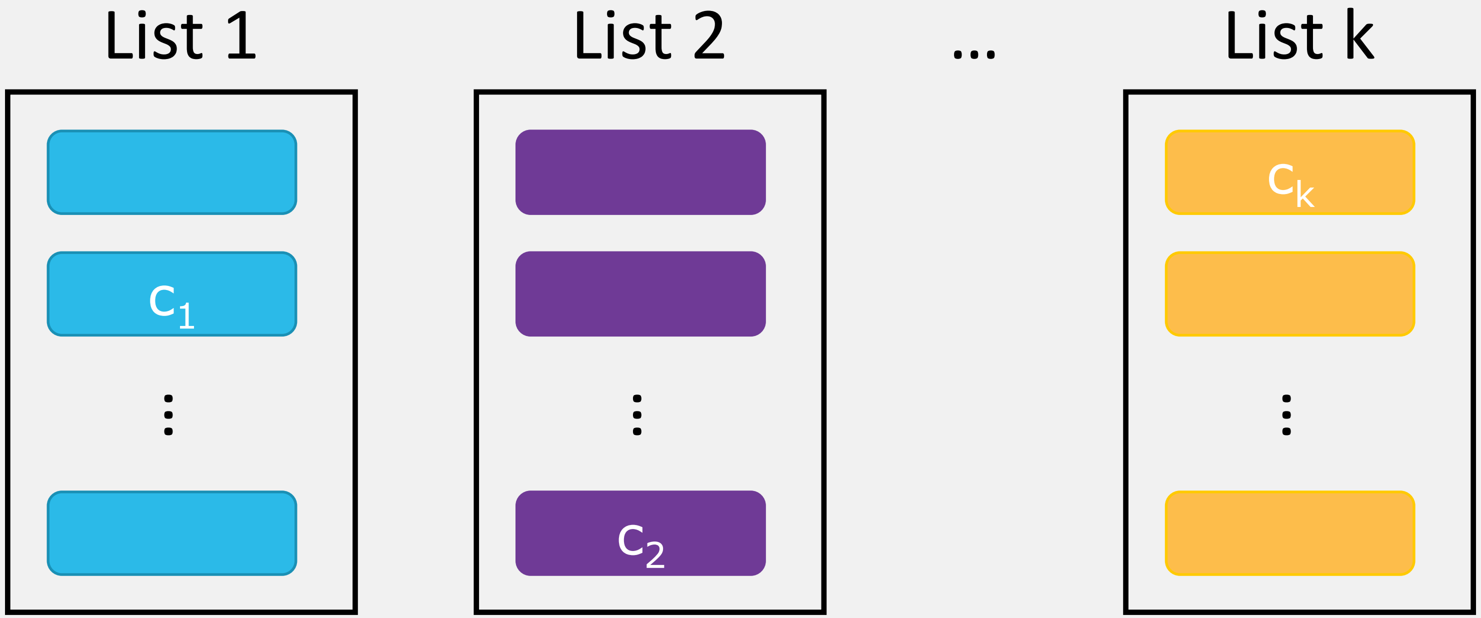
Attacks and non-provability

Wagner's generalized birthday attack [W02]

k-sum problem in Z_q:

Given k lists of random elements in Z_q

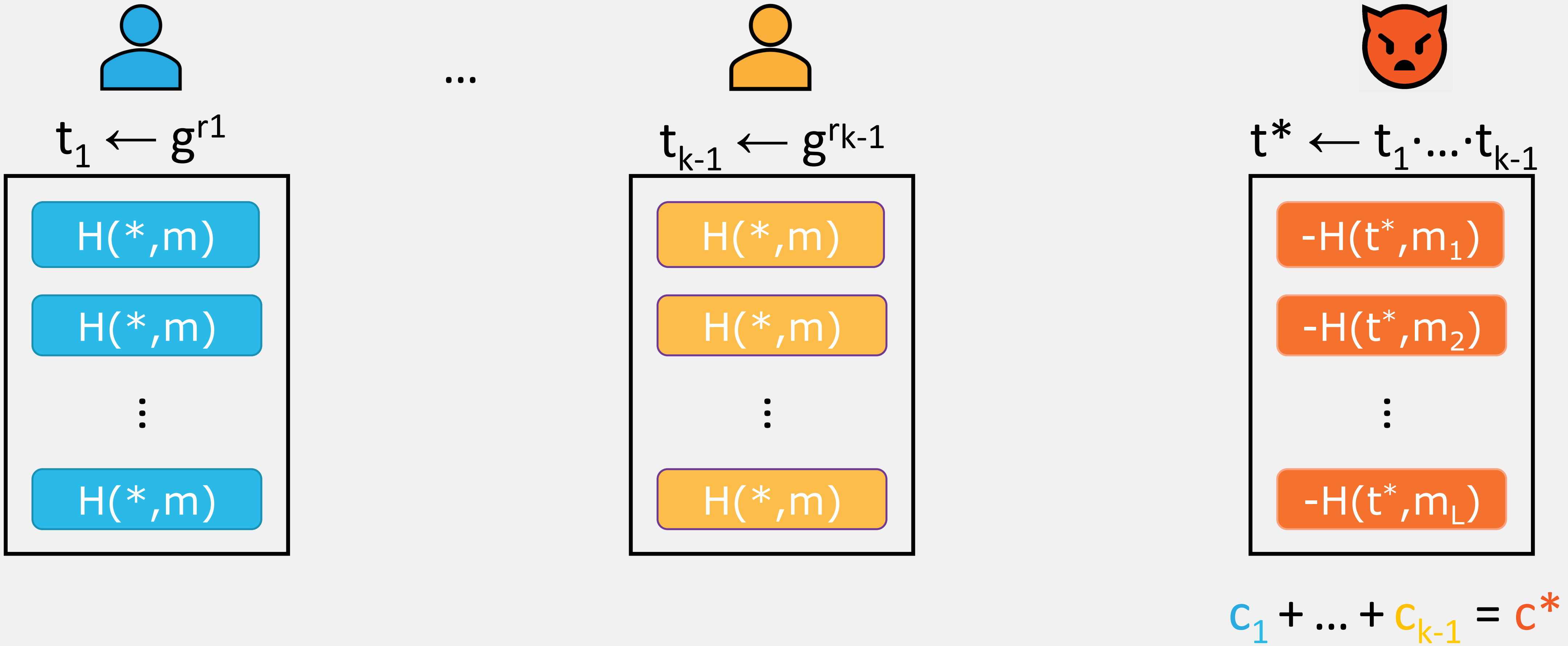
Find $(c_1,\dots,c_k)$ in lists such that $c_1 + \dots + c_k = 0 \bmod q$



Subexponential solution: Solved for $k = 2^{\sqrt{n}}$ in time $O(2^{2\sqrt{n}})$ where $n = |q|$.

Application to “plain” Schnorr and CoSi

- sk only appears in signature in $s = r + c * sk$, with $c = H(g^r, m)$
- If we have signatures with $c_1 + \ldots + c_{k-1} = H(t^*, m)$, we can forge a signature on m^*!



Attacks on two-round multi-signature schemes

- Attack applies to all previously* known two-round schemes
  - BCJ-1 and BCJ-2
  - MWLD
  - CoSi
  - MuSig-1
- Sub-exponential but practical

  (for 256-bit q)
  - 15 parallel signing queries: 2^{62} steps
  - 127 parallel signing queries: 2^{45} steps
- Prevented by increasing |q|

  …any hope for provable (asymptotic) security?

* before first version of this paper

Non-provability of two-round schemes

Theorem: One-more discrete logarithm problem is hard

$\Downarrow$

BCJ/MWLD/CoSi/MuSig-1 cannot be proved secure
under one-more discrete logarithm

(through algebraic black-box reductions in random-oracle model)

Essentially excludes all known proof techniques (including rewinding)
under likely assumptions.

Subtle flaws in proofs of BCJ/MWLD/MuSig-1
(CoSi was never proved secure)

Secure schemes

Modified BCJ multi-signature

- 2 round, secure under discrete logarithm, same efficiency as BCJ
- Large scale deployment:
    - 16,384 signers generate signature within 2 seconds
    - 20% bandwidth, 75% computation increase compared to CoSi (plain schnorr)

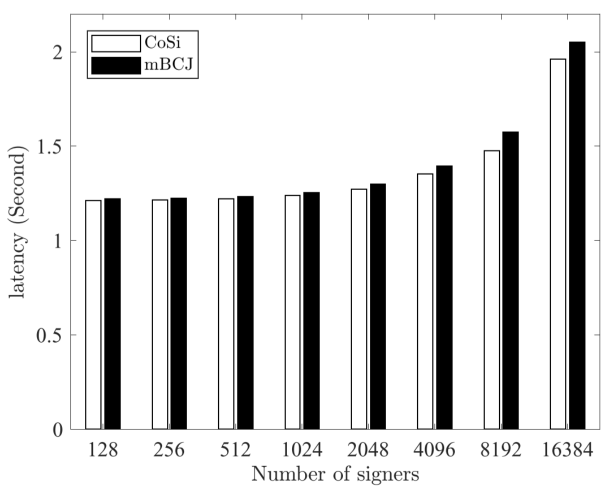


Fig. 4. Comparing end-to-end latency of CoSi and mBCJ signing with varying amounts of signers.

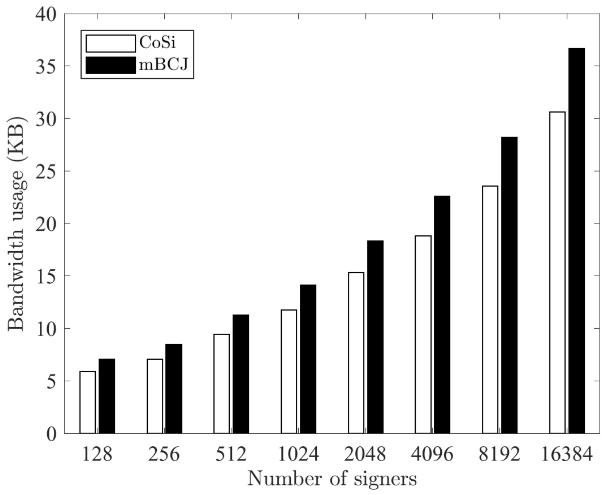


Fig. 5. Bandwidth consumption (sent and received combined) of CoSi and mBCJ with varying amounts of signers.

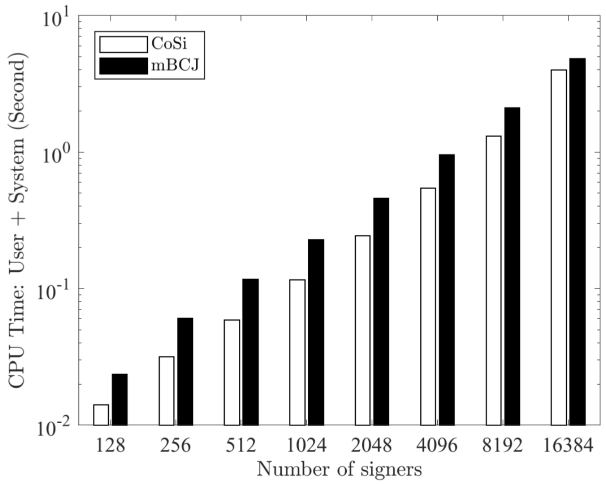


Fig. 6. CPU time (User + System) of CoSi and mBCJ with varying amounts of signers.

Other secure schemes

- Three-round scheme [BDN18, MPSW19]
  - Secure under discrete-log assumption

- Non-interactive scheme from BLS [BLS01,Bol03,RY07,BDN18]
  - Smaller signatures
  - Non-interactive aggregation
  - Requires bilinear pairings

Lessons learned

Lessons learned

- Cryptographic schemes need security proofs
  - Don’t drop steps that look like they’re “just to make the proof work”
- Security proofs must be reviewed
  - Proofs can be subtle, especially with rewinding arguments
  - Tool support for checking proofs?
- *Provable security is not perfect, but best tool we have*

Thank you!

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References

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[BCJ08] Bagherzandi, Cheon, Jarecki: Multisignatures secure under the discrete logarithm assumption and a generalized forking lemma. CCS 2008

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[STV+16] Syta et al.: Keeping Authorities "Honest or Bust" with Decentralized Witness Cosigning. IEEE S&P 2016

[MPSW18a] Maxwell, Poelstra, Soerin, Wuille: Simple Schnorr Multi-Signatures with Applications to Bitcoin. ePrint report /2018/068/20180118:124757

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[BDN18] Boneh, Drijvers, Neven: Compact Multi-signatures for Smaller Blockchains. ASIACRYPT 2018

[RY07] Ristenpart, Yilek: The Power of Proofs-of-Possession: Securing Multiparty Signatures against Rogue-Key Attacks. EUROCRYPT 2007

Modified BCJ multi-signatures

$pk_i = g^{sk_i}$ + PoP

$(g_2,h_1,h_2) \leftarrow H'(m)$

$r,\alpha_1,\alpha_2 \leftarrow_R Z_q$

$t_{i,1} \leftarrow g_1^{\alpha 1} h_1^{\alpha 2}$

$t_{i,2} \leftarrow g_2^{\alpha 1} h_2^{\alpha 2} g_1^{r}$

$t_{i,1}, t_{i,2}$

$t_1 \leftarrow \Pi t_{i,1}$; $t_2 \leftarrow \Pi t_{i,2}$

$c \leftarrow H(t_1,t_2,\Pi pk_i,m)$

$s_i \leftarrow r + c \cdot sk_i + \Sigma s_i \bmod q$

$s_i, \alpha_{i,1}, \alpha_{i,2}$

$s \leftarrow \Sigma s_i \bmod q$

$\alpha_1 \leftarrow \Sigma \alpha_{i,1} \bmod q$

$\alpha_2 \leftarrow \Sigma \alpha_{i,2} \bmod q$

$\sigma \leftarrow (t_1,t_2,s,\alpha_1,\alpha_2)$

KAgg: Check PoPs, $apk \leftarrow \Pi pk_i$

Verify: $c \leftarrow H(t_1,t_2,apk,m)$

Check $t_1 = g_1^{\alpha 1} h_1^{\alpha 2}$

and $t_2 = g_2^{\alpha 1} h_2^{\alpha 2} g_1^{s} apk^{-c}$

Efficiency

Sign: 1 $mexp^2$ + 1 $mexp^3$

plain Schnorr: 1 exp

Verify: 3 $mexp^2$

plain Schnorr: 1 $mexp^2$

Signature size: 160 B

plain Schnorr: 64 B

Application to "plain" Schnorr and CoSi

Query on m_1

$r_1 \leftarrow_R Z_q$

$t_1 \leftarrow g^{r1}$

$c_1 \leftarrow H(t_1, m_1)$

$s_1 \leftarrow r_1 + c_1 \cdot sk$

$\sigma_1 \leftarrow (c_1, s_1)$

Query on m_2

$r_2 \leftarrow_R Z_q$

$t_2 \leftarrow g^{r2}$

$c_2 \leftarrow H(t_2, m_2)$

$s_2 \leftarrow r_2 + c_2 \cdot sk$

$\sigma_2 \leftarrow (c_2, s_2)$

Forgery on m_3

$t_3 \leftarrow t_1 \cdot t_2$

$c_3 \leftarrow H(t_3, m_3)$ such that $c_3 = c_1 + c_2$

$s_3 \leftarrow s_1 + s_2$

$\sigma_3 \leftarrow (c_3, s_3)$

Lessons learned

- Provable security! 🤔
- Review security proofs! 🤔

- Proofs can be subtle, especially forking
- Tool support for checking proofs?
- Don’t drop steps that look like they’re “just to make the proof work”

- Provable security is not perfect, but best tool we have

Application to "plain" Schnorr and CoSi

…

$t_1 \leftarrow g^{r1}$

H(*,m)

H(*,m)

⋮

H(*,m)

$s_1 \leftarrow r_1 + c_1 \cdot sk^* \bmod q$

$t_{k-1} \leftarrow g^{r_{k-1}}$

H(*,m)

H(*,m)

⋮

H(*,m)

$s_{k-1} \leftarrow r_{k-1} + c_{k-1} \cdot sk^* \bmod q$

$t^* \leftarrow t_1 \cdot \ldots \cdot t_{k-1}$

$-H(t^*, m_1)$

$-H(t^*, m_2)$

⋮

$-H(t^*, m_L)$

$c_1 + \ldots + c_{k-1} = c^* \bmod q$

$s^* \leftarrow s_1 + \ldots + s_{k-1} \bmod q$

$pk^* = g^{sk^*}$

$g^{s^*} = g^{\Sigma si} = g^{\Sigma ri + \Sigma ci \cdot sk^*} = \Pi t_i \cdot pk^{*c^*} = t \cdot pk^{*c^*}$

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