

DATA RECOVERY ON ENCRYPTED DATABASES WITH k-NEAREST NEIGHBOR QUERY LEAKAGE

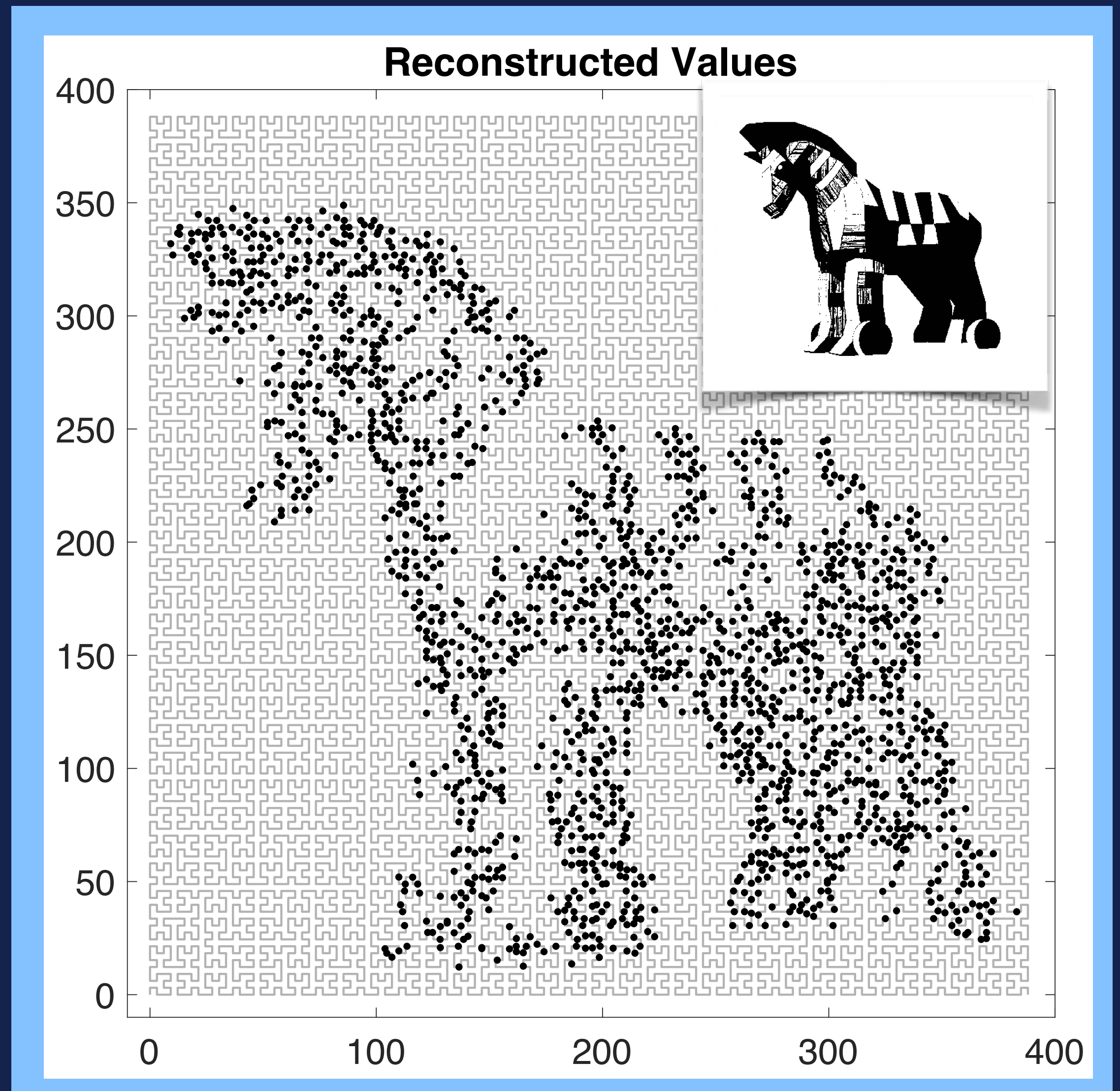
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CHARALAMPOS PAPAMANTHOU
ROBERTO TAMASSIA



BROWN



UNIVERSITY OF
MARYLAND





INTRO

WHO CARES ABOUT k-NN?

COLUMN-ORIENTED DBMS

18.2. Processors

18.2.1. ArrowConversionProcess

18.2.2. BinConversionProcess

18.2.3. DensityProcess

18.2.4. DateOffsetProcess

18.2.5. HashAttributeProcess

18.2.6. HashAttributeColorProcess

18.2.7. JoinProcess

18.2.8. KNearestNeighborProcess

18.2.9. Point2PointProcess

18.2.10. ProximitySearchProcess

18.2.11. RouteSearchProcess

18.2.12. SamplingProcess

18.2.13. StatsProcess

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18.2.15. TubeSelectProcess

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18.2.17. UniqueProcess

18.2.18. Chaining Processes

19. GeoMesa GeoJSON

GeoMesa Documentation

18.2.8. KNearestNeighborProcess

The `KNearestNeighborProcess` performs a K Nearest Neighbor search on a Geomesa feature collection using another feature collection as input. Return k neighbors for each point in the input data set. If a point is the nearest neighbor of multiple points of the input data set, it is returned only once.

Parameters	Description
inputFeatures	Input feature collection that defines the KNN search.
dataFeatures	The data set to query for matching features.
numDesired	K : number of nearest neighbors to return.
estimatedDistance	Estimate of Search Distance in meters for K neighbors—used to set the granularity of the search.
maxSearchDistance	Maximum search distance in meters—used to prevent runaway queries of the entire table.

18.2.8.1. K-Nearest-Neighbor example (XML)

`KNNProcess_wps.xml` is a geoserver WPS call to the GeoMesa `KNearestNeighborProcess`. It is here chained with a Query process (see **Chaining Processes**) in order to avoid points related to the same Id to be matched by the request. It can be run with the following curl call:

```
curl -v -u admin:geoserver -H "Content-Type: text/xml" -d@KNNProcess_wps.xml localhost:8080/geoserver/wps
```




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Process

as a K Nearest Neighbor search on a Geomesa feature collection

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27.2. Index-based KNN

“KNN” stands for “K nearest neighbours”, where “K” is the number of neighbours you are looking for.

KNN is a pure index based nearest neighbour search. By walking up and down the index, the search can find the nearest candidate geometries without using any magical search radius numbers, so the technique is suitable and high performance even for very large tables with highly variable data densities.

Note

The KNN feature is only available on PostGIS 2.0 with PostgreSQL 9.1 or greater.

The KNN system works by evaluating distances between bounding boxes inside the PostGIS R-Tree index.

Because the index is built using the bounding boxes of geometries, the distances between any geometries that are not points will be inexact: they will be the distances between the bounding boxes of geometries.

The syntax of the index-based KNN query places a special “index-based distance operator” in the ORDER BY clause of the query, in this case “<->”. There are two index-based distance operators,

- <-> means “distance between box centers”
- <#> means “distance between box edges”

One side of the index-based distance operator must be a literal geometry value. We can get away with a subquery that returns as single geometry, or we could include a *WKT* geometry instead.

```
-- Closest 10 streets to Broad Street station are ?
SELECT
  streets.gid,
  streets.name
FROM
```



INTRO

WHO CARES ABOUT k-NN?

COLUMN-ORIENTED DBMS

CLOUD SERVICES

OBJECT-RELATIONAL DBMS

IBM Cloud

CatalogDocs

IBM Cloudant

LEARN

Getting started tutorial

Overview

IBM Cloud Public

Pricing

Security and Compliance

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HOW TO

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Recovery and backup

Nearest neighbor search

IBM Cloudant Geo supports Nearest Neighbor search, which is known as NN search. If provided, the `nearest=true` search returns all results by sorting their distances to the center of the query geometry. This geometric relation `nearest=true` can be used either with all the geometric relations described earlier, or alone.

For example, one police officer might search five crimes that occurred near a specific location by typing the query in the following example.

Example query to find nearest five crimes against a specific location:

```
https://education.cloudant.com/crimes/_design/geodd/_geo/geoidx?g=POINT(-71.053712
```

Tip: The `nearest=true` search can change the semantics of an IBM Cloudant Geo search. For example, without `nearest=true` in the example query, the results include only GeoJSON documents that have coordinates equal to the query point `(-71.0537124 42.3681995)` or an empty results set. However, by using the `nearest=true` search, the results include all GeoJSON documents in the database whose order is measured by the distance to the query point.

GeoMesa Documentation

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18.2.2. BinConversionProcess

process

a K Nearest Neighbor search on a Geomesa feature collection

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FROM

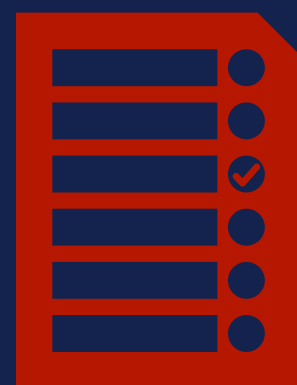


SETUP

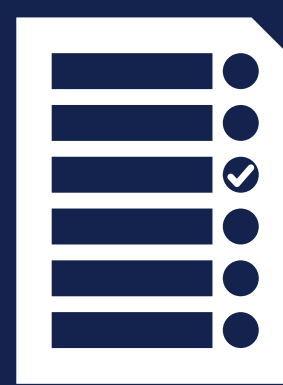
k-NEAREST NEIGHBORS

Records:

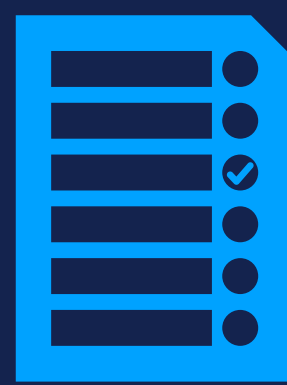
s_0



s_1



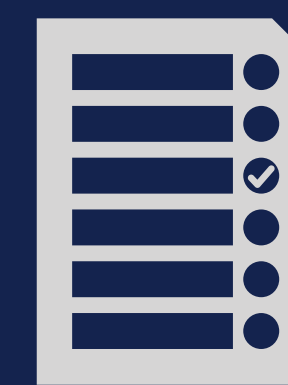
s_2



s_3



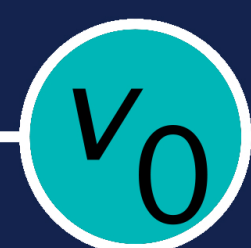
s_4



s_5



α



β

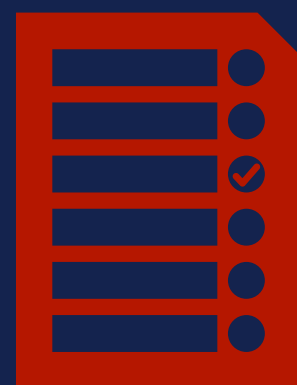


SETUP

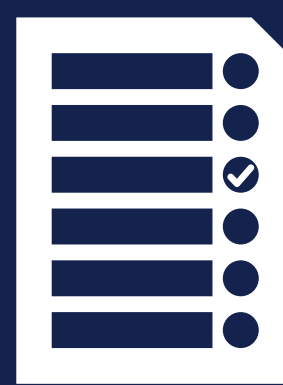
k-NEAREST NEIGHBORS

Records:

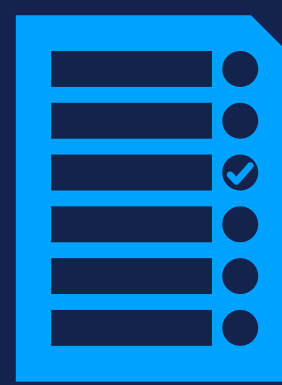
s_0



s_1



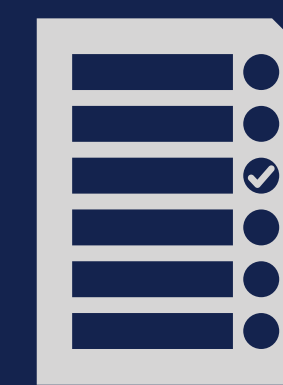
s_2



s_3



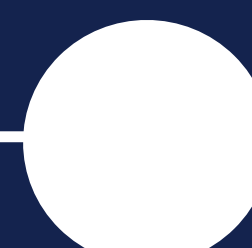
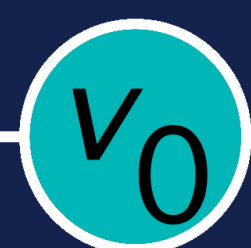
s_4



s_5



α



β

q

SETUP k-NEAREST NEIGHBORS

Records:

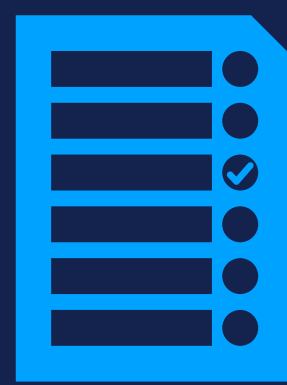
s_0



s_1



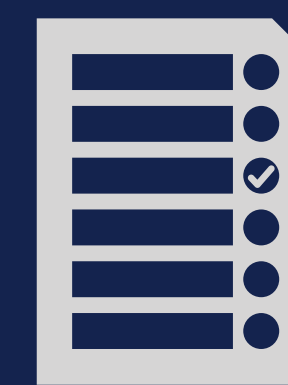
s_2



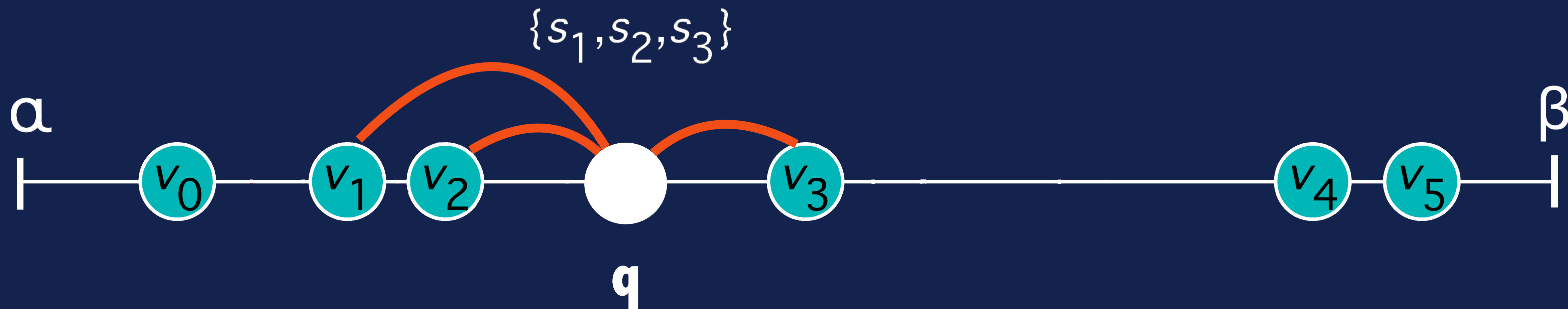
s_3



s_4

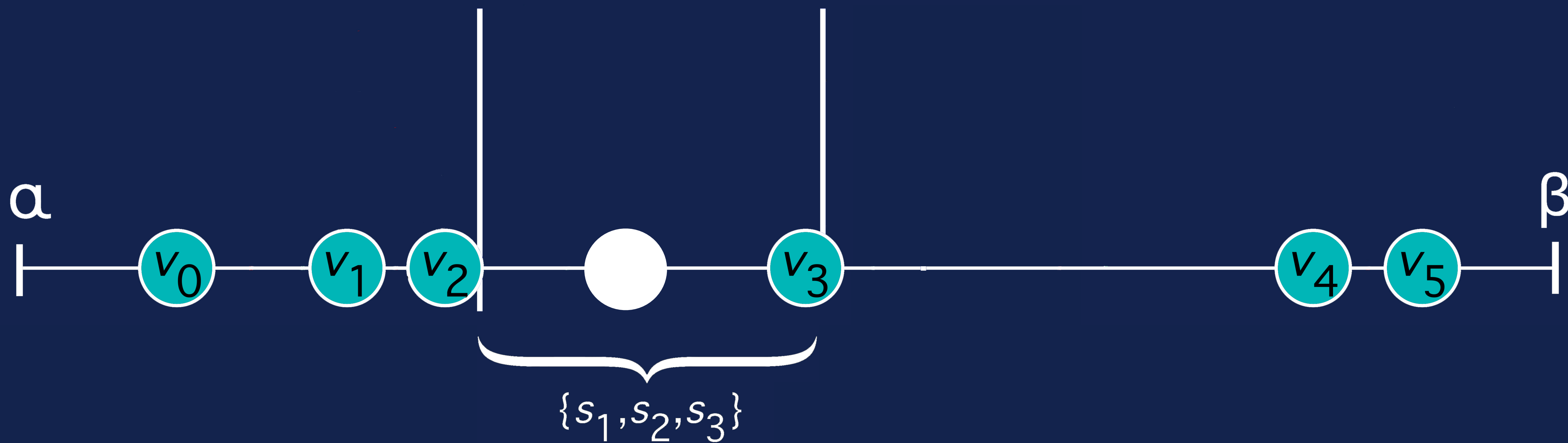


s_5



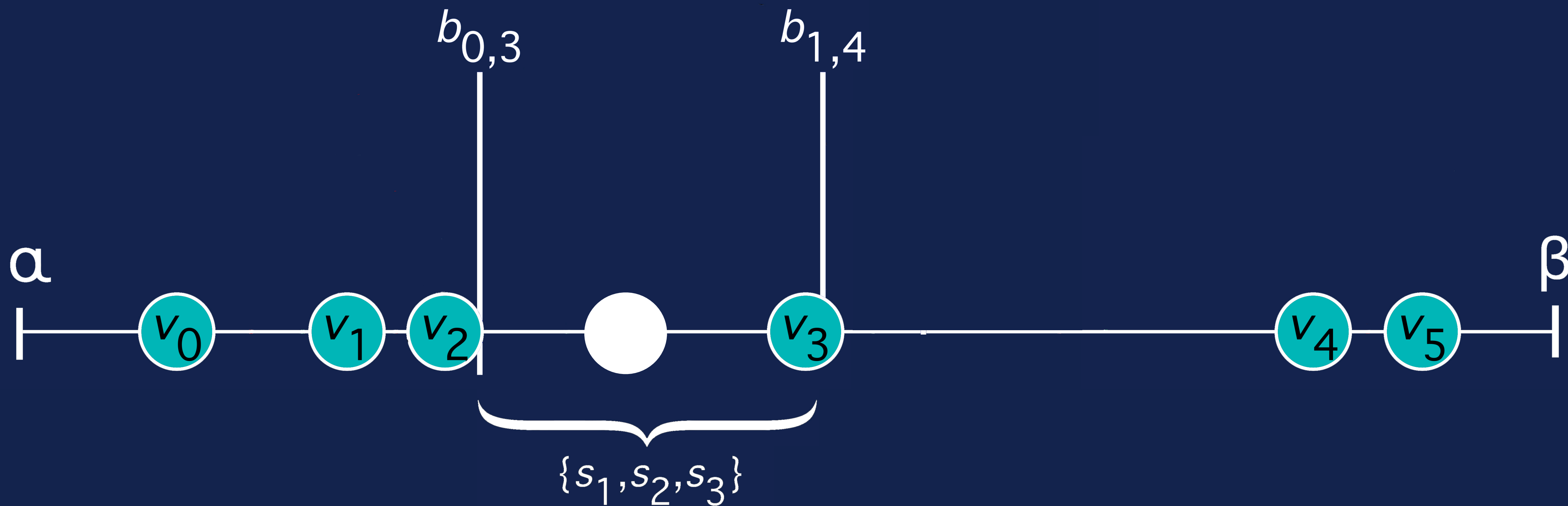


SETUP VORONOI DIAGRAMS



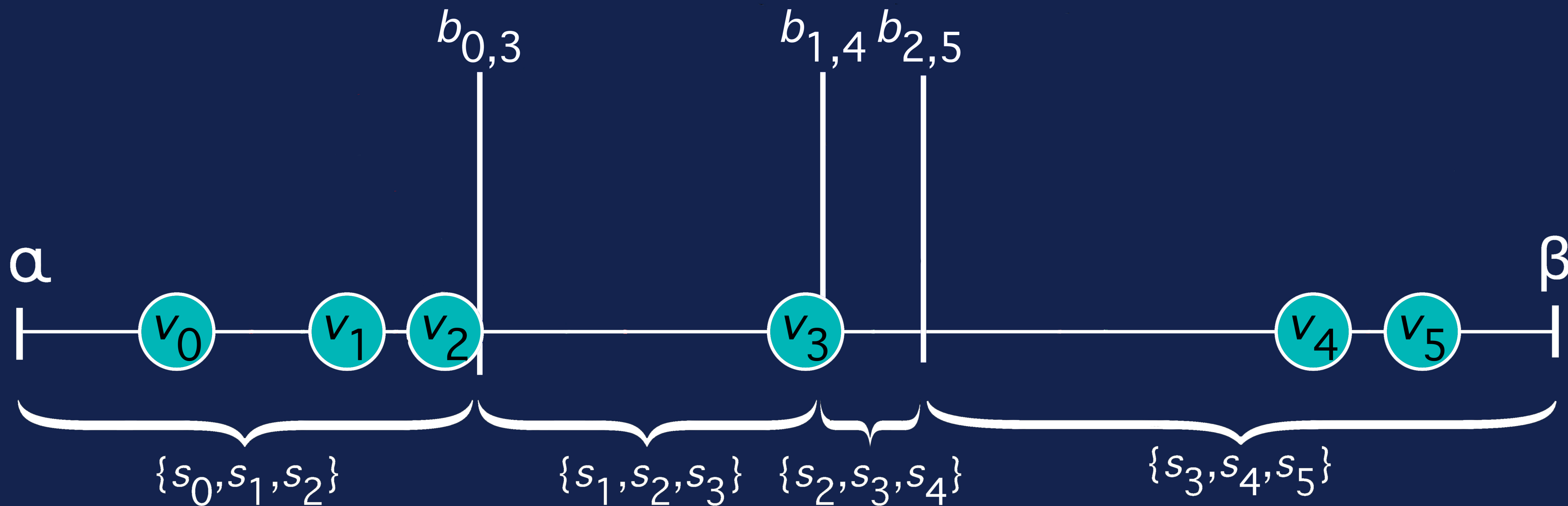


SETUP VORONOI DIAGRAMS





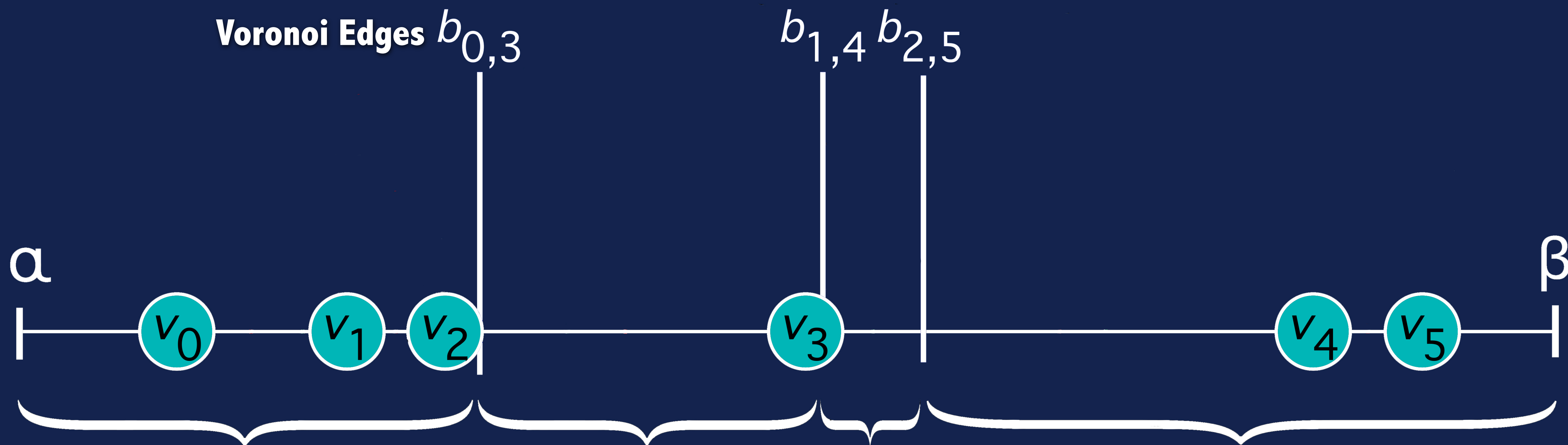
SETUP VORONOI DIAGRAMS





SETUP VORONOI DIAGRAMS

Voronoi Diagram



Voronoi Segment

Response $\{s_0, s_1, s_2\}$

$\{s_1, s_2, s_3\}$

$\{s_2, s_3, s_4\}$

$\{s_3, s_4, s_5\}$

SETUP ENCRYPTED SEARCH

Client



Server



Tokens

$$\text{PRF}_K(\bullet) = t$$

$$\text{PRF}_K(\bullet) = t'$$

$$\text{PRF}_K(\bullet) = t''$$

$$\text{PRF}_K(\bullet) = t$$

Responses



SETUP ENCRYPTED SEARCH

Client



Server



Tokens

$$\text{PRF}_K(\text{grey circle}) = t$$

$$\text{PRF}_K(\text{green circle}) = t'$$

$$\text{PRF}_K(\text{red circle}) = t''$$

$$\text{PRF}_K(\text{grey circle}) = t$$

**Search Pattern
Leakage**

Responses



Access Pattern Leakage



k-NN EXACT RECONSTRUCTION

ORDERED RESPONSES: Possible when all encrypted queries are issued

UNORDERED RESPONSES: Impossible due to many reconstructions

k-NN APPROXIMATE RECONSTRUCTION

ORDERED RESPONSES: Approximate reconstruction when not all encrypted queries are issued

UNORDERED RESPONSES: Even with many reconstructions approximate with bounded error



k-NN EXACT RECONSTRUCTION

ORDERED RESPONSES: Possible when all encrypted queries are issued

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k-NN APPROXIMATE RECONSTRUCTION

ORDERED RESPONSES: Approximate reconstruction when not all encrypted queries are issued

UNORDERED RESPONSES: Even with many reconstructions approximate with bounded error



UNORDERED RESPONSES

ASSUMPTIONS OF THE ATTACK

BOUNDARIES:

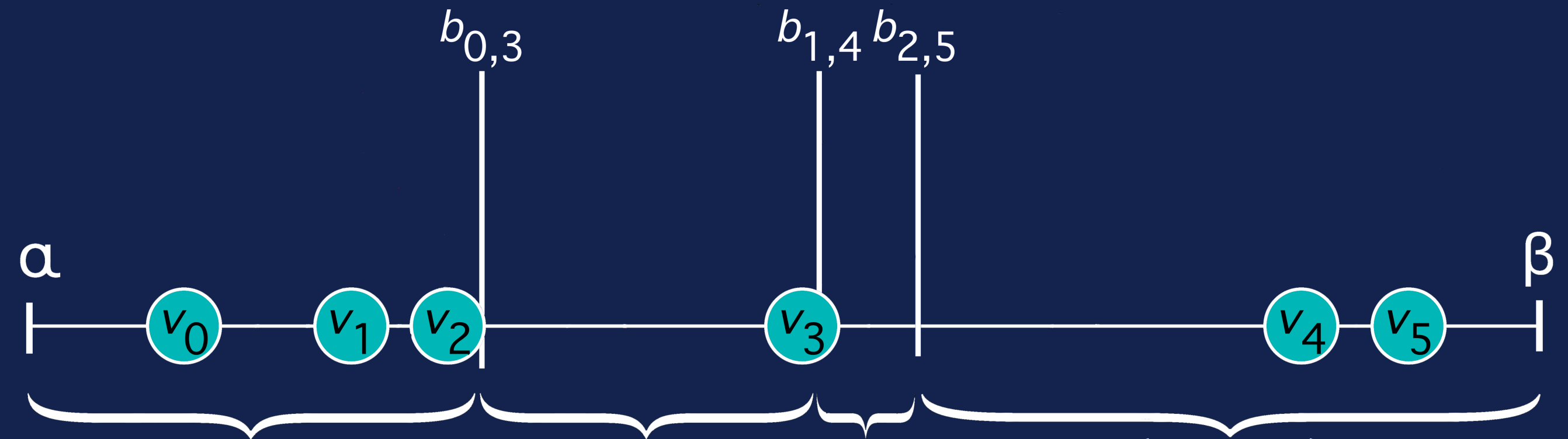
Known boundaries α and β

STATIC:

No updates in the database

UNIFORMITY:

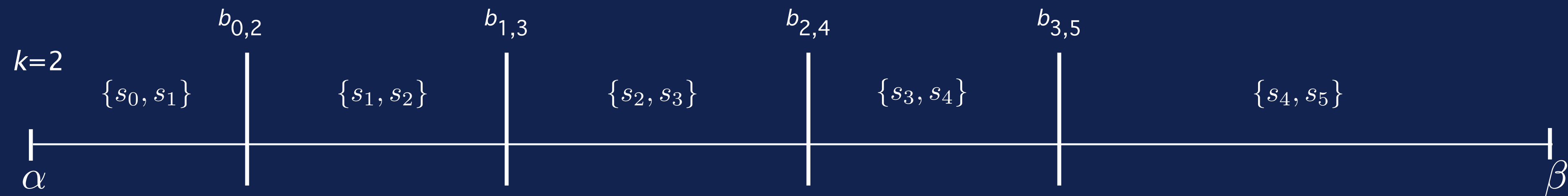
Queries are generated uniformly at random from $[\alpha, \beta]$



UNORDERED RESPONSES
EXACT RECONSTRUCTION

Impossible to achieve Exact Reconstruction

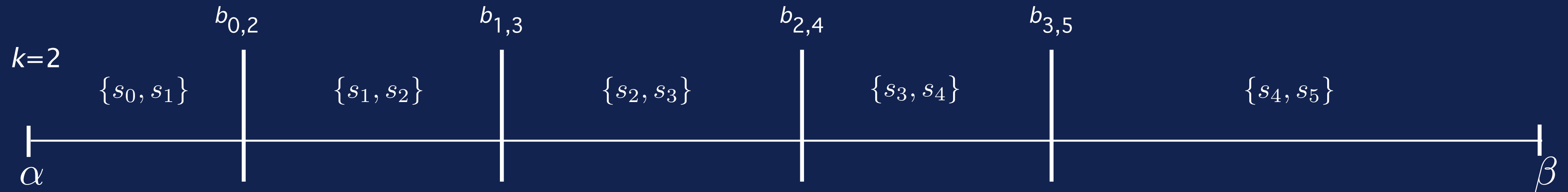
Best Case Scenario
for the Adversary



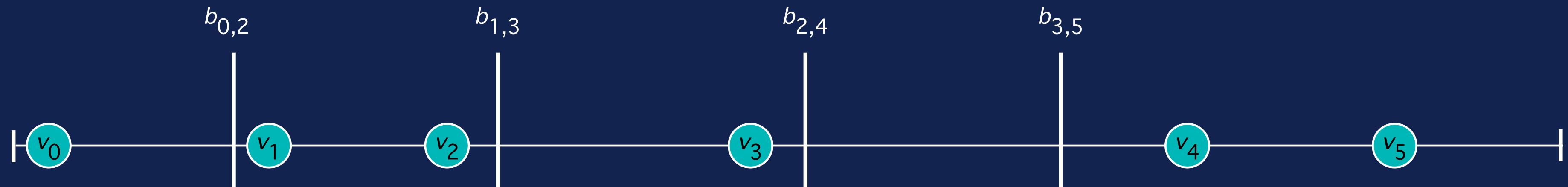
UNORDERED RESPONSES
EXACT RECONSTRUCTION

Impossible to achieve Exact Reconstruction

Best Case Scenario
for the Adversary



Valid Reconstruction
 DB_1

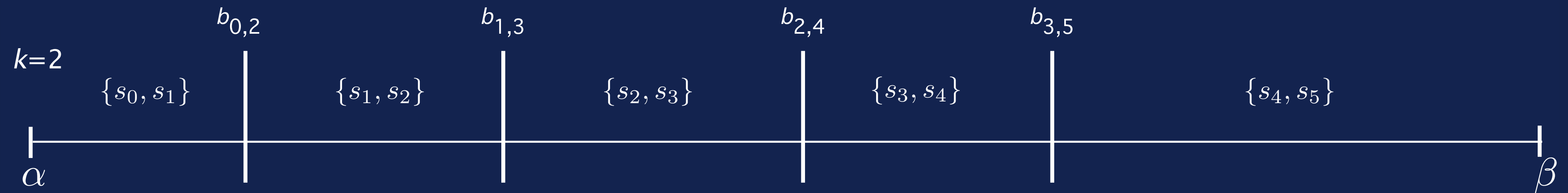




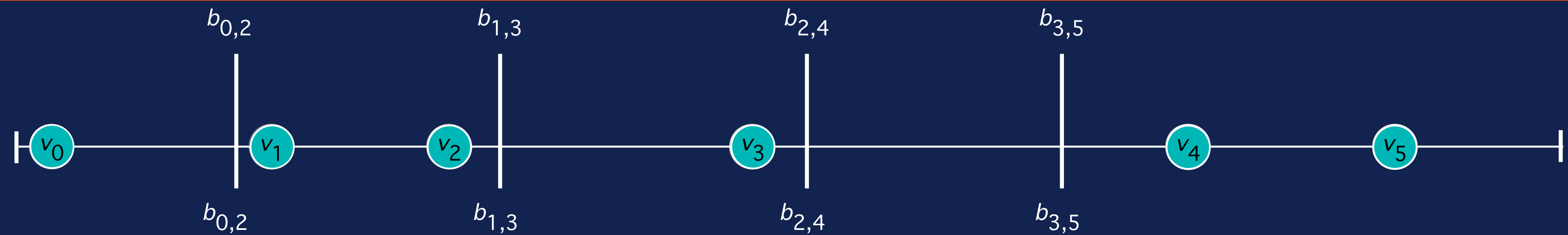
UNORDERED RESPONSES EXACT RECONSTRUCTION

Impossible to achieve Exact Reconstruction

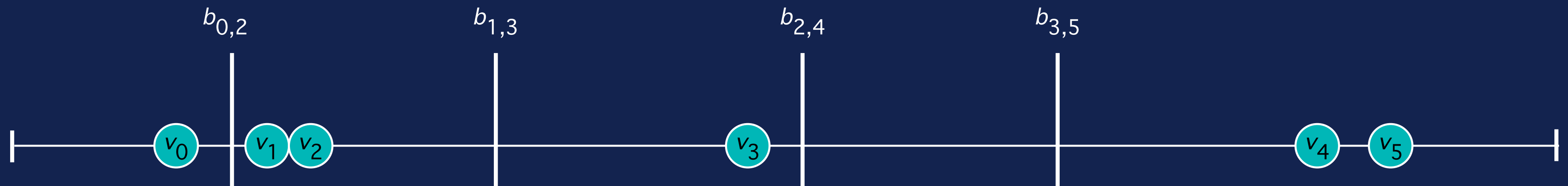
Best Case Scenario
for the Adversary



Valid Reconstruction
 DB_1



Valid Reconstruction
 DB_2

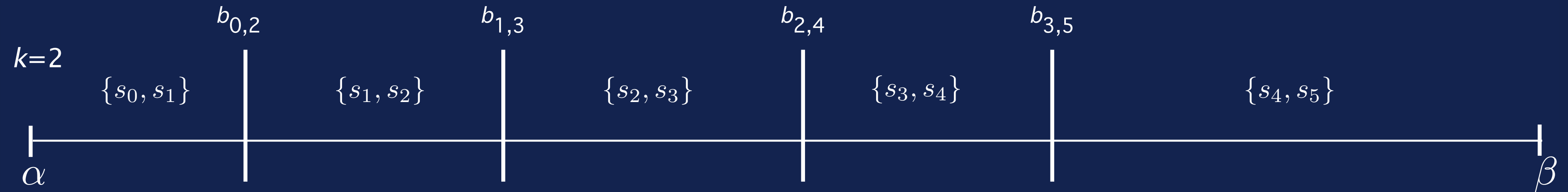




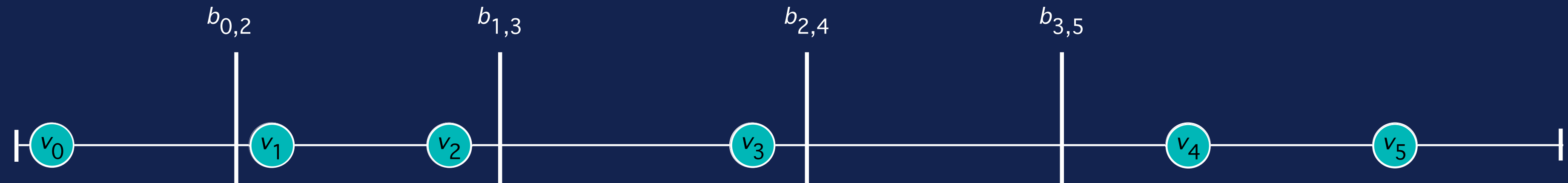
UNORDERED RESPONSES EXACT RECONSTRUCTION

Impossible to achieve Exact Reconstruction

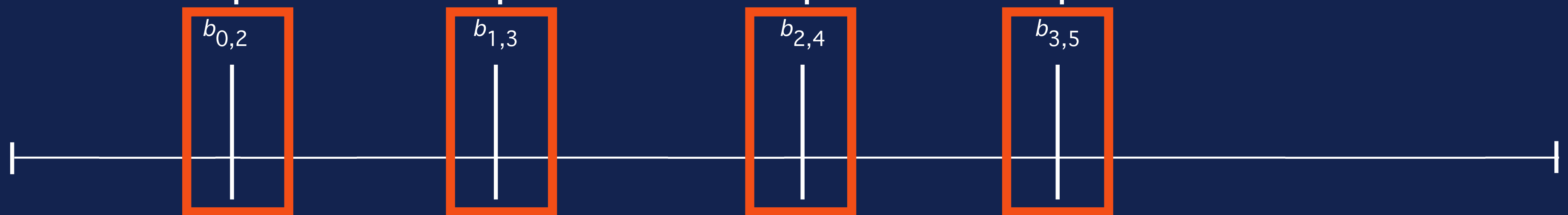
Best Case Scenario
for the Adversary



Valid Reconstruction
 DB_1



Valid Reconstruction
 DB_2

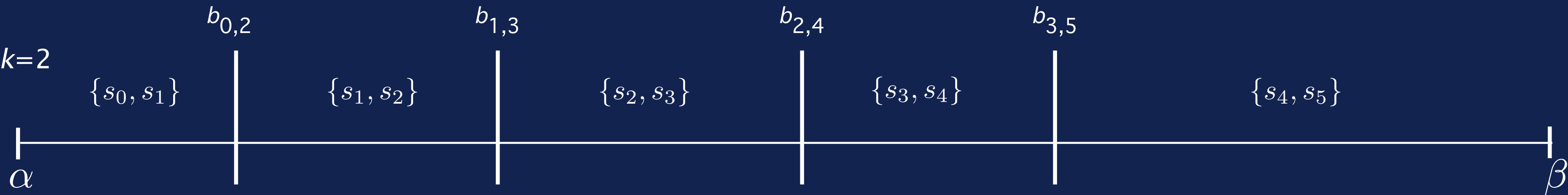




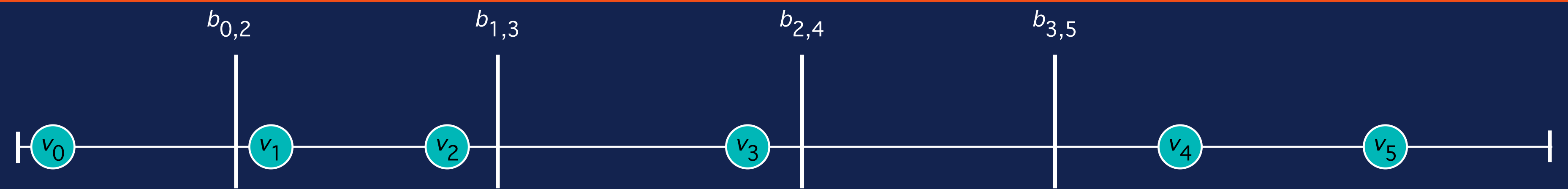
UNORDERED RESPONSES EXACT RECONSTRUCTION

Impossible to achieve Exact Reconstruction

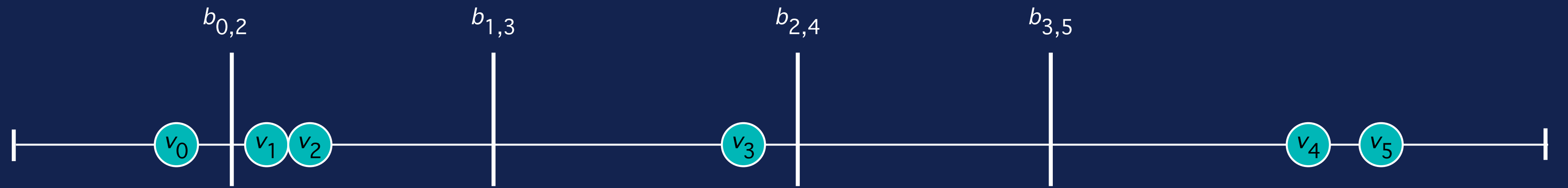
Best Case Scenario
for the Adversary



Valid Reconstruction
DB₁

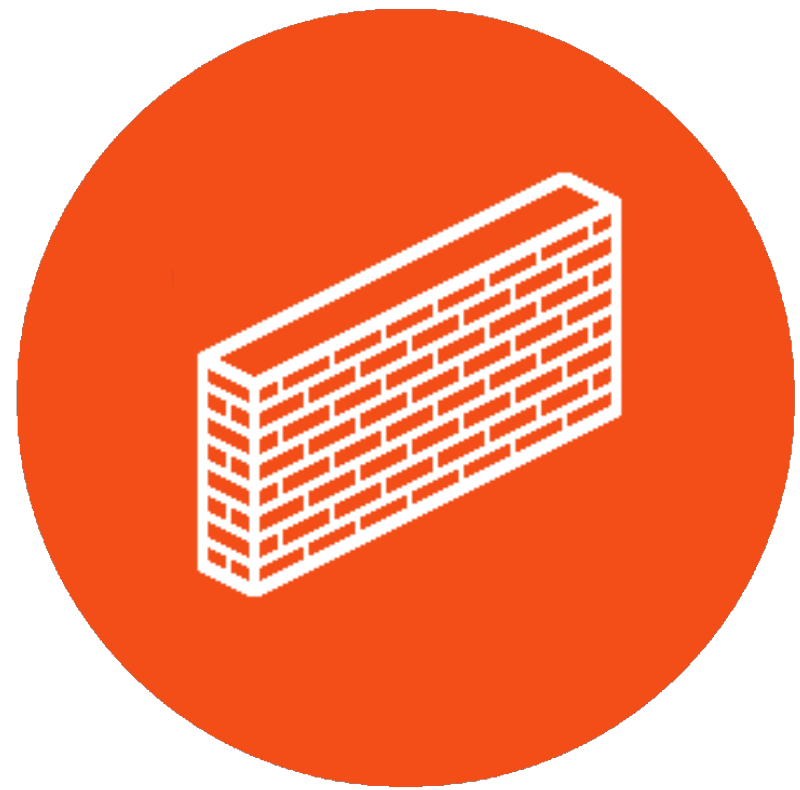


Valid Reconstruction
DB₂

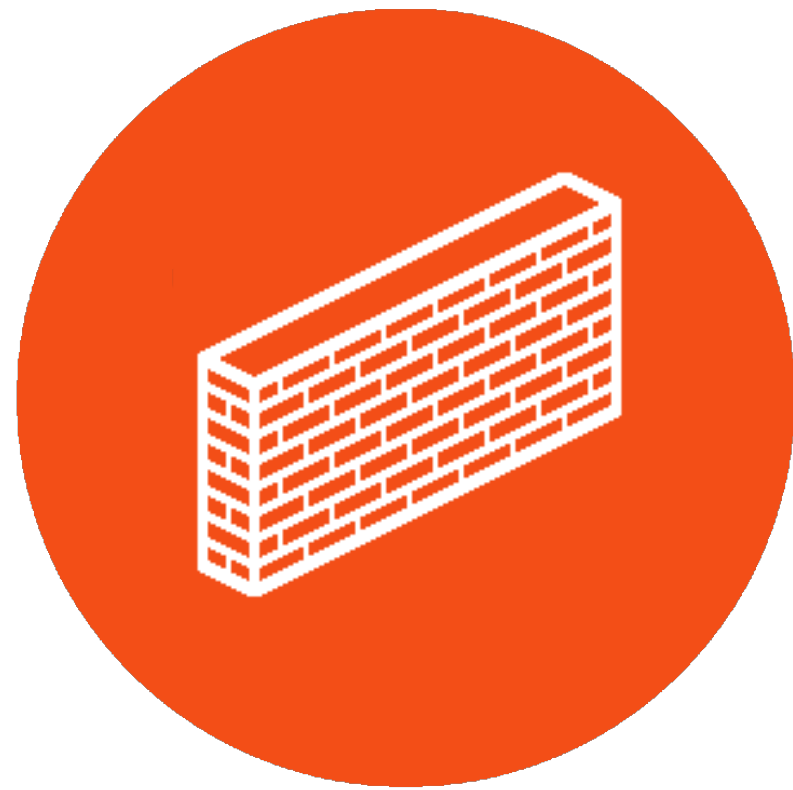


$\text{Vor}(\text{DB}_1) = \text{Vor}(\text{DB}_2) = \dots$

Many reconstructions that explain the Voronoi Diagram



Since there are **MANY** reconstructions and the exact recovery is **IMPOSSIBLE**, the encrypted values must be safe...



Since there are **MANY** reconstructions and the exact recovery is **IMPOSSIBLE**, the encrypted values must be safe...

Data Recovery on Encrypted Databases With k -Nearest Neighbor Query Leakage

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Abstract—Recent works by Kellaris *et al.* (CCS’16) and Lacharité *et al.* (SP’18) demonstrated attacks of data recovery for encrypted databases that support rich queries such as range queries. In this paper, we develop the first data recovery attacks on encrypted databases supporting one-dimensional k -nearest neighbor (k -NN) queries, which are widely used in spatial data management. Our attacks exploit a generic k -NN query leakage profile: the attacker observes the identifiers of matched records. We consider both unordered responses, where the leakage is a set, and ordered responses, where the leakage is a k -tuple ordered by distance from the query point.

As a first step, we perform a theoretical feasibility study on *exact reconstruction*, i.e., recovery of the exact plaintext values of the encrypted database. For ordered responses, we show that exact reconstruction is *feasible* if the attacker has additional access to some auxiliary information that is normally not available in practice. For unordered responses, we prove that exact reconstruction is *impossible* due to the infinite number of valid reconstructions. As a next step, we propose practical and more realistic *approximate reconstruction attacks* so as to recover an approximation of the plaintext values. For ordered

al. [46], demonstrate how an attacker can utilize access patterns to launch *query-recovery* attacks under various assumptions.

However, in the case of richer queries (e.g., range [16], [22], [37] and SQL [36], [38]), more severe *data-recovery* attacks are possible due to the expressiveness of the query. In particular, the work by Kellaris, Kollios, Nissim, and O’Neill [25] attacks SE-type systems that support range queries (e.g., [16], [21], [29]) by observing record identifiers whose plaintext values belong to the queried range. Similarly, a recent work by Lacharité, Minaud, and Paterson [27] further explores range query leakage to achieve exact and approximate reconstruction for the case of dense datasets with *orders of magnitude fewer queries* (when compared to [25]). Finally, order-preserving encryption based systems (e.g., CryptDB [38]) supporting even more expressive queries (such as SQL) have been shown to be vulnerable to data-recovery attacks [14], [20], [33] even without observing *any queries*, just by the setup leakage.

Answer: We can still compute an reconstruction that is **VERY CLOSE to the encrypted DB**

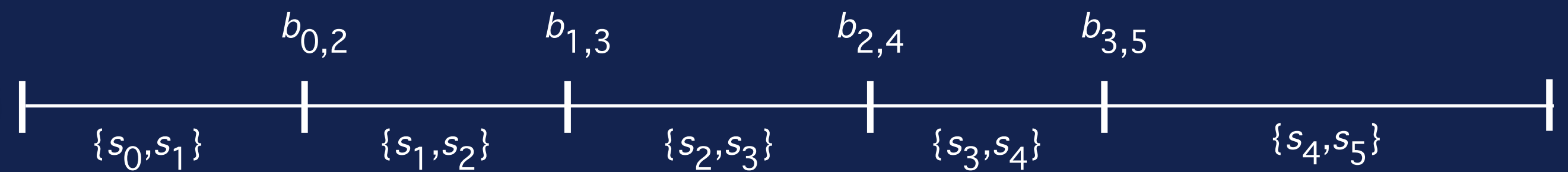


UNORDERED RESPONSES

APPROXIMATE RECONSTRUCTION*

In case all queries are issued:

The length of each Voronoi segments



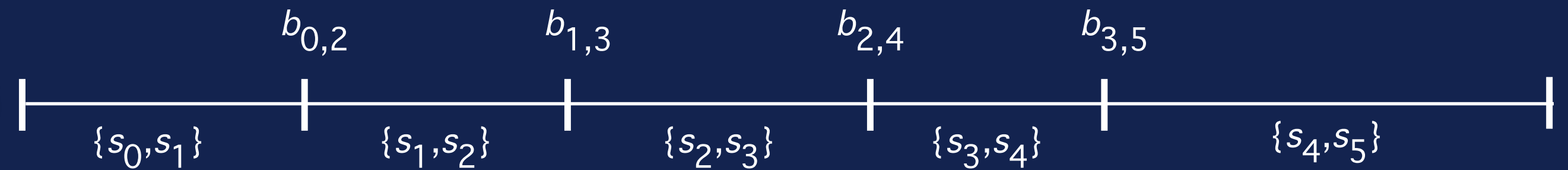
Uniform Query Distribution: Estimate via Concentration Bounds on Multinomials



UNORDERED RESPONSES APPROXIMATE RECONSTRUCTION*

In case all queries are issued:

The length of each Voronoi segments



Goal:

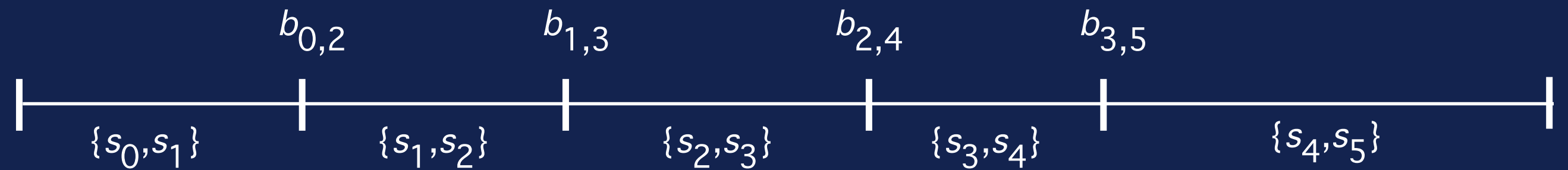
Characterize the set of all valid reconstructions that explain the Voronoi Diagram



UNORDERED RESPONSES APPROXIMATE RECONSTRUCTION*

In case all queries are issued:

The length of each Voronoi segments



Goal:

Characterize the set of all valid reconstructions that explain the Voronoi Diagram

What's Next:

Intuitive characterization = rigorous reconstruction guarantees



UNORDERED RESPONSES **APPROXIMATE RECONSTRUCTION***

Modeling All Reconstructions:



UNORDERED RESPONSES

APPROXIMATE RECONSTRUCTION*

Modeling All Reconstructions:

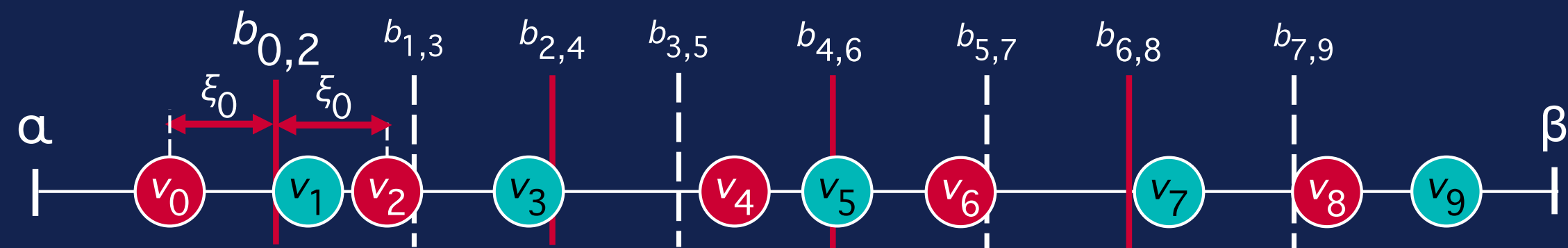
Use geometry of bisectors to define unknowns



UNORDERED RESPONSES APPROXIMATE RECONSTRUCTION*

Modeling All Reconstructions:

Use geometry of bisectors to define unknowns



$$v_0 = b_{0,2} - \xi_0$$

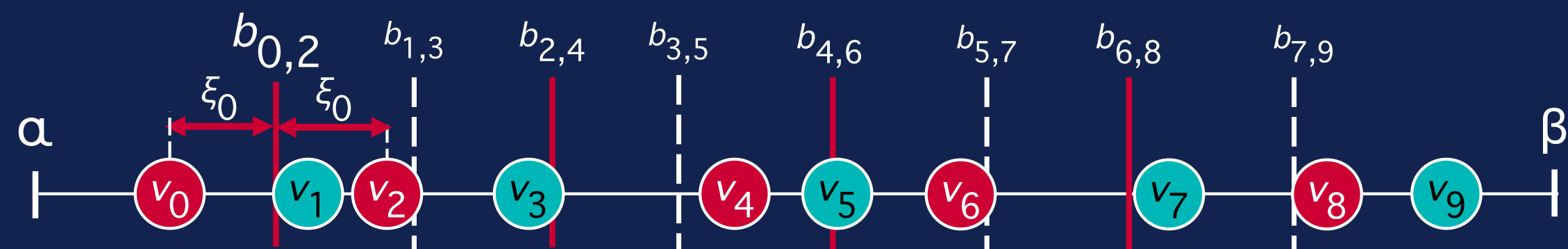
$$v_2 = b_{0,2} + \xi_0$$



UNORDERED RESPONSES APPROXIMATE RECONSTRUCTION*

Modeling All Reconstructions:

Use geometry of bisectors to define unknowns



$$v_0 = b_{0,2} - \xi_0$$

$$v_2 = b_{0,2} + \xi_0$$

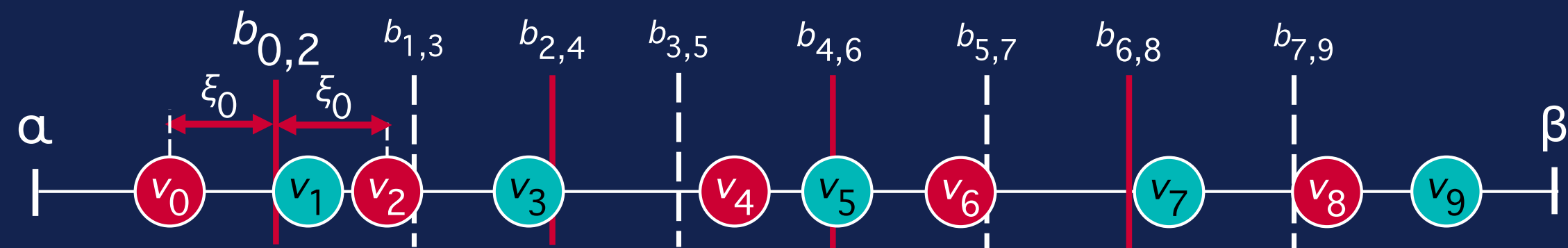
$$v_4 = 2b_{2,4} - v_2$$



UNORDERED RESPONSES APPROXIMATE RECONSTRUCTION*

Modeling All Reconstructions:

Use geometry of bisectors to define unknowns



$$v_0 = b_{0,2} - \xi_0$$

$$v_2 = b_{0,2} + \xi_0$$

$$v_4 = 2b_{2,4} - v_2$$

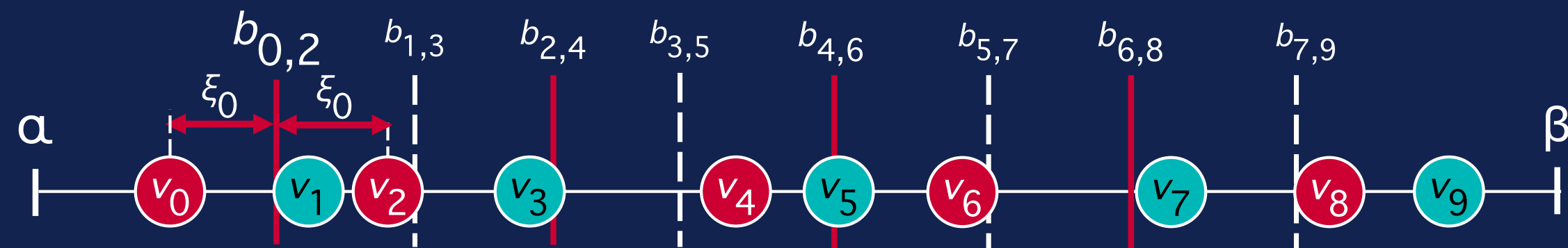




UNORDERED RESPONSES APPROXIMATE RECONSTRUCTION*

Modeling All Reconstructions:

Use geometry of bisectors to define unknowns



$$v_0 = b_{0,2} - \xi_0$$

$$v_2 = b_{0,2} + \xi_0$$

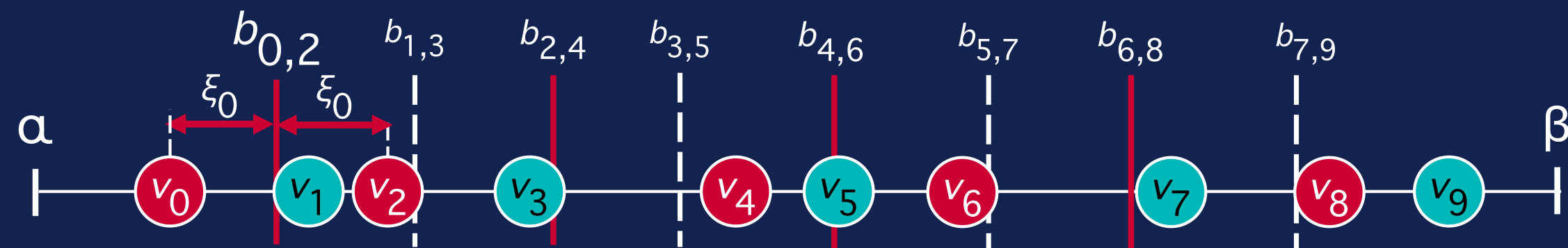
$$v_4 = 2b_{2,4} - v_2 = 2b_{2,4} - b_{0,2} - \xi_0$$



UNORDERED RESPONSES APPROXIMATE RECONSTRUCTION*

Modeling All Reconstructions:

Use geometry of bisectors to define unknowns



$$v_0 = b_{0,2} - \xi_0$$

$$v_2 = b_{0,2} + \xi_0$$

$$v_4 = 2b_{2,4} - v_2 = 2b_{2,4} - b_{0,2} - \xi_0$$

$$v_6 = 2b_{4,6} - v_4 = 2b_{4,6} - 2b_{2,4} + b_{0,2} + \xi_0$$

$$v_8 = 2b_{6,8} - v_6 = 2b_{6,8} - 2b_{4,6} + 2b_{2,4} - b_{0,2} - \xi_0$$

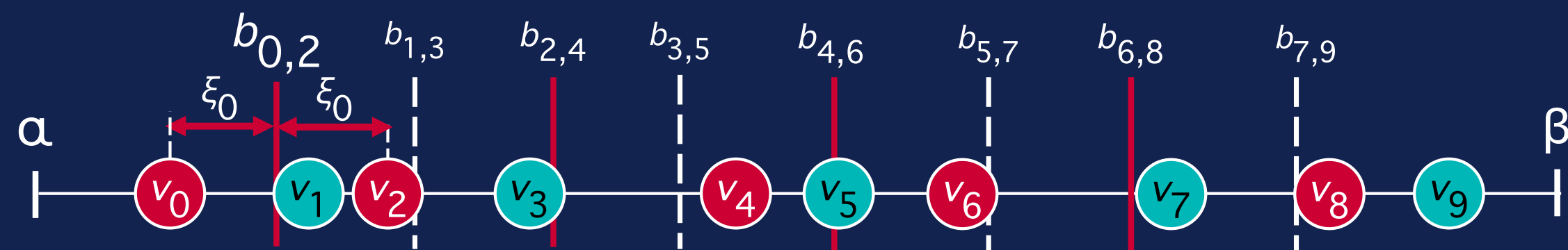
Half of the v_i as a function of unknown ξ_0



UNORDERED RESPONSES APPROXIMATE RECONSTRUCTION*

Modeling All Reconstructions:

Use geometry of bisectors to define unknowns



$$v_0 = b_{0,2} - \xi_0$$

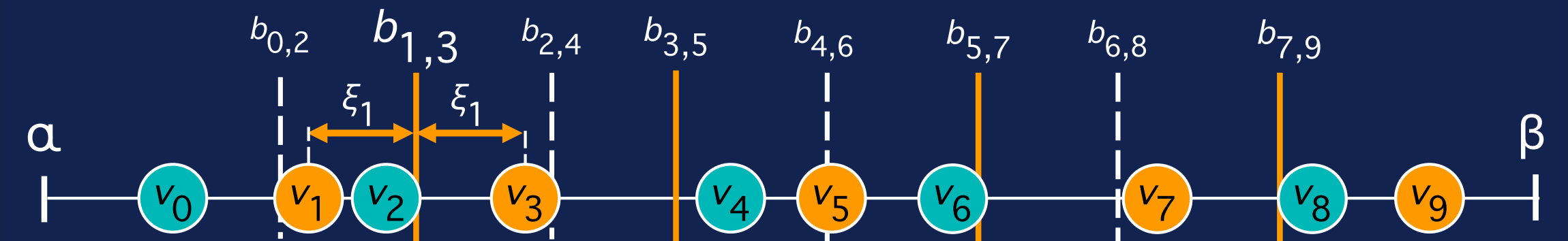
$$v_2 = b_{0,2} + \xi_0$$

$$v_4 = 2b_{2,4} - v_2 = 2b_{2,4} - b_{0,2} - \xi_0$$

$$v_6 = 2b_{4,6} - v_4 = 2b_{4,6} - 2b_{2,4} + b_{0,2} + \xi_0$$

$$v_8 = 2b_{6,8} - v_6 = 2b_{6,8} - 2b_{4,6} + 2b_{2,4} - b_{0,2} - \xi_0$$

Half of the v_i as a function of **unknown** ξ_0



$$v_1 = b_{1,3} - \xi_1$$

$$v_3 = b_{1,3} + \xi_1$$

$$v_5 = 2b_{3,5} - v_3 = 2b_{3,5} - b_{1,3} - \xi_1$$

$$v_7 = 2b_{5,7} - v_5 = 2b_{5,7} - 2b_{3,5} + b_{1,3} + \xi_1$$

$$v_9 = 2b_{7,9} - v_7 = 2b_{7,9} - 2b_{5,7} + 2b_{3,5} - b_{1,3} - \xi_1$$

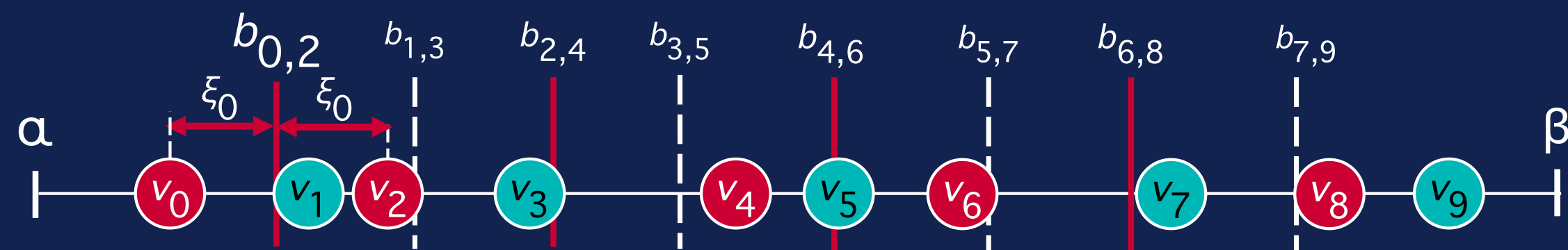
Other half of the v_i as a function of **unknown** ξ_1



UNORDERED RESPONSES APPROXIMATE RECONSTRUCTION*

Modeling All Reconstructions:

Use geometry of bisectors to define unknowns



$$v_0 = b_{0,2} - \xi_0$$

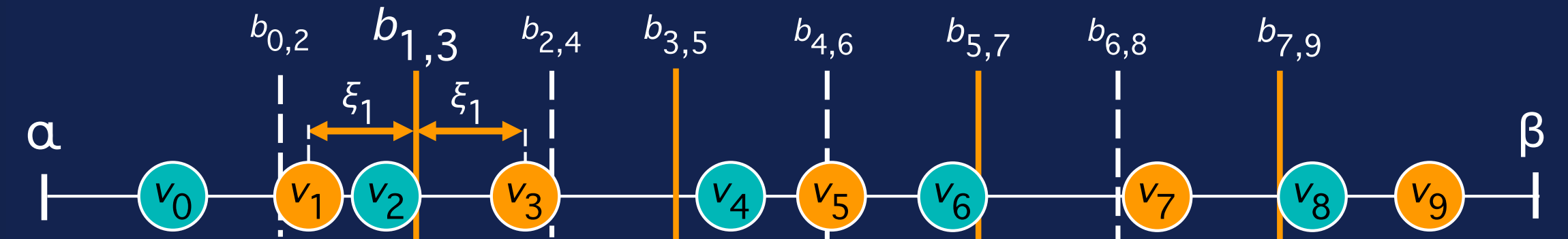
$$v_2 = b_{0,2} + \xi_0$$

$$v_4 = 2b_{2,4} - v_2 = 2b_{2,4} - b_{0,2} - \xi_0$$

$$v_6 = 2b_{4,6} - v_4 = 2b_{4,6} - 2b_{2,4} + b_{0,2} + \xi_0$$

$$v_8 = 2b_{6,8} - v_6 = 2b_{6,8} - 2b_{4,6} + 2b_{2,4} - b_{0,2} - \xi_0$$

Half of the v_i as a function of **unknown** ξ_0



$$v_1 = b_{1,3} - \xi_1$$

$$v_3 = b_{1,3} + \xi_1$$

$$v_5 = 2b_{3,5} - v_3 = 2b_{3,5} - b_{1,3} - \xi_1$$

$$v_7 = 2b_{5,7} - v_5 = 2b_{5,7} - 2b_{3,5} + b_{1,3} + \xi_1$$

$$v_9 = 2b_{7,9} - v_7 = 2b_{7,9} - 2b_{5,7} + 2b_{3,5} - b_{1,3} - \xi_1$$

Other half of the v_i as a function of **unknown** ξ_1

Reduced the space of reconstructions from n-dimensions to 2-dimensions



UNORDERED RESPONSES APPROXIMATE RECONSTRUCTION*

Modeling All Reconstructions:

Ordering Constraints:

$$v_0 < v_1$$



UNORDERED RESPONSES APPROXIMATE RECONSTRUCTION*

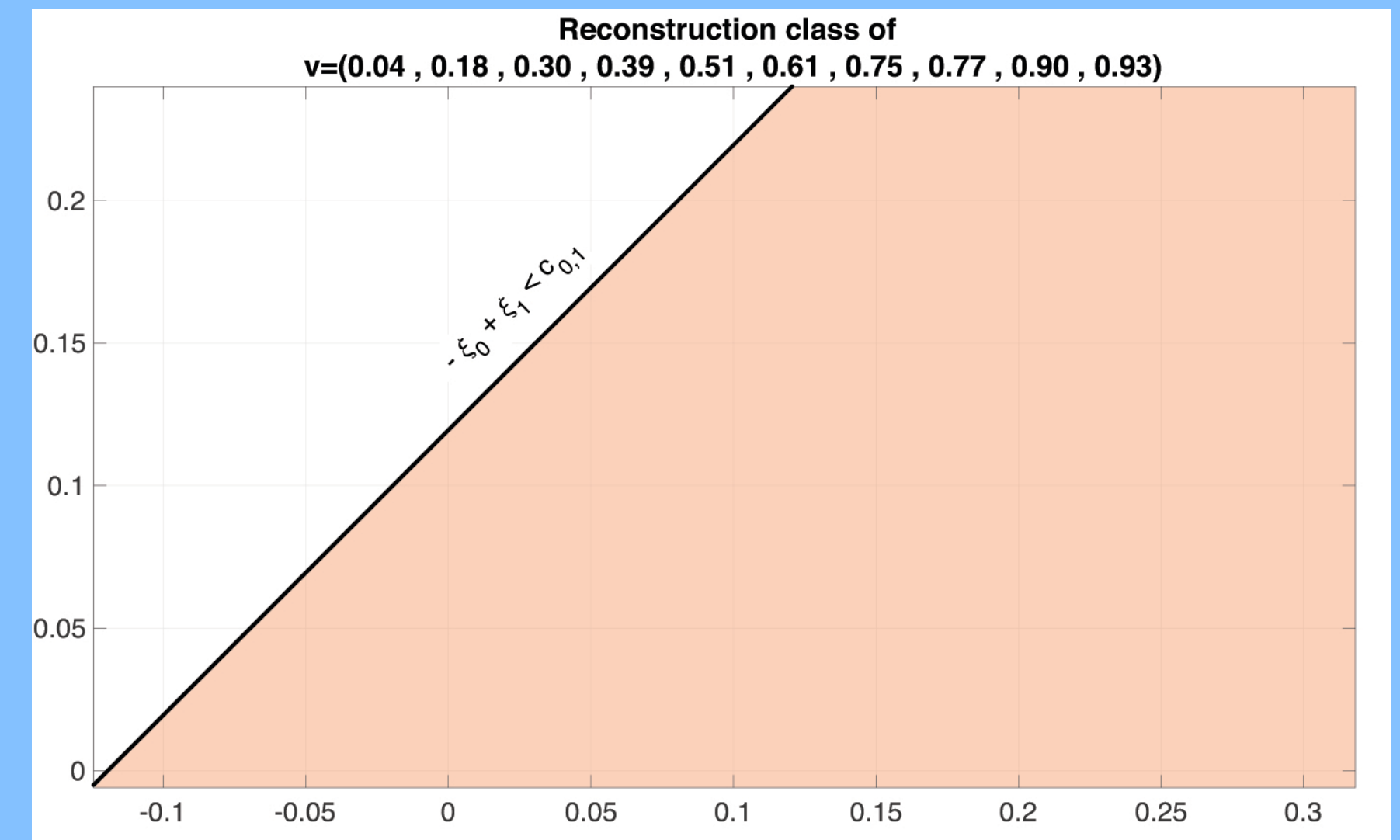
Modeling All Reconstructions:

Ordering Constraints:

$v_0 < v_1 \Rightarrow -\xi_0 + \xi_1 < c_{0,1}$, where $c_{0,1} = (b_{1,3} - b_{0,2})$

Geometric Characterization

ξ_1





UNORDERED RESPONSES APPROXIMATE RECONSTRUCTION*

Modeling All Reconstructions:

Ordering Constraints:

$$v_0 < v_1 \Rightarrow -\xi_0 + \xi_1 < c_{0,1}, \text{ where } c_{0,1} = (b_{1,3} - b_{0,2})$$

$$v_1 < v_2 \Rightarrow -\xi_0 - \xi_1 < c_{1,2}, \text{ where } c_{1,2} = -(b_{1,3} - b_{0,2})$$

$$v_2 < v_3 \Rightarrow \xi_0 - \xi_1 < c_{2,3}, \text{ where } c_{2,3} = (b_{1,3} - b_{0,2})$$

$$v_3 < v_4 \Rightarrow \xi_0 + \xi_1 < c_{3,4}, \text{ where } c_{3,4} = (b_{2,4} - b_{1,3}) + (b_{2,4} - b_{0,2})$$

$$v_4 < v_5 \Rightarrow -\xi_0 + \xi_1 < c_{4,5}, \text{ where } c_{4,5} = 2(b_{3,5} - b_{2,4}) - (b_{1,3} - b_{0,2})$$

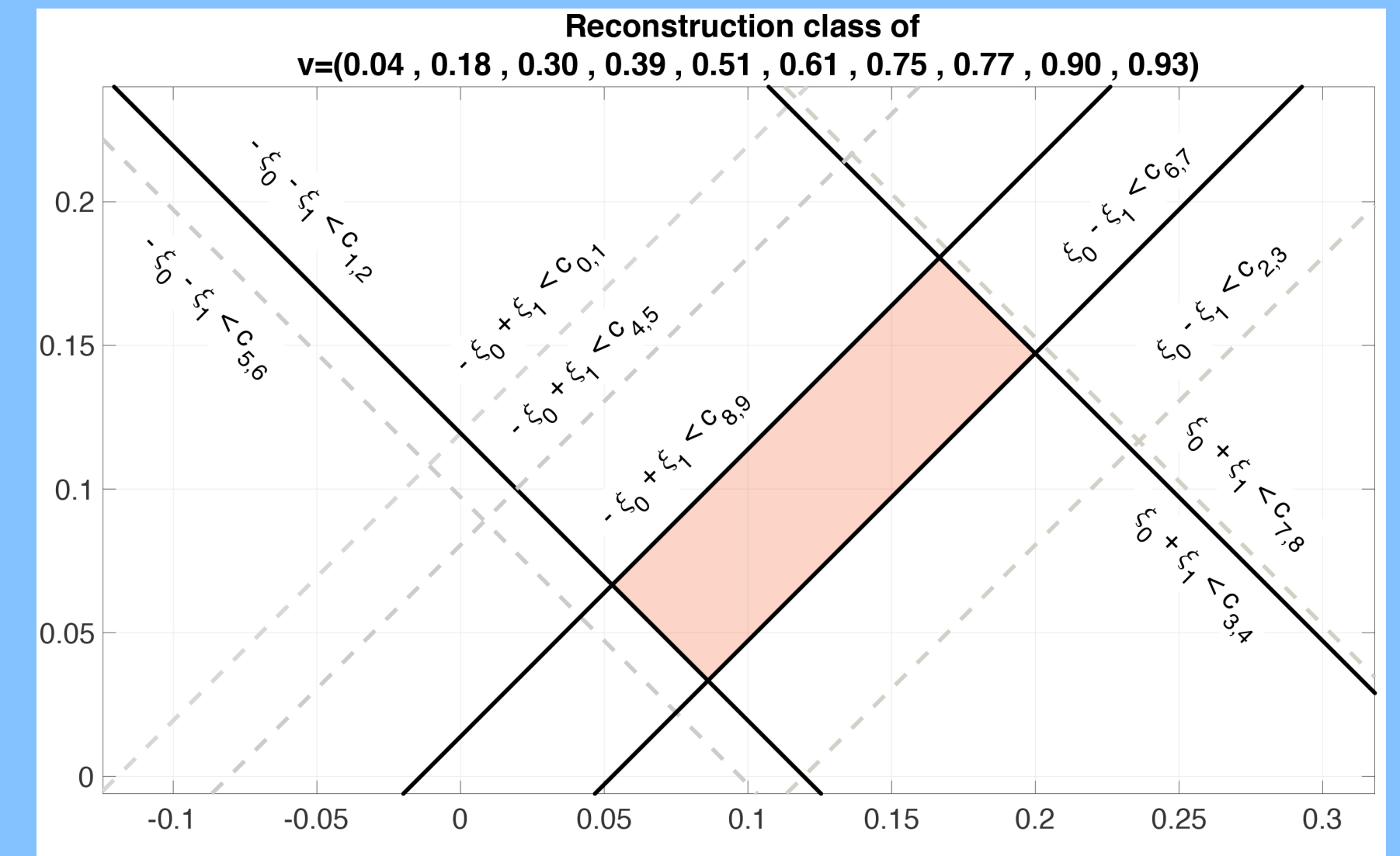
$$v_5 < v_6 \Rightarrow -\xi_0 - \xi_1 < c_{5,6}, \text{ where } c_{5,6} = 2(b_{4,6} - b_{3,5}) - (b_{2,4} - b_{0,2}) - (b_{2,4} - b_{1,3})$$

$$v_6 < v_7 \Rightarrow \xi_0 - \xi_1 < c_{6,7}, \text{ where } c_{6,7} = 2(b_{5,7} - b_{4,6}) - 2(b_{3,5} - b_{2,4}) + (b_{1,3} - b_{0,2})$$

$$v_7 < v_8 \Rightarrow \xi_0 + \xi_1 < c_{7,8}, \text{ where } c_{7,8} = 2(b_{6,8} - b_{5,7}) - 2(b_{4,6} - b_{3,5}) + (b_{2,4} - b_{1,3}) + (b_{2,4} - b_{0,2})$$

$$v_8 < v_9 \Rightarrow -\xi_0 + \xi_1 < c_{8,9}, \text{ where } c_{8,9} = 2(b_{7,9} - b_{6,8}) - 2(b_{5,7} - b_{4,6}) + 2(b_{3,5} - b_{2,4}) - (b_{1,3} - b_{0,2})$$

Geometric Characterization

 ξ_1  ξ_0



UNORDERED RESPONSES APPROXIMATE RECONSTRUCTION*

Modeling All Reconstructions:

Ordering Constraints:

$$v_0 < v_1 \Rightarrow -\xi_0 + \xi_1 < c_{0,1}, \text{ where } c_{0,1} = (b_{1,3} - b_{0,2})$$

$$v_1 < v_2 \Rightarrow -\xi_0 - \xi_1 < c_{1,2}, \text{ where } c_{1,2} = -(b_{1,3} - b_{0,2})$$

$$v_2 < v_3 \Rightarrow \xi_0 - \xi_1 < c_{2,3}, \text{ where } c_{2,3} = (b_{1,3} - b_{0,2})$$

$$v_3 < v_4 \Rightarrow \xi_0 + \xi_1 < c_{3,4}, \text{ where } c_{3,4} = (b_{2,4} - b_{1,3}) + (b_{2,4} - b_{0,2})$$

$$v_4 < v_5 \Rightarrow -\xi_0 + \xi_1 < c_{4,5}, \text{ where } c_{4,5} = 2(b_{3,5} - b_{2,4}) - (b_{1,3} - b_{0,2})$$

$$v_5 < v_6 \Rightarrow -\xi_0 - \xi_1 < c_{5,6}, \text{ where } c_{5,6} = 2(b_{4,6} - b_{3,5}) - (b_{2,4} - b_{0,2}) - (b_{2,4} - b_{1,3})$$

$$v_6 < v_7 \Rightarrow \xi_0 - \xi_1 < c_{6,7}, \text{ where } c_{6,7} = 2(b_{5,7} - b_{4,6}) - 2(b_{3,5} - b_{2,4}) + (b_{1,3} - b_{0,2})$$

$$v_7 < v_8 \Rightarrow \xi_0 + \xi_1 < c_{7,8}, \text{ where } c_{7,8} = 2(b_{6,8} - b_{5,7}) - 2(b_{4,6} - b_{3,5}) + (b_{2,4} - b_{1,3}) + (b_{2,4} - b_{0,2})$$

$$v_8 < v_9 \Rightarrow -\xi_0 + \xi_1 < c_{8,9}, \text{ where } c_{8,9} = 2(b_{7,9} - b_{6,8}) - 2(b_{5,7} - b_{4,6}) + 2(b_{3,5} - b_{2,4}) - (b_{1,3} - b_{0,2})$$

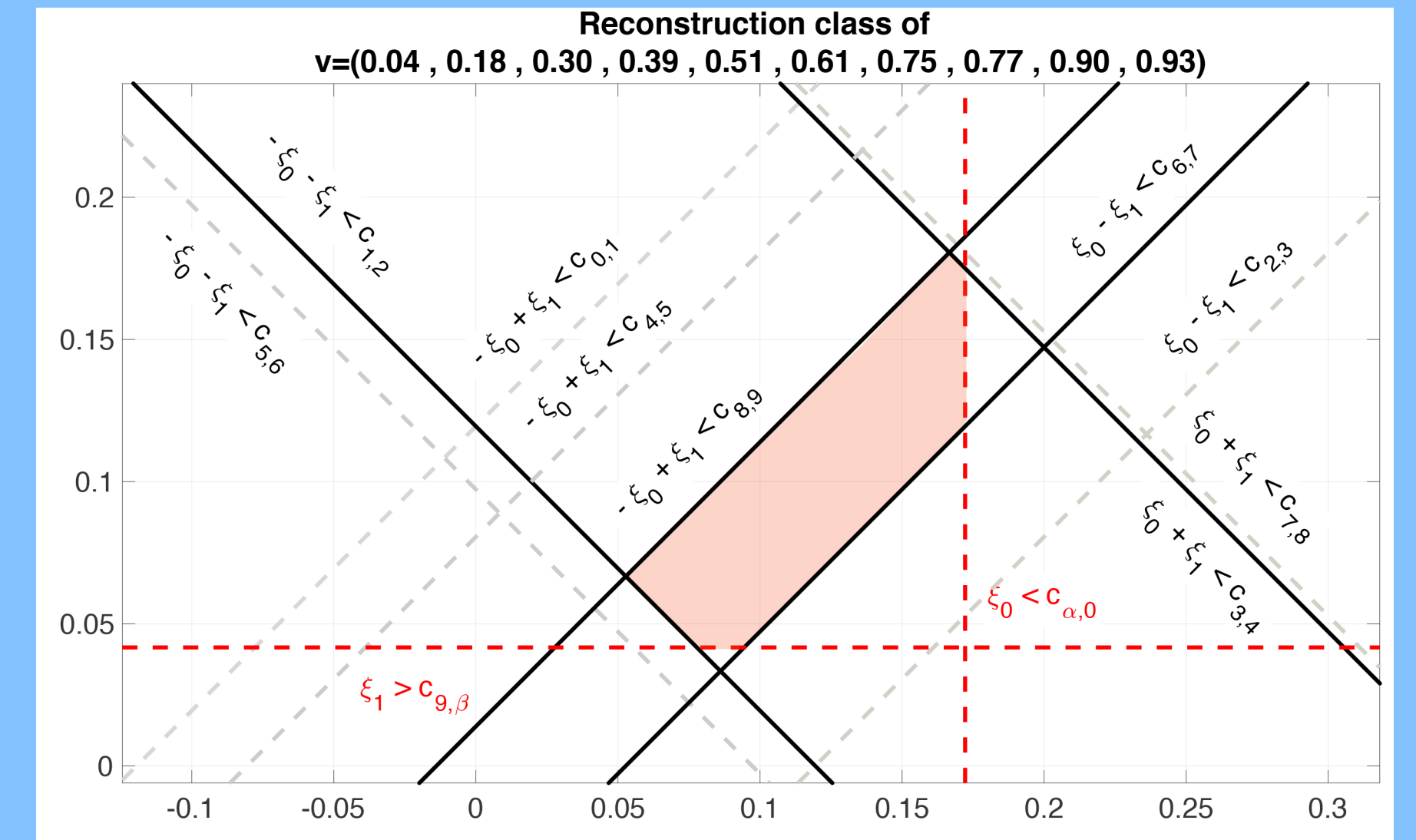
Boundary Constraints:

$$\alpha < v_0 \Rightarrow \xi_0 < c_{\alpha,0}, \text{ where } c_{\alpha,0} = b_{0,2} - \alpha$$

$$v_9 < \beta \Rightarrow \xi_1 > c_{9,\beta}, \text{ where } c_{9,\beta} = 2b_{7,9} - 2b_{5,7} + 2b_{3,5} - b_{1,3} - \beta$$

Geometric Characterization

ξ_1

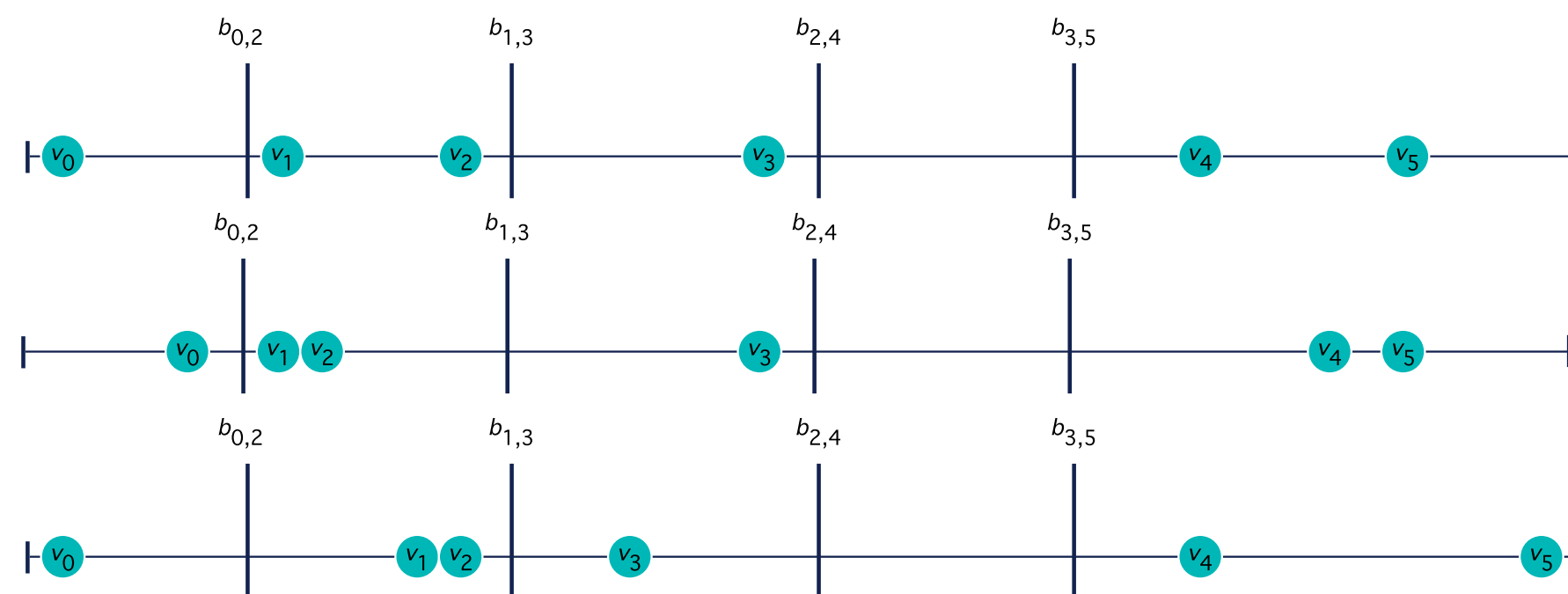


ξ_0

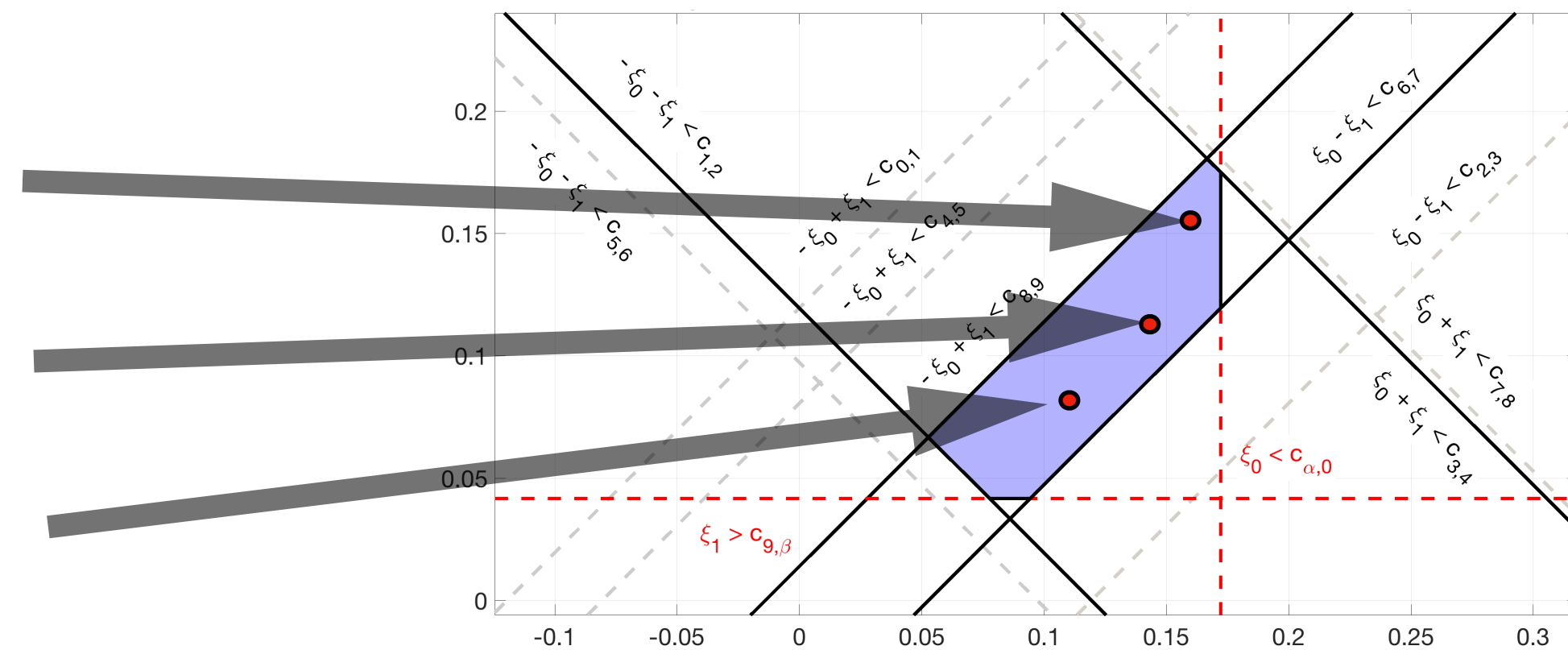


“Squeezed” the seemingly large space of valid reconstructions into a small polygon

Valid Reconstructions



Geometric Characterization

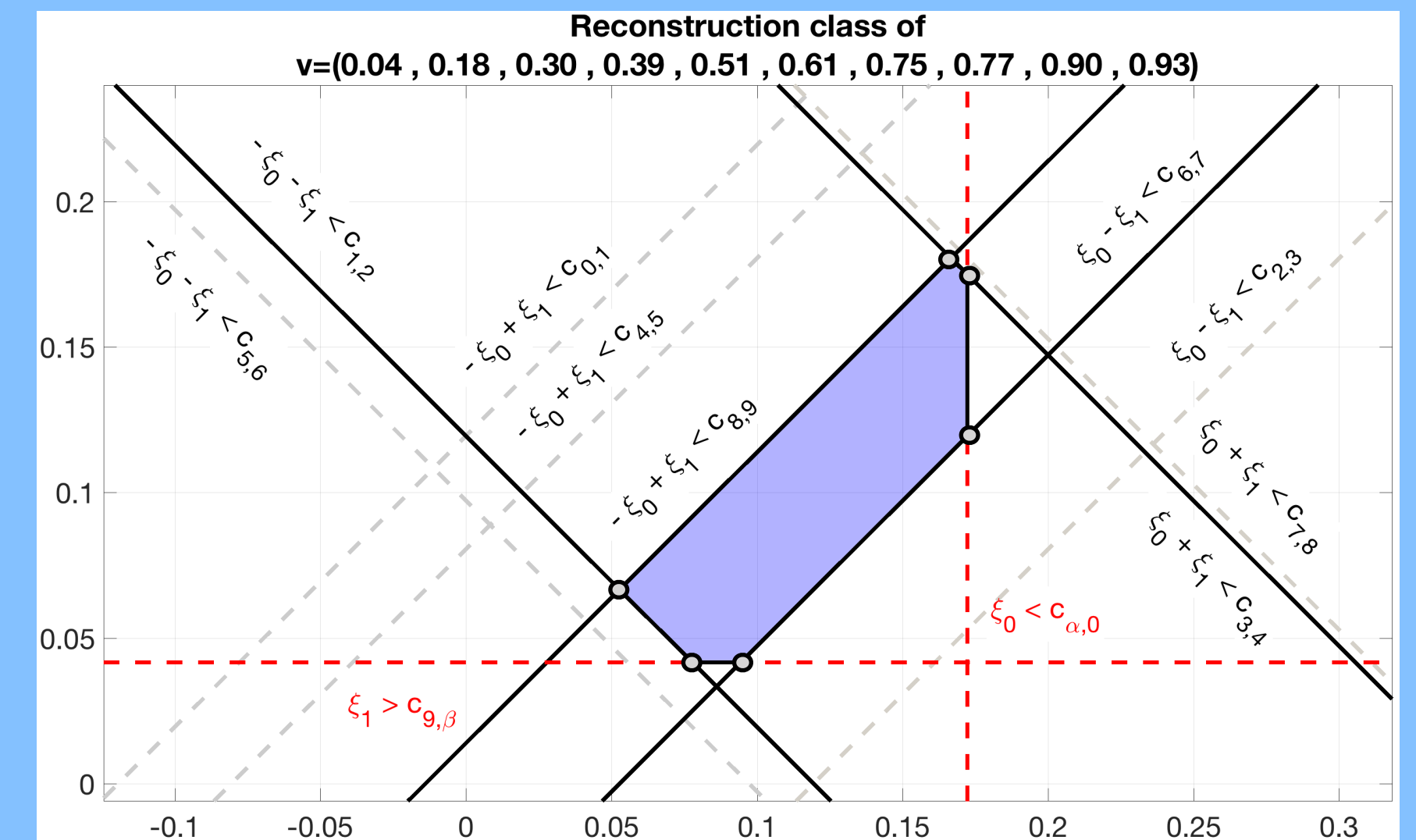




UNORDERED RESPONSES APPROXIMATE RECONSTRUCTION*

Original DB: $v' = (v'_0, \dots, v'_{n-1})$

Reconstr. DB: $v'' = (v''_0, \dots, v''_{n-1})$





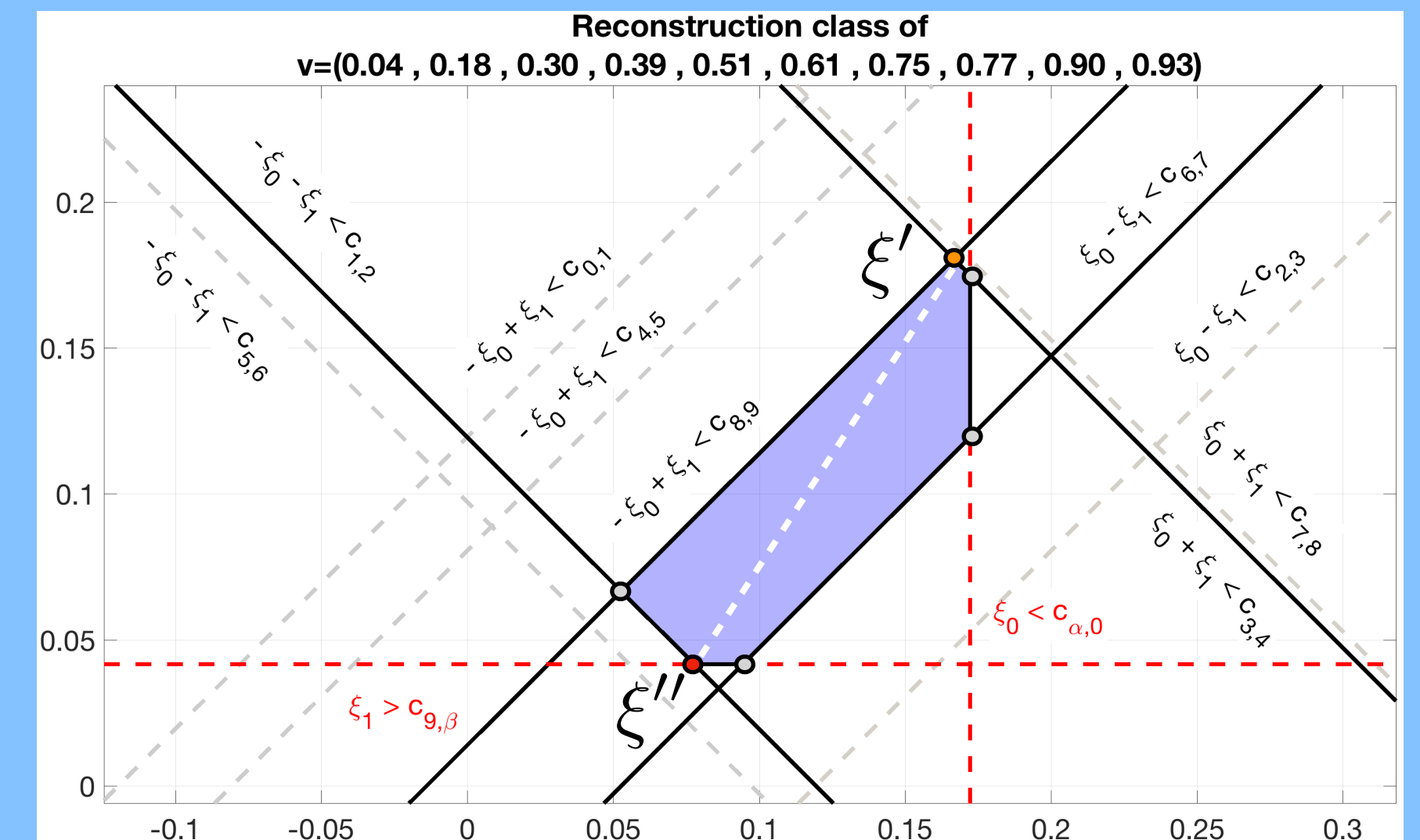
UNORDERED RESPONSES APPROXIMATE RECONSTRUCTION*

Reconstruction Error between v', v''

$$\max_{i \in [0, n-1]} |v'_i - v''_i| \leq \text{diam}(F_v)$$

Original DB: $v' = (v'_0, \dots, v'_{n-1})$

Reconstr. DB: $v'' = (v''_0, \dots, v''_{n-1})$



Maximum Error



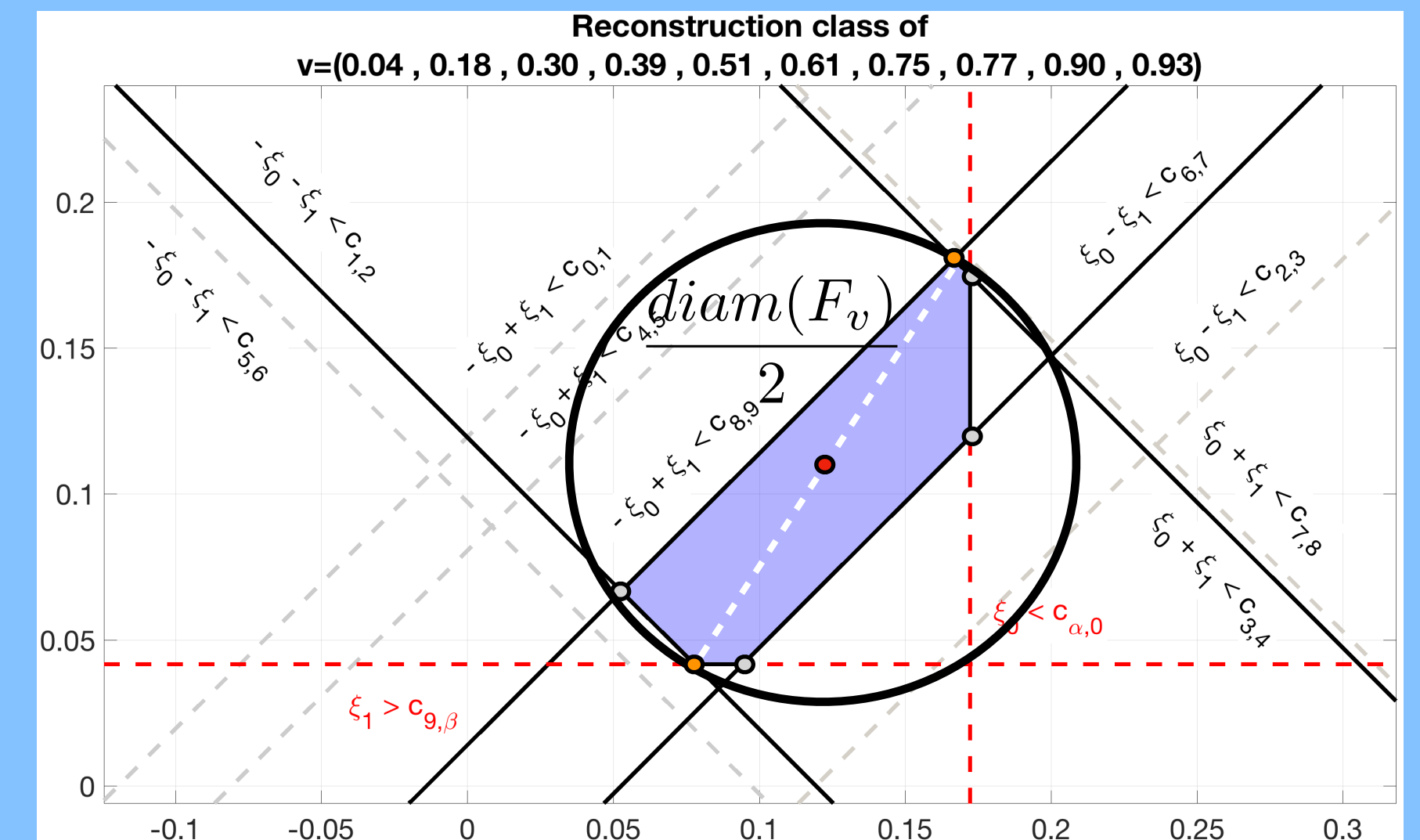
UNORDERED RESPONSES APPROXIMATE RECONSTRUCTION*

Reconstruction Error between v', v''

$$\max_{i \in [0, n-1]} |v'_i - v''_i| \leq \text{diam}(F_v)$$

Original DB: $v' = (v'_0, \dots, v'_{n-1})$

Reconstr. DB: $v'' = (v''_0, \dots, v''_{n-1})$





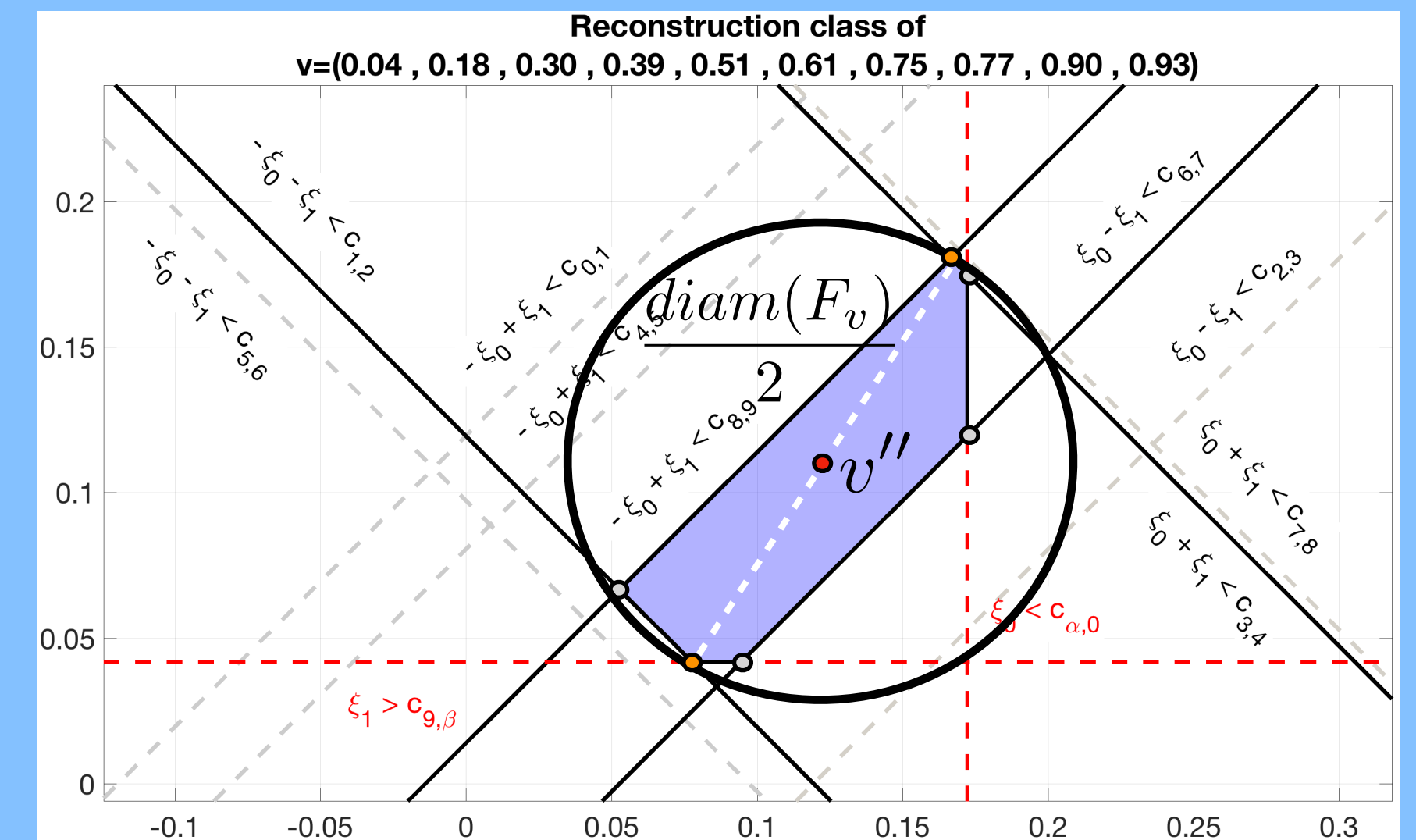
UNORDERED RESPONSES APPROXIMATE RECONSTRUCTION*

Reconstruction Error between v', v''

$$\max_{i \in [0, n-1]} |v'_i - v''_i| \leq \text{diam}(F_v)$$

Original DB: $v' = (v'_0, \dots, v'_{n-1})$

Reconstr. DB: $v'' = (v''_0, \dots, v''_{n-1})$



Our Reconstruction



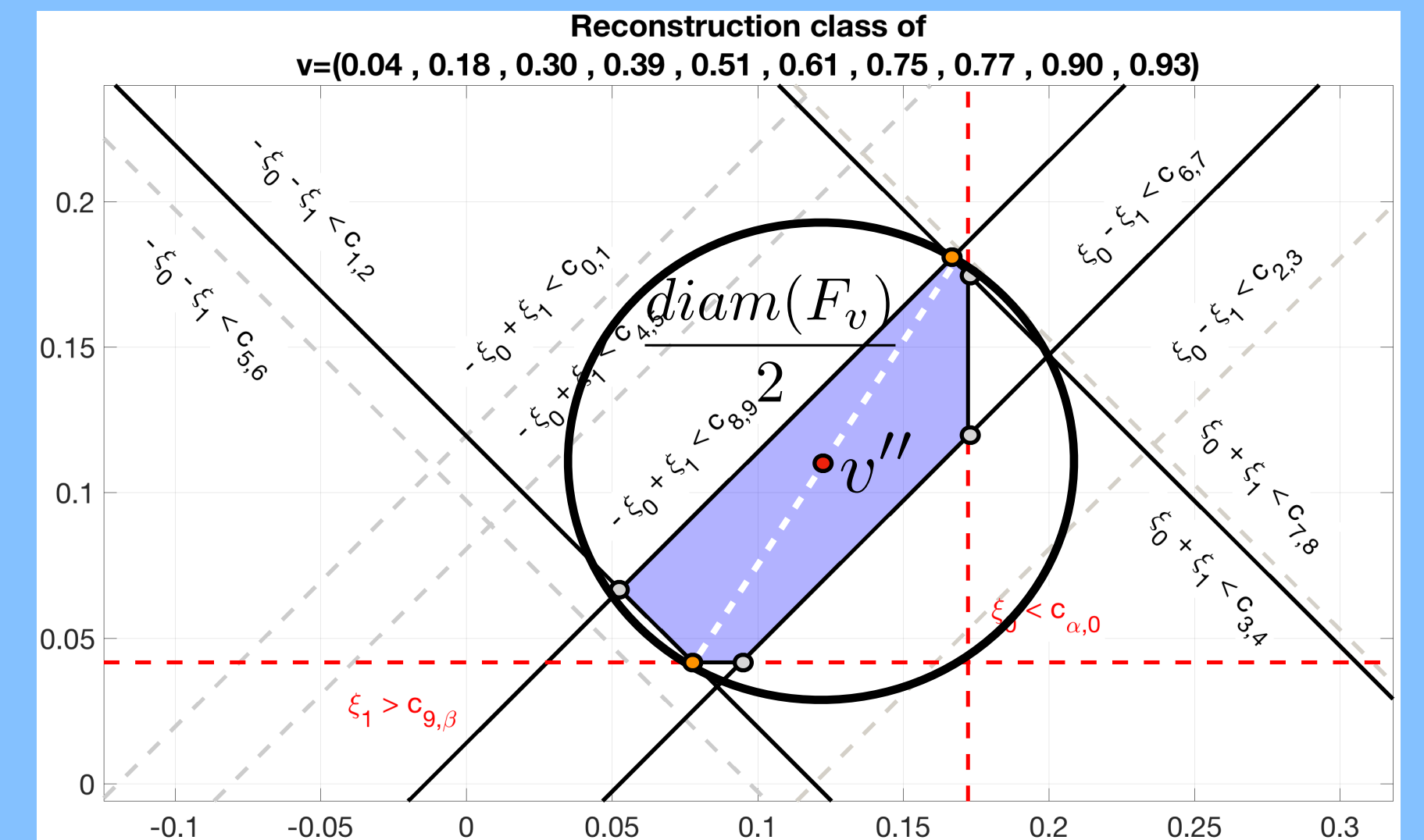
UNORDERED RESPONSES APPROXIMATE RECONSTRUCTION*

Reconstruction Error between v', v''

$$\max_{i \in [0, n-1]} |v'_i - v''_i| \leq \text{diam}(F_v)$$

Original DB: $v' = (v'_0, \dots, v'_{n-1})$

Reconstr. DB: $v'' = (v''_0, \dots, v''_{n-1})$



Our Reconstruction

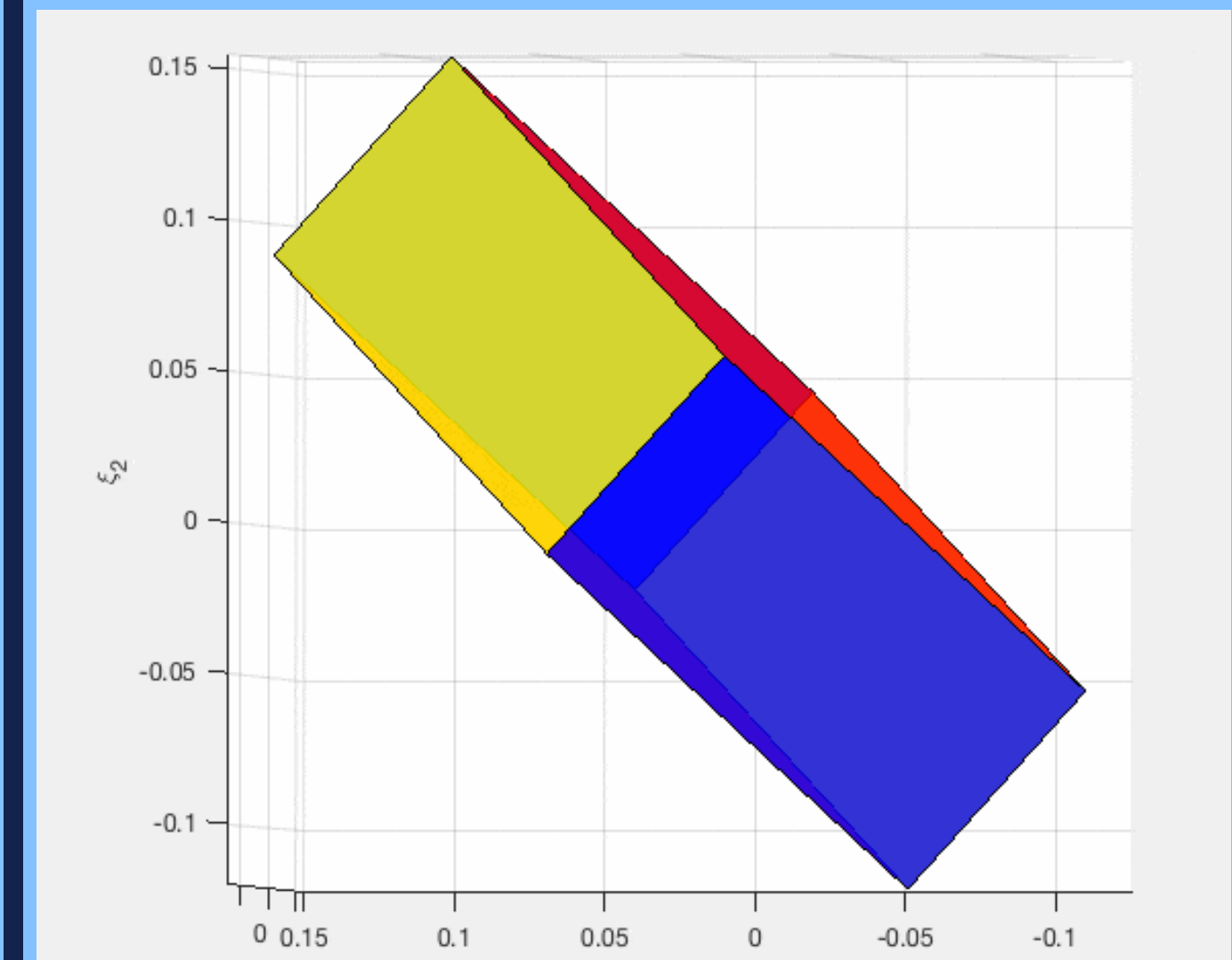
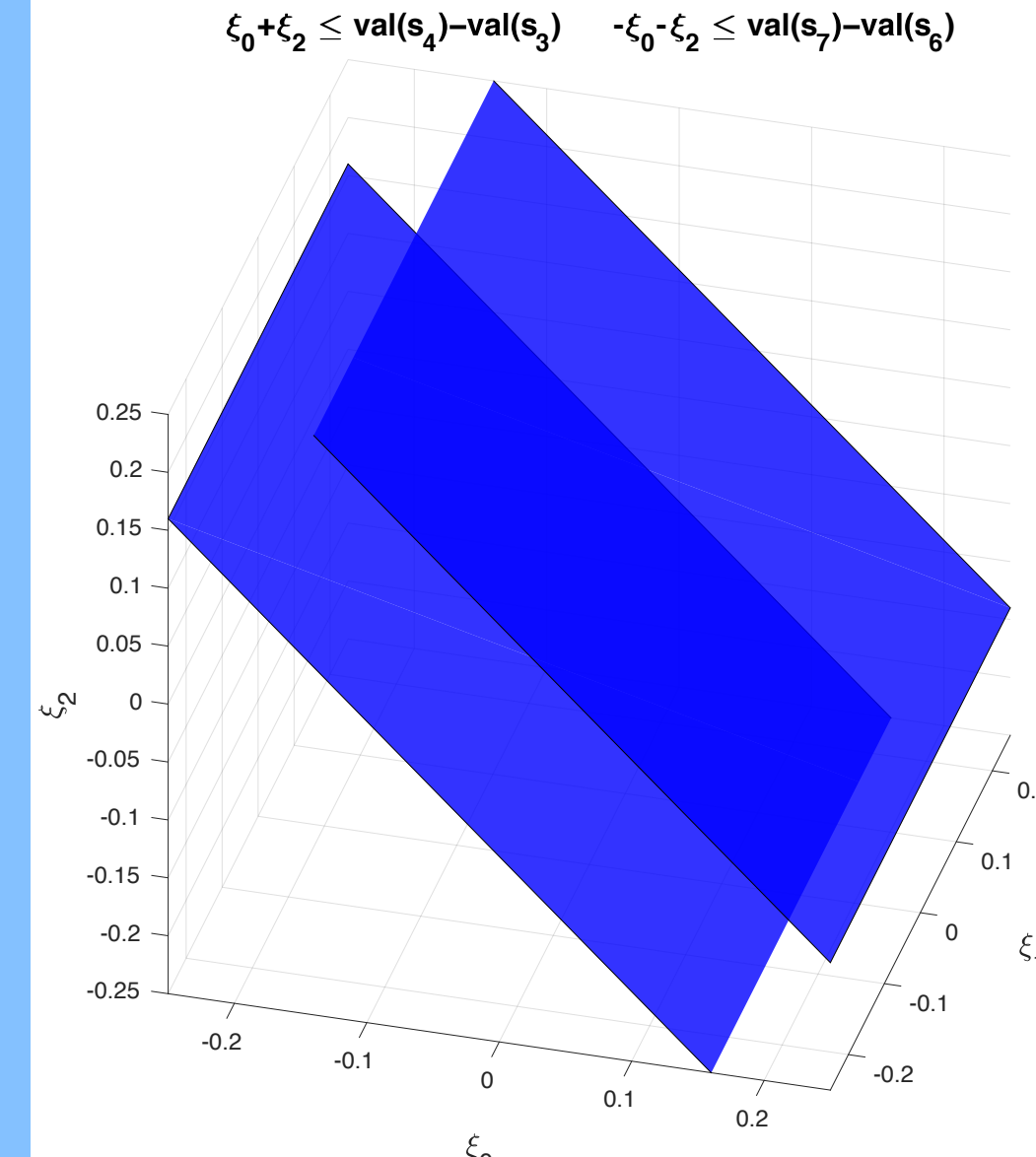
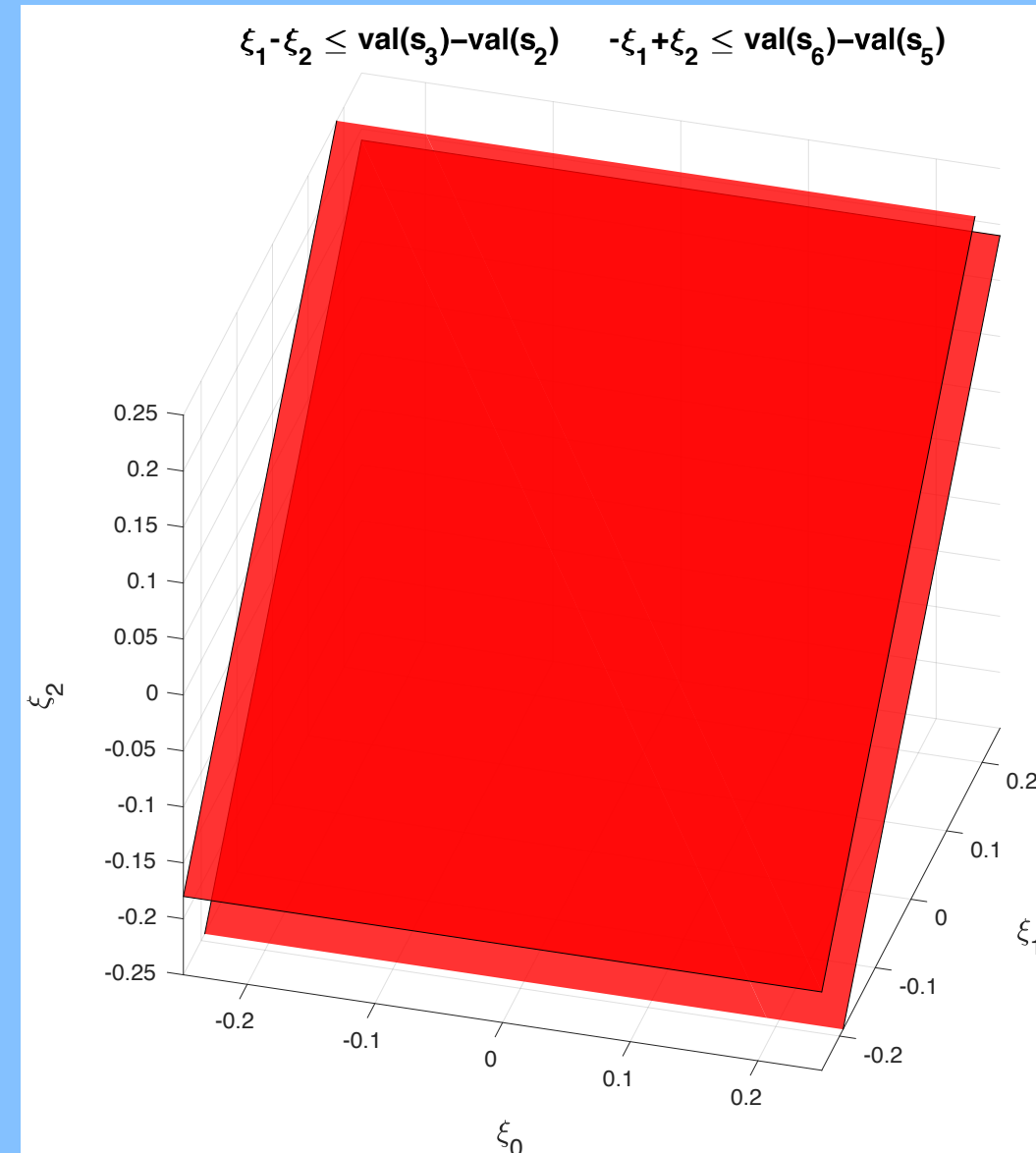
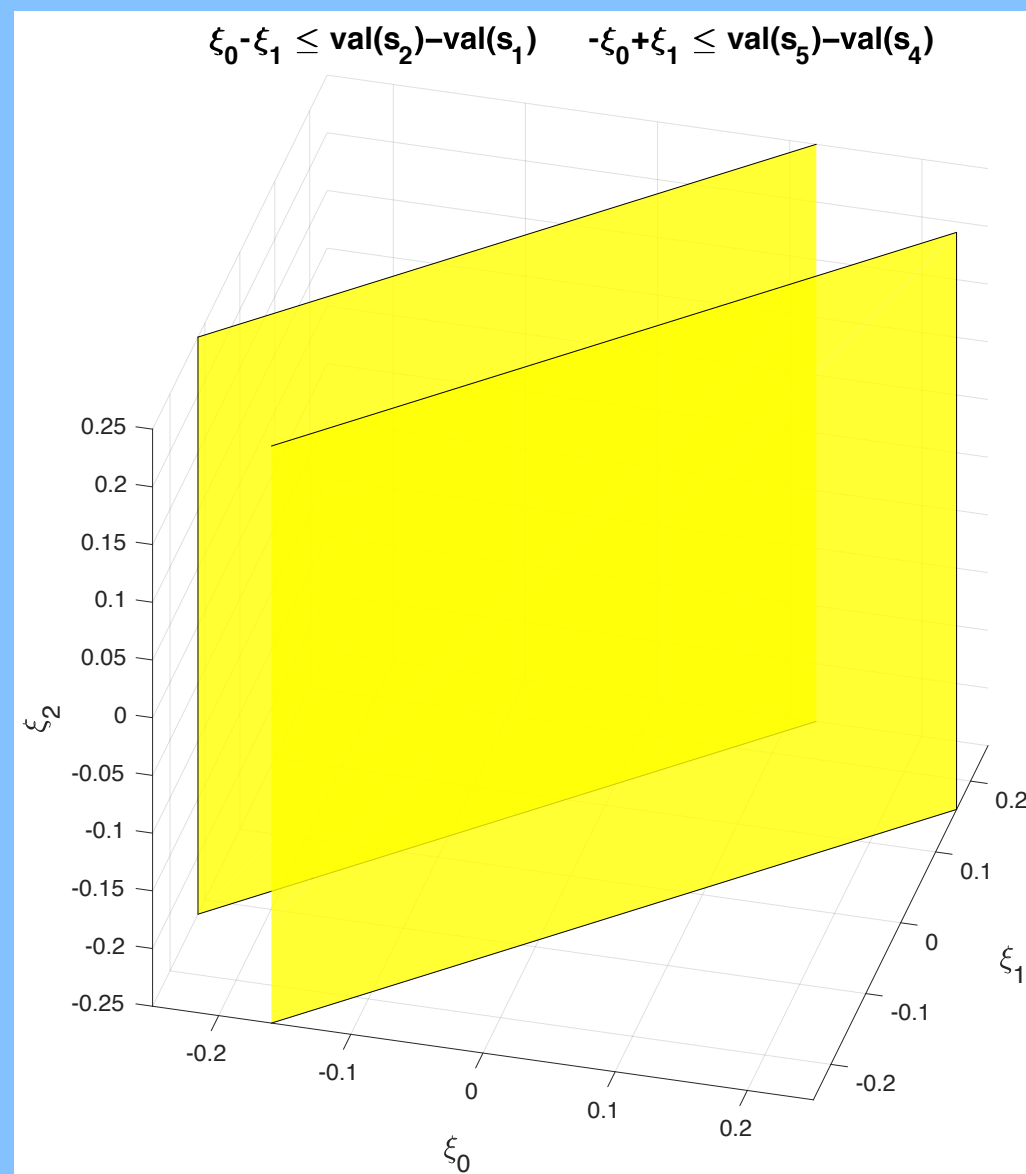
The worst case reconstruction between v'' and **every DB** in F_v is upper-bounded by $\frac{\text{diam}(F_v)}{2}$



UNORDERED RESPONSES APPROXIMATE RECONSTRUCTION*

Case k=3

F_v



k-NN queries $\rightarrow F_v$ is a polytope in k-dimensional space



EVALUATION

ORDERED & UNORDERED RESPONSES

1-31 October 2009



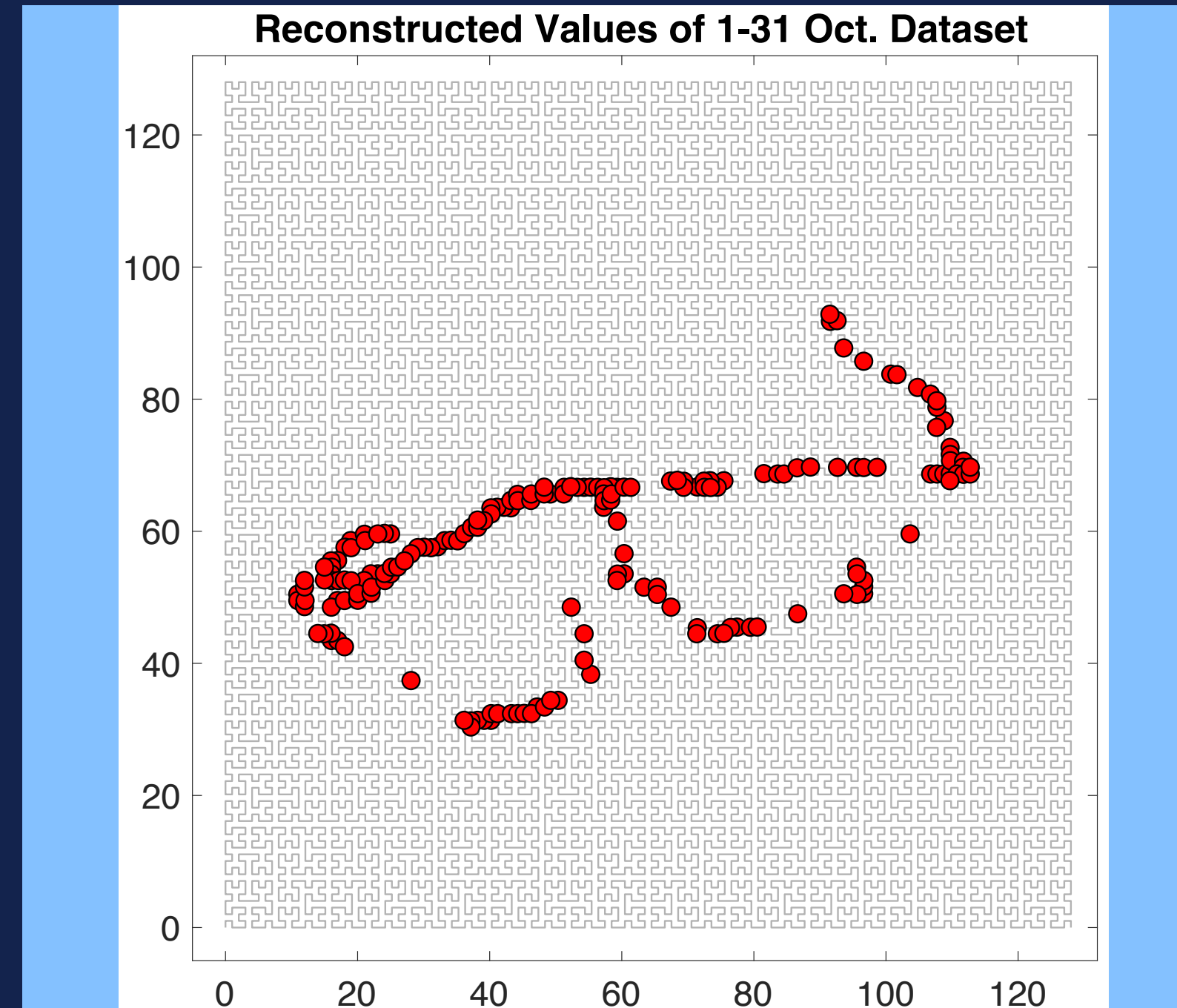
-Geolocation
of politician Spitz

-Simulated k-NN
Leakage from
queries on his
location DB



EVALUATION ORDERED & UNORDERED RESPONSES

1-31 October 2009



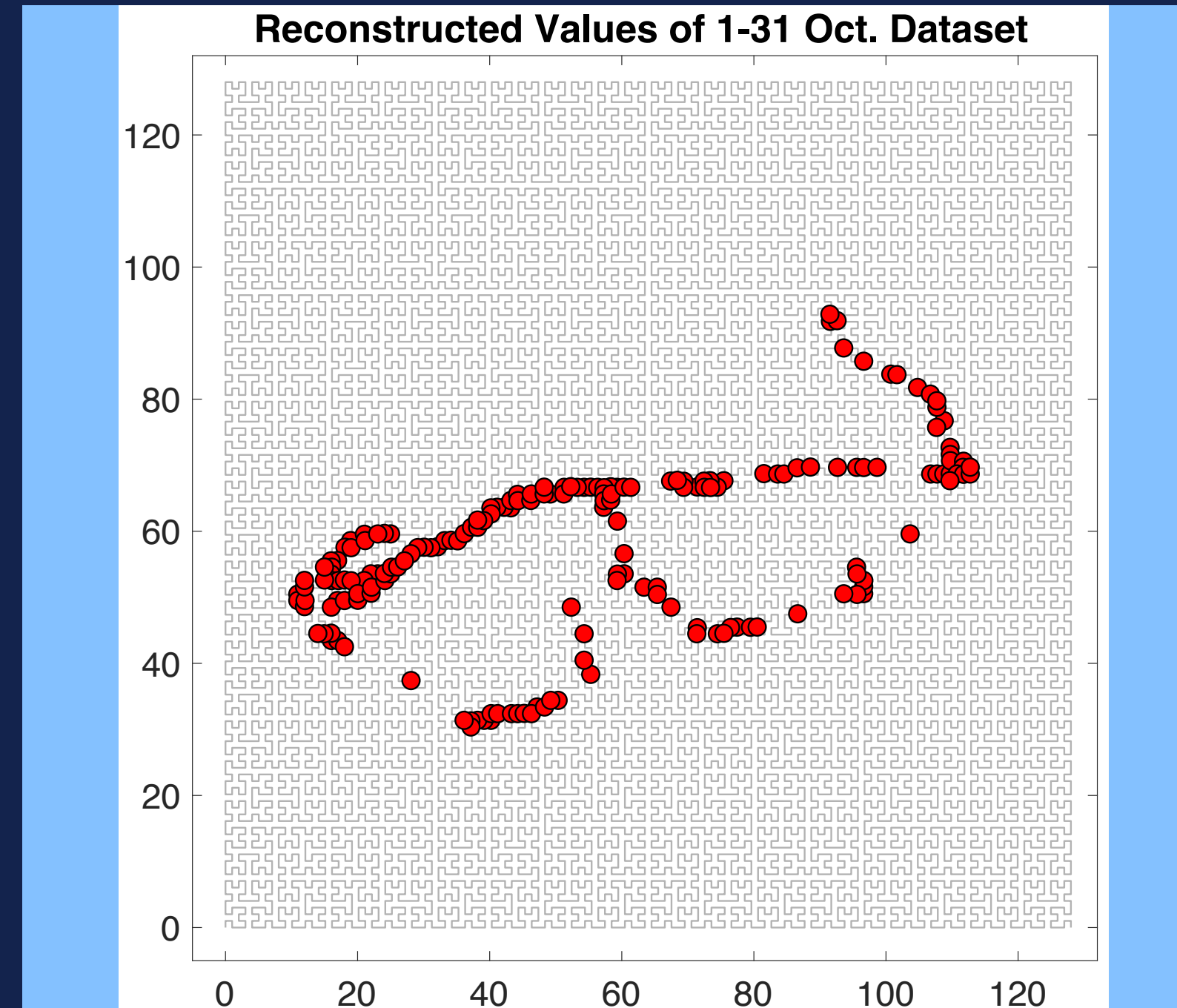
-Geolocation
of politician Spitz

-Simulated k-NN
Leakage from
queries on his
location DB



EVALUATION ORDERED & UNORDERED RESPONSES

1-31 October 2009



-Geolocation
of politician Spitz

-Simulated k-NN
Leakage from
queries on his
location DB

	1-31 October, $m = 250 \cdot 10^6$, $n = 183$		
	diameter	Absolute Error	Success
$k = 2$	1.8	1.0	70%
$k = 5$	6.4	1.4	95%
$k = 8$	12.8	1.4	95%



k-NN EXACT RECONSTRUCTION

ORDERED RESPONSES: Possible when all encrypted queries are issued

UNORDERED RESPONSES: Impossible due to many reconstructions

k-NN APPROXIMATE RECONSTRUCTION

ORDERED RESPONSES: Approximate reconstruction when not all encrypted queries are issued

UNORDERED RESPONSES: Even with many reconstructions approximate with bounded error

Thank you!

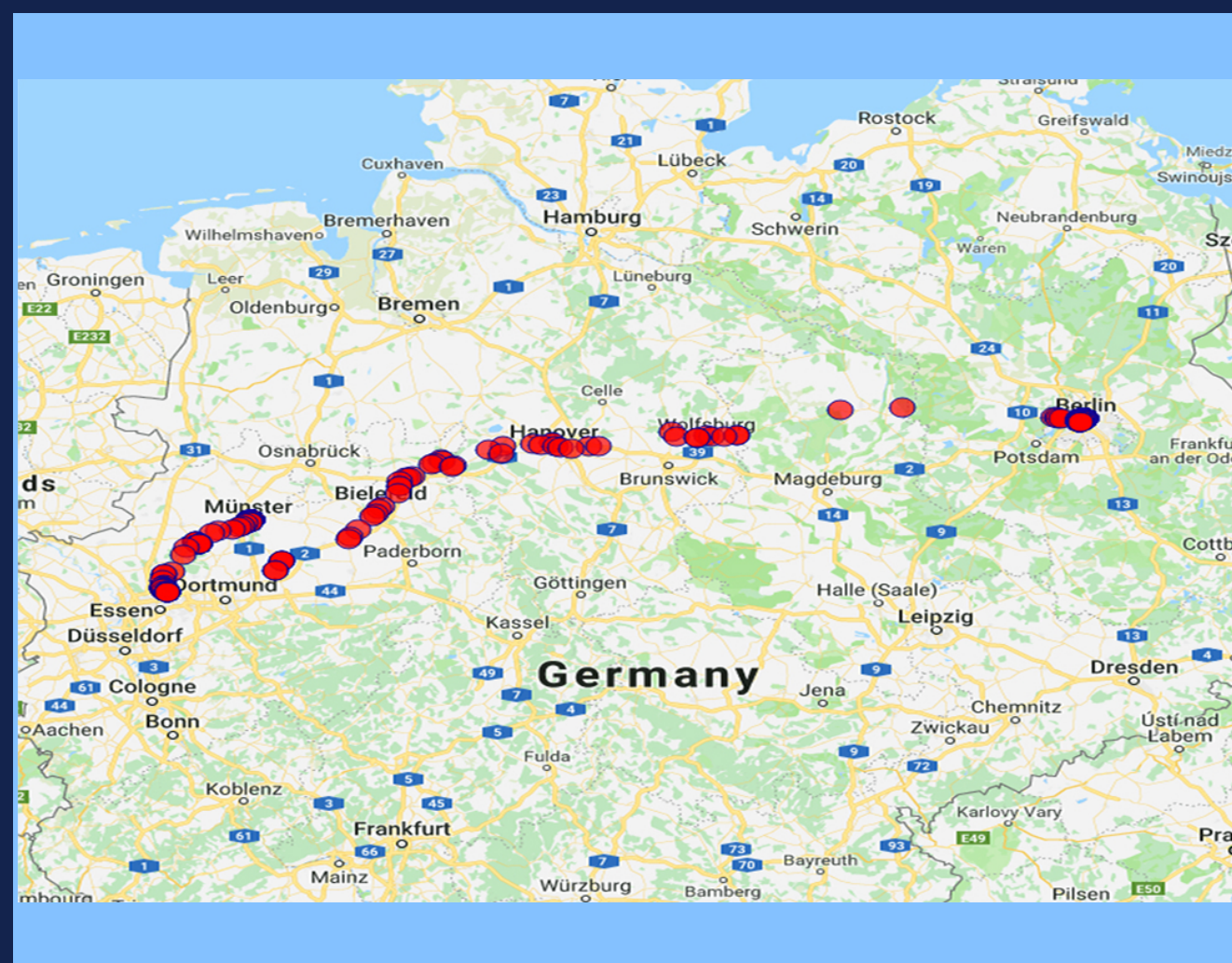


EVALUATION ORDERED & UNORDERED RESPONSES

1-5 October

1-15 October

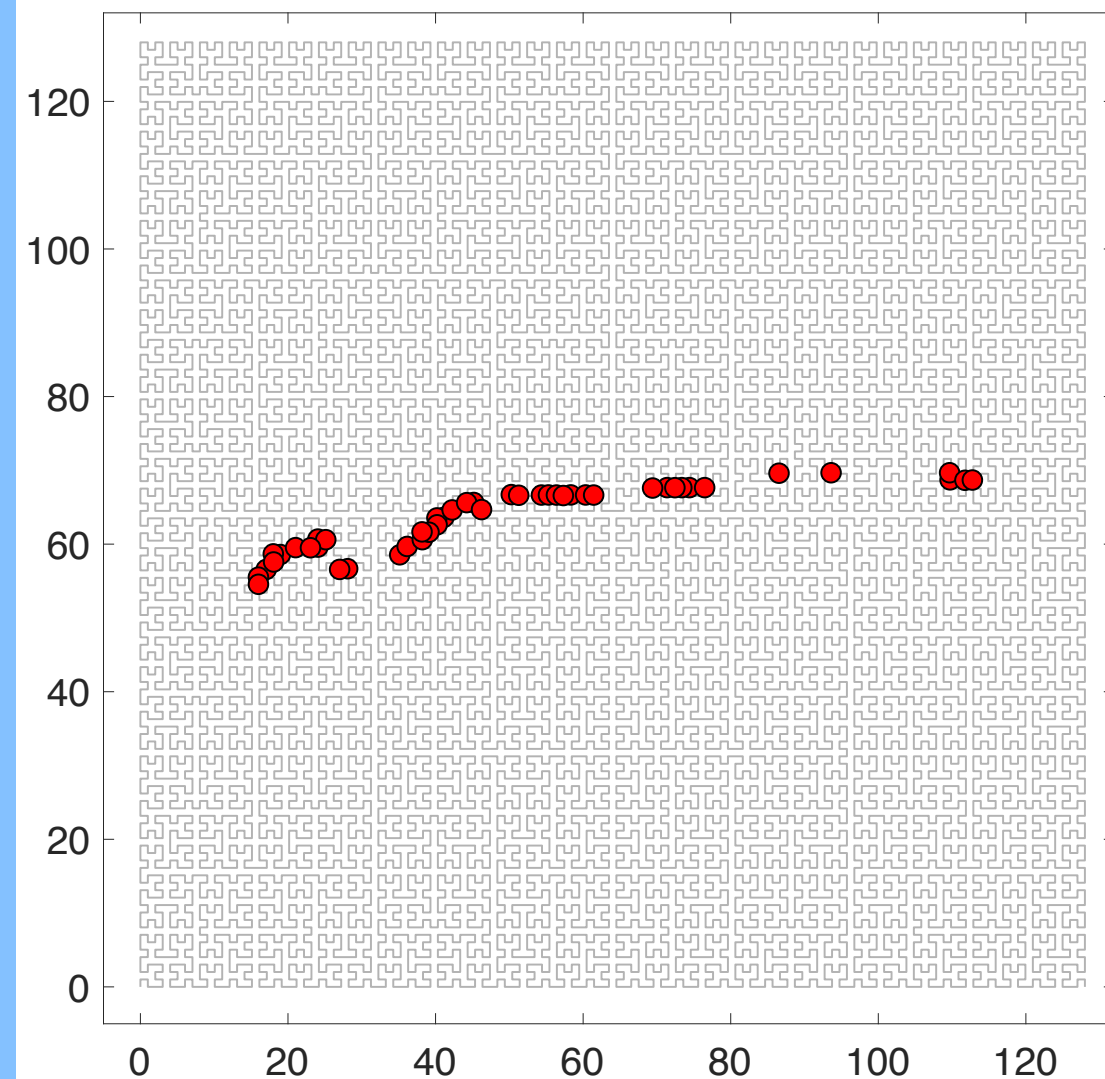
1-31 October



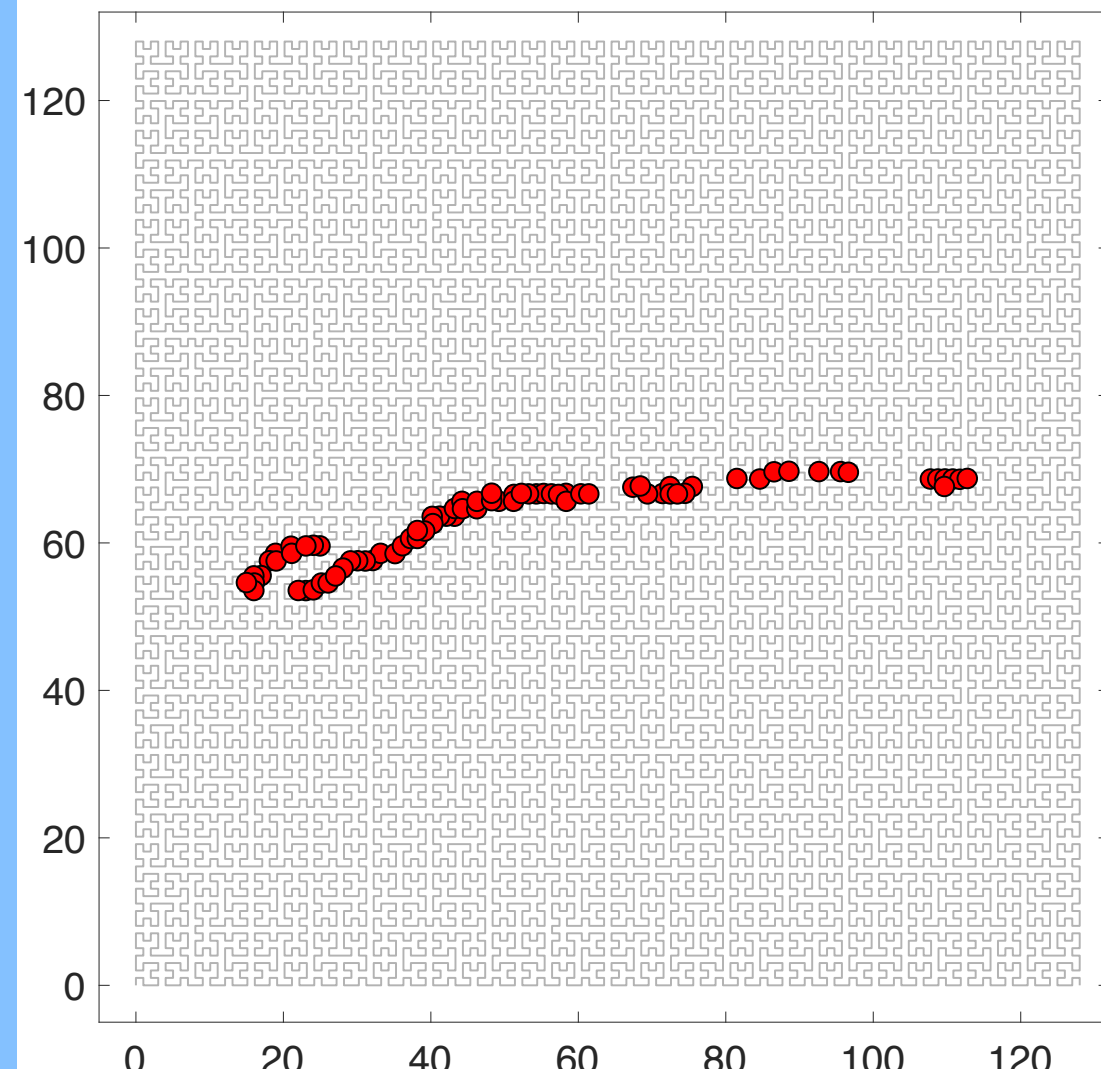
-Geolocation
of politician Malte Spitz

-Simulated k-NN
Leakage from queries
on his location DB

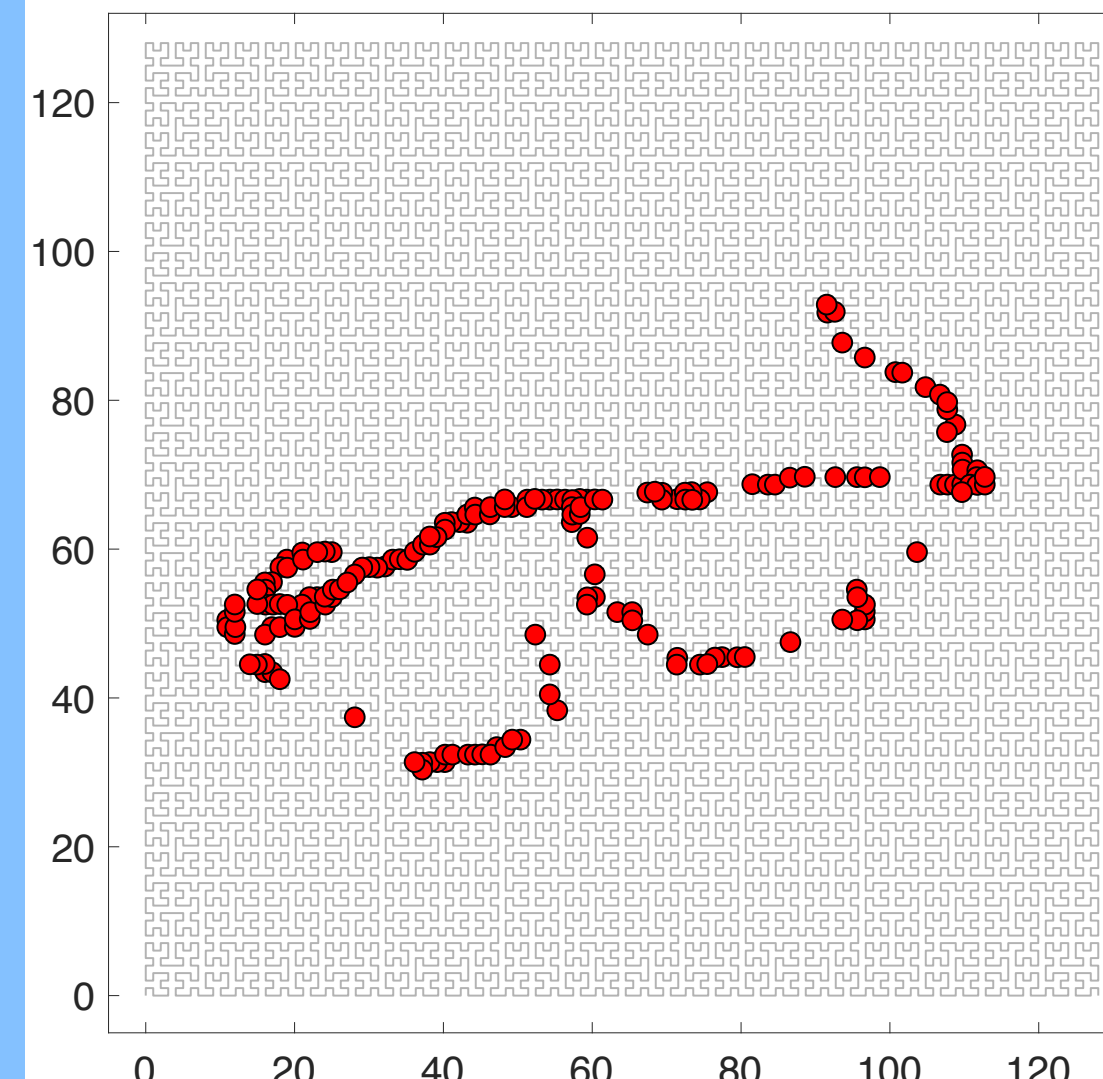
Reconstructed Values of 1-5 Oct. Dataset



Reconstructed Values of 1-15 Oct. Dataset



Reconstructed Values of 1-31 Oct. Dataset





EVALUATION UNORDERED RESPONSES

	1-5 October, $m = 25 \cdot 10^6$, $n = 46$								1-5 October, $m = 800 \cdot 10^6$, $n = 46$						
	diameter		Abs. Error-1D		Rel. Error-1D	Abs. Error-2D	Success		diameter		Abs. Error-1D		Rel. Error-1D	Abs. Error-2D	Success
	exact	est	avg	std	avg	max			exact	est	avg	std	avg	max	
$k = 2$	1.8	1.1	3.6	1.1	0.02%	3.0	40%		1.8	1.7	0.5	0.1	0.003%	0.9	100%
$k = 5$	18.3	17.9	5.7	1.6	0.03%	5.0	80%		18.3	18.3	3.4	0.2	0.02%	2.9	100%
$k = 8$	79.9	78.3	16.9	1.4	0.1%	7.4	100%		79.9	79.5	14.6	0.15	0.09%	6.5	100%
	1-15 October, $m = 70 \cdot 10^6$, $n = 79$								1-15 October, $m = 800 \cdot 10^6$, $n = 79$						
	diameter		Abs. Error-1D		Rel. Error-1D	Abs. Error-2D	Success		diameter		Abs. Error-1D		Rel. Error-1D	Abs. Error-2D	Success
	exact	est	avg	std	avg	max			exact	est	avg	std	avg	max	
$k = 2$	1.9	0.8	1.8	0.7	0.010%	3.0	45%		1.9	1.4	0.6	0.1	0.003%	0.8	100%
$k = 5$	6.6	6.0	1.9	0.6	0.011%	2.5	80%		6.6	6.7	0.6	0.2	0.003%	1.3	100%
$k = 8$	15.4	14.6	2.5	0.6	0.015%	2.9	80%		15.4	15.1	1.0	0.1	0.006%	1.2	100%
	1-31 October, $m = 250 \cdot 10^6$, $n = 183$								1-31 October, $m = 800 \cdot 10^6$, $n = 183$						
	diameter		Abs. Error-1D		Rel. Error-1D	Abs. Error-2D	Success		diameter		Abs. Error-1D		Rel. Error-1D	Abs. Error-2D	Success
	exact	est	avg	std	avg	max			exact	est	avg	std	avg	max	
$k = 2$	1.8	1.0	1.0	0.2	0.006%	1.4	70%		1.8	1.1	0.7	0.1	0.004%	1.0	95%
$k = 5$	6.4	5.0	1.4	0.3	0.008%	2.0	95%		6.4	5.6	0.7	0.1	0.004%	1.1	100%
$k = 8$	12.8	11.6	1.4	0.3	0.008%	2.0	95%		12.8	12.2	0.8	0.2	0.004%	1.0	100%