Relational Analysis of Sensor Attacks on Cyber-Physical Systems

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Sensor Attacks

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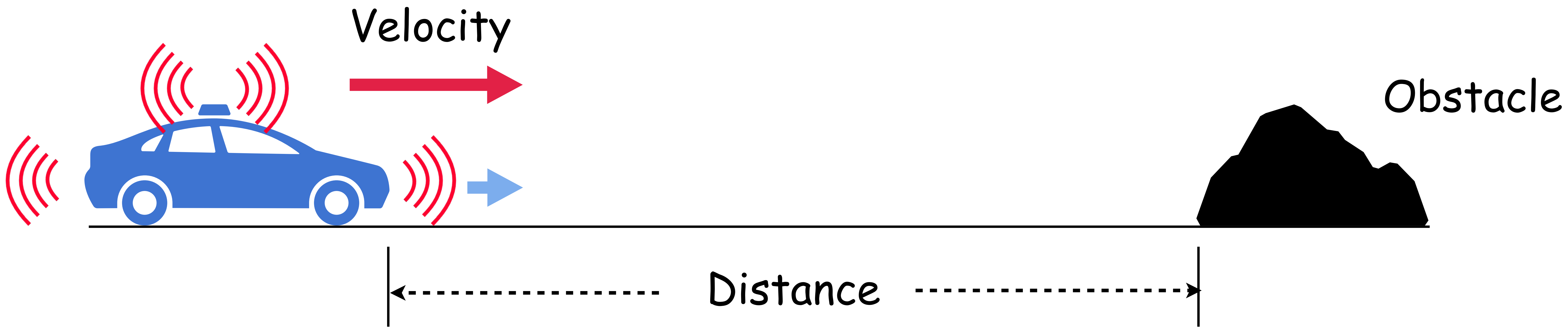

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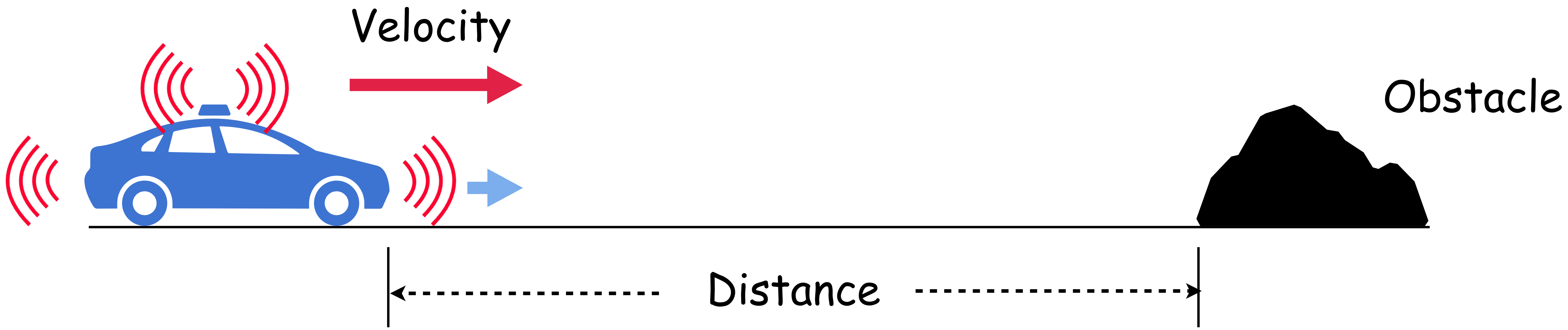
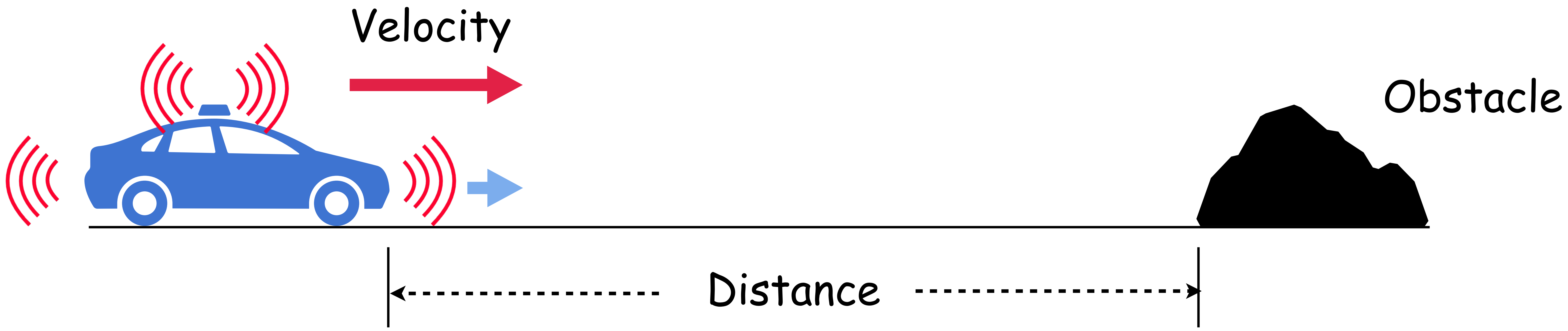


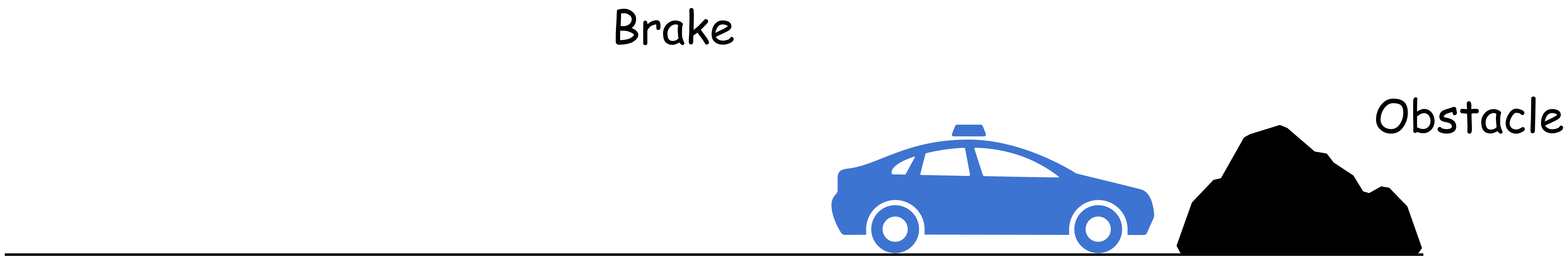
accelerate or brake?

Sensor Attacks



accelerate or brake? Decision is calculated based on sensed velocity and distance

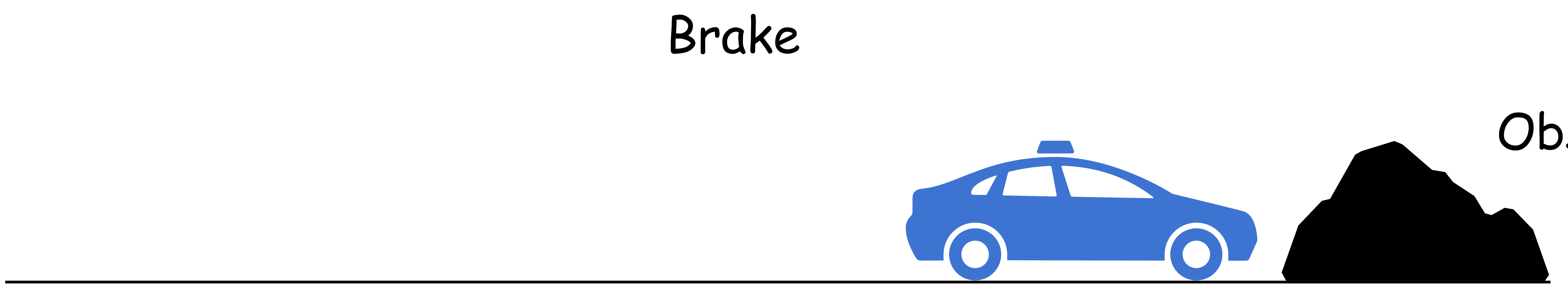
Sensor Attacks



Safe operation

accelerate or brake? Decision is calculated based on sensed velocity and distance

Sensor Attacks



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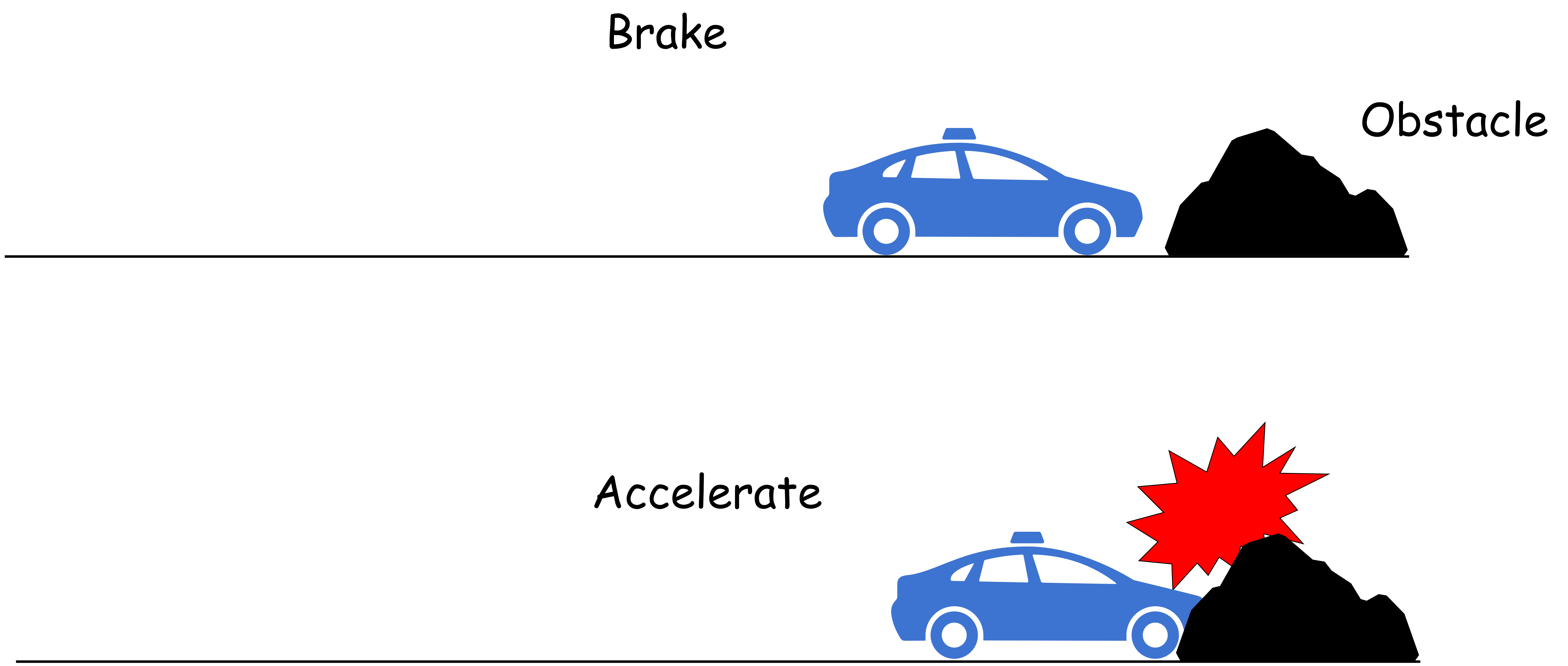

Safe operation

Sensor compromised

accelerate or brake? Decision is calculated based on sensed velocity and distance

Our View

1. Formal approach is needed
2. Understanding the impact of a compromised sensor is important
3. Need to reason about relational properties

Contributions

A formal framework to model and analyze sensor attacks on cyber-physical systems

Methodology

Contributions

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A threat model

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A formal framework to model and analyze sensor attacks on cyber-physical systems

A threat model	→	Two robustness properties

Methodology

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A threat model → Two robustness properties → An equivalence relation

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A threat model → Two robustness properties → An equivalence relation → Two proof techniques

Methodology

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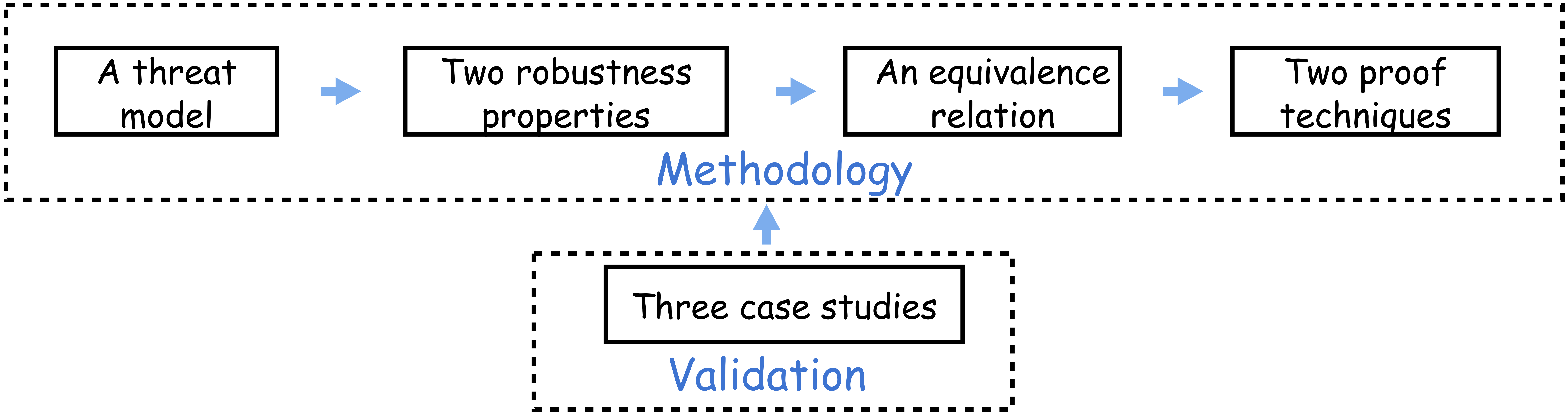
A formal framework to model and analyze sensor attacks on cyber-physical systems



Real-valued terms θ	
x	Program variable
c	Constant
$\theta_1 \oplus \theta_2$	Computation on terms

Hybrid Program α, β	
$x := \theta$	Deterministic assignment
$x := *$	Nondeterministic assignment
$x' = \theta \& \phi$	Continuous evolution
$?\phi$	Test if formula ϕ is true
$\alpha; \beta$	Sequential composition
$\alpha \cup \beta$	Nondeterministic choice
α^*	Nondeterministic repetition

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General form: (ctrl; plant)*

Differential Dynamic Logic ϕ, ψ

$\theta_1 \sim \theta_2$	Comparison between terms
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Safety: $\phi \rightarrow [\alpha]\psi$

Example Hybrid Program Model

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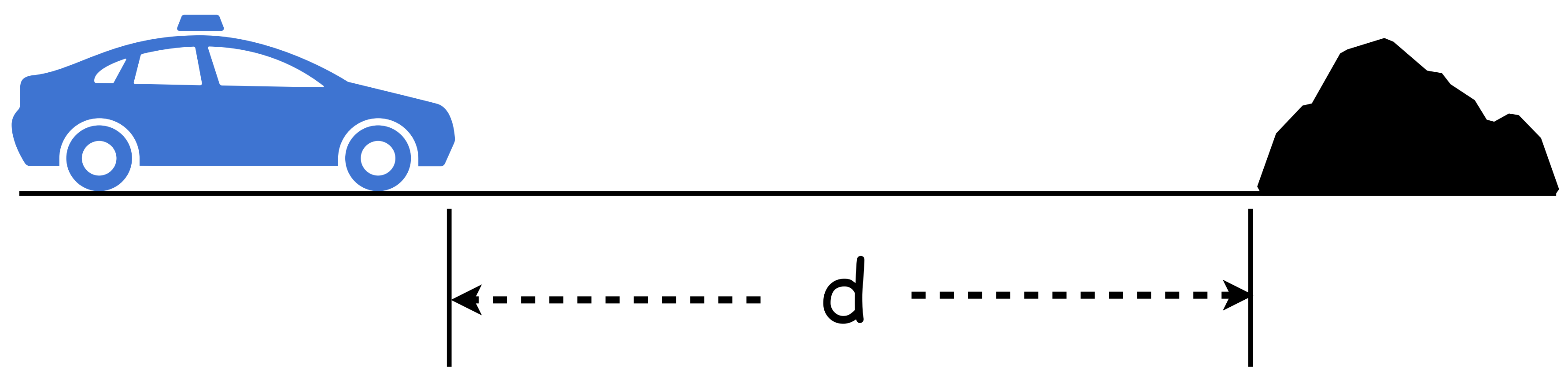

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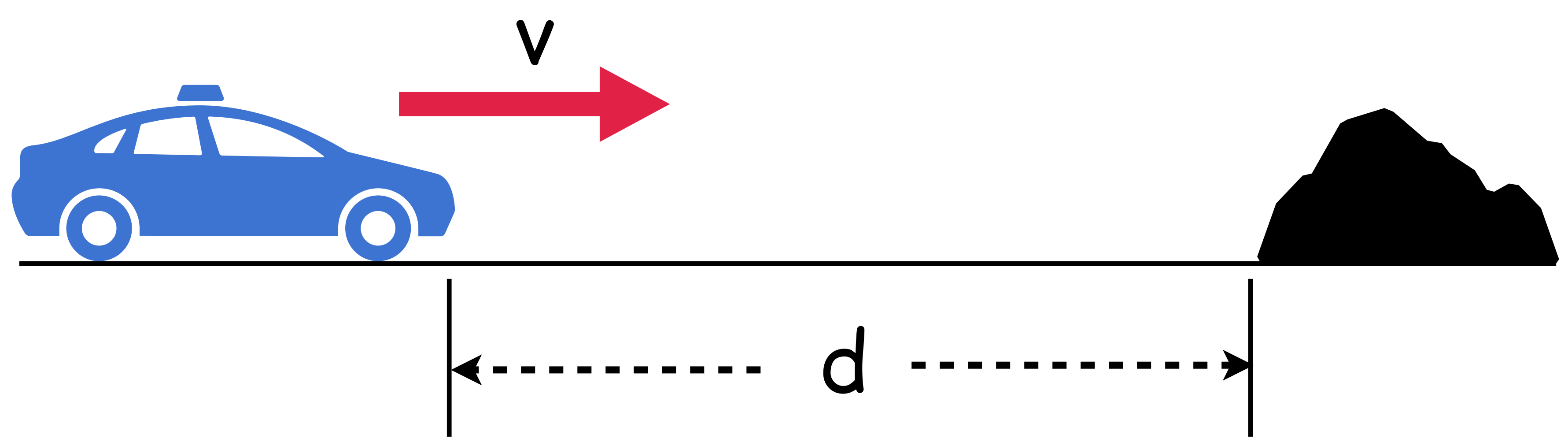

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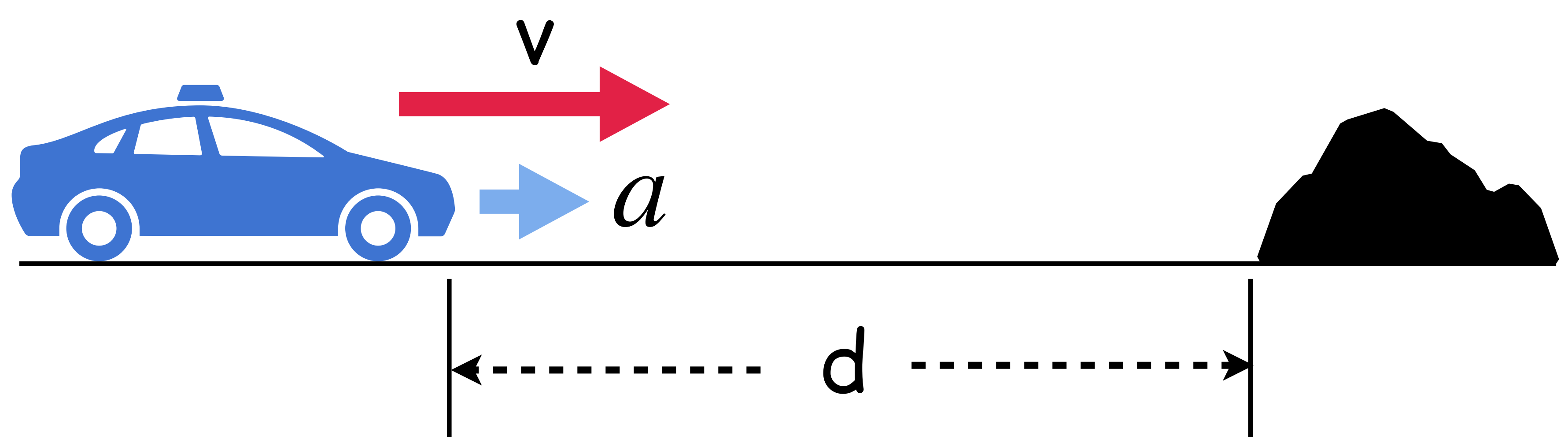

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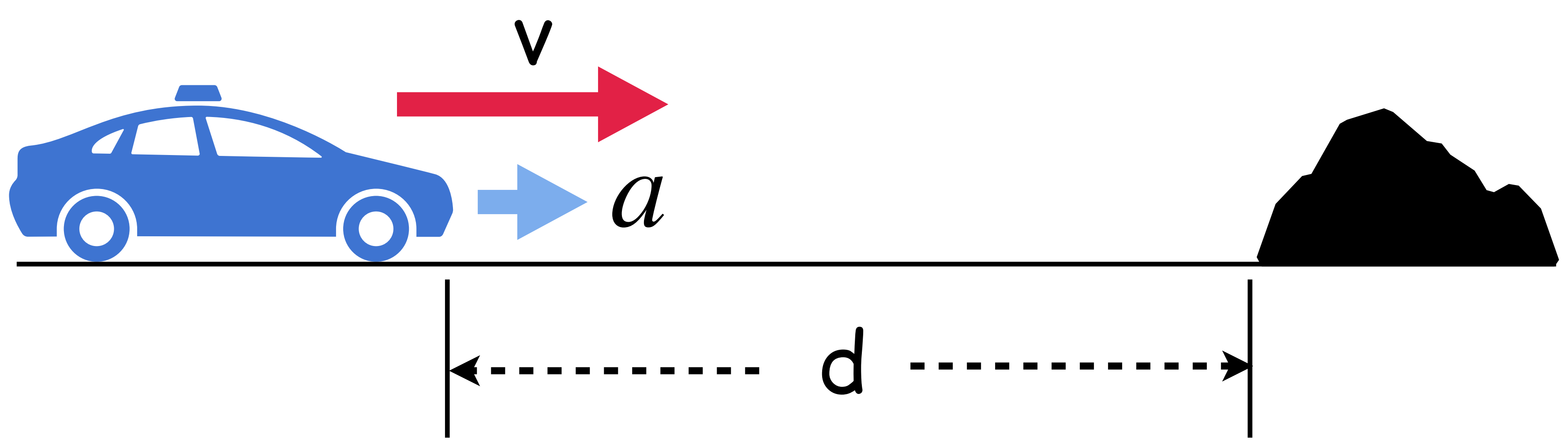

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t : clock variable

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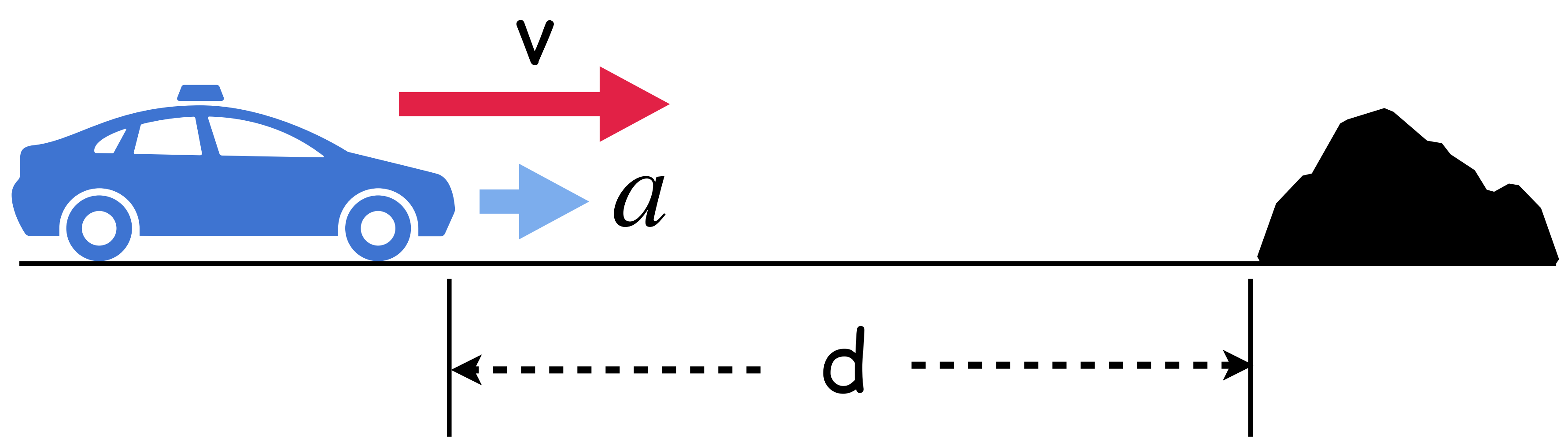

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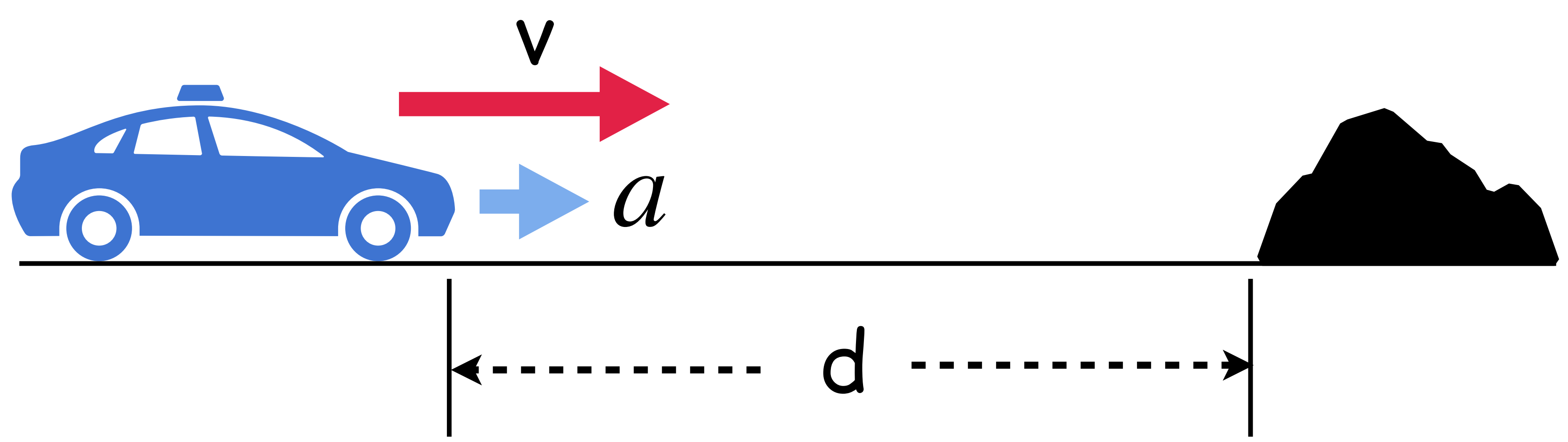

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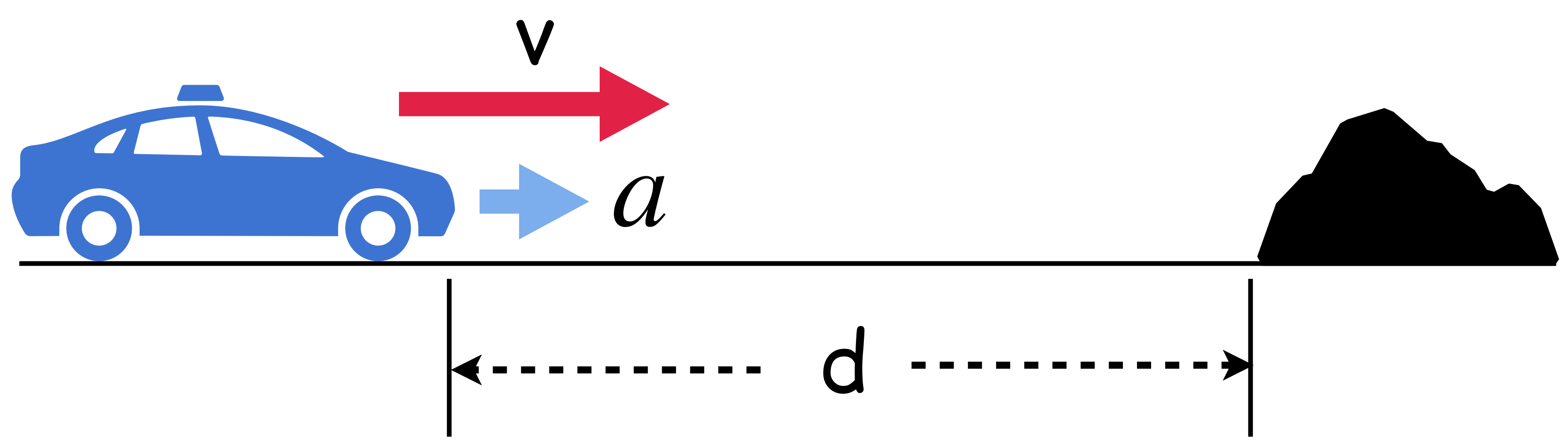

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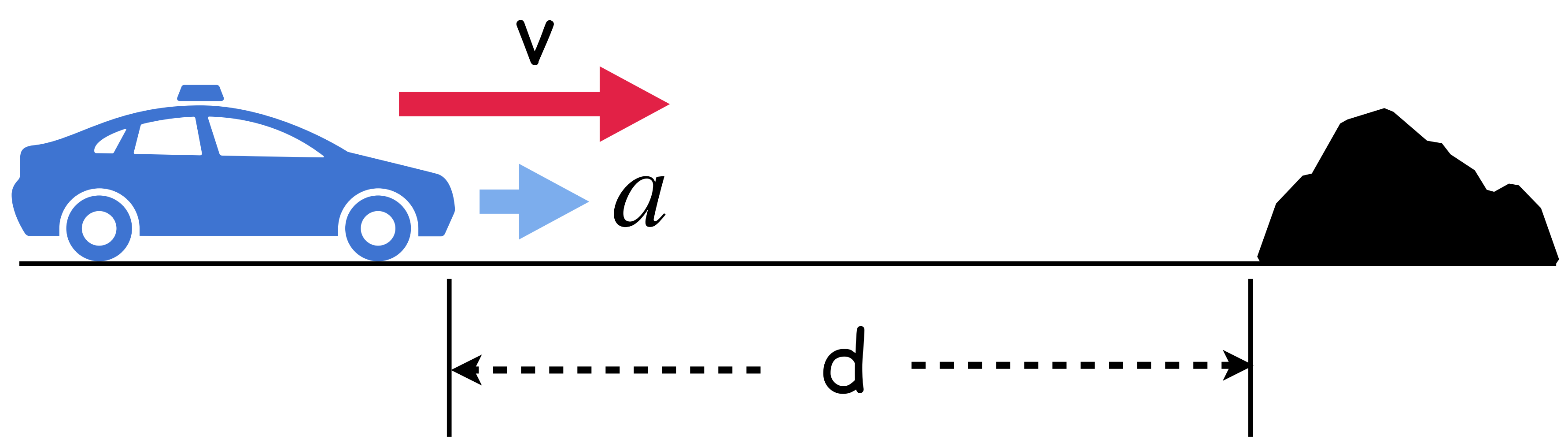

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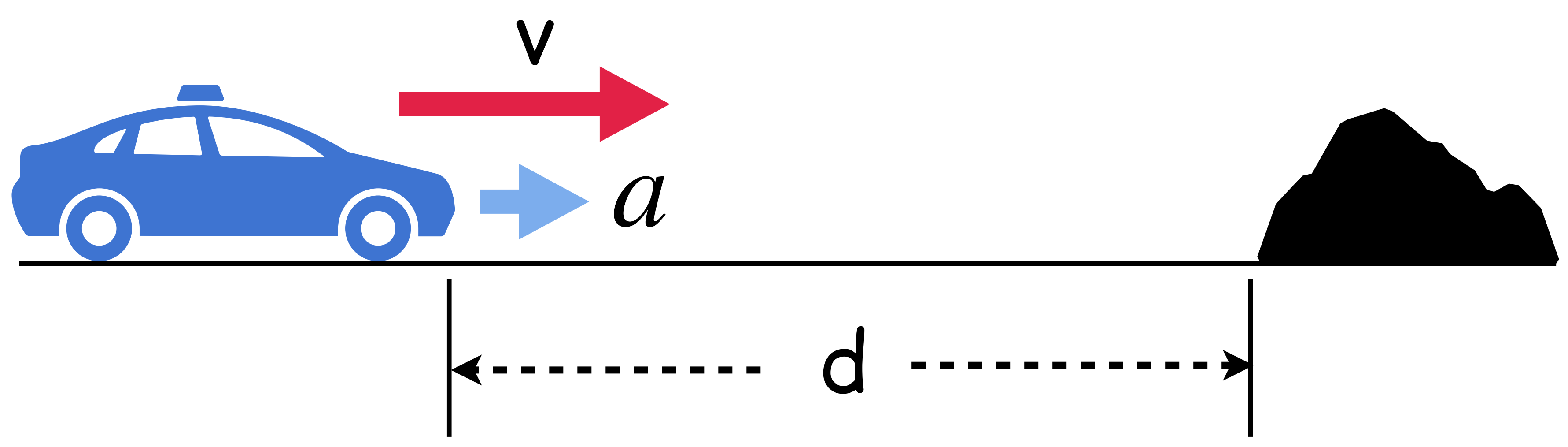

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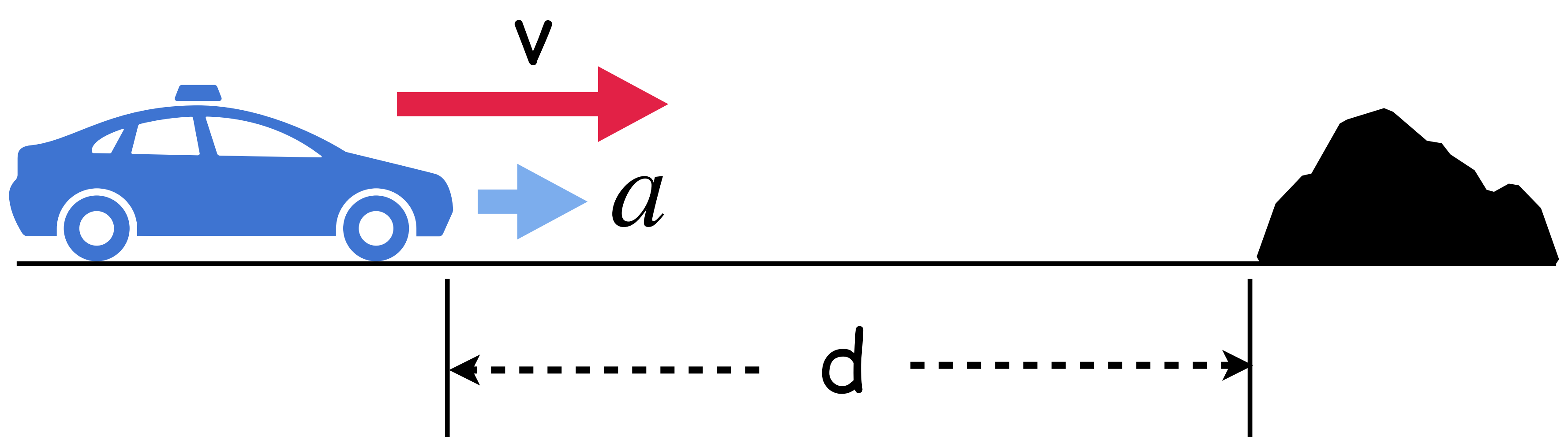

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$$(A \geq 0 \wedge B \geq 0 \wedge 2Bd > v^2) \rightarrow [(\text{ctrl}; \text{plant})^*]$$

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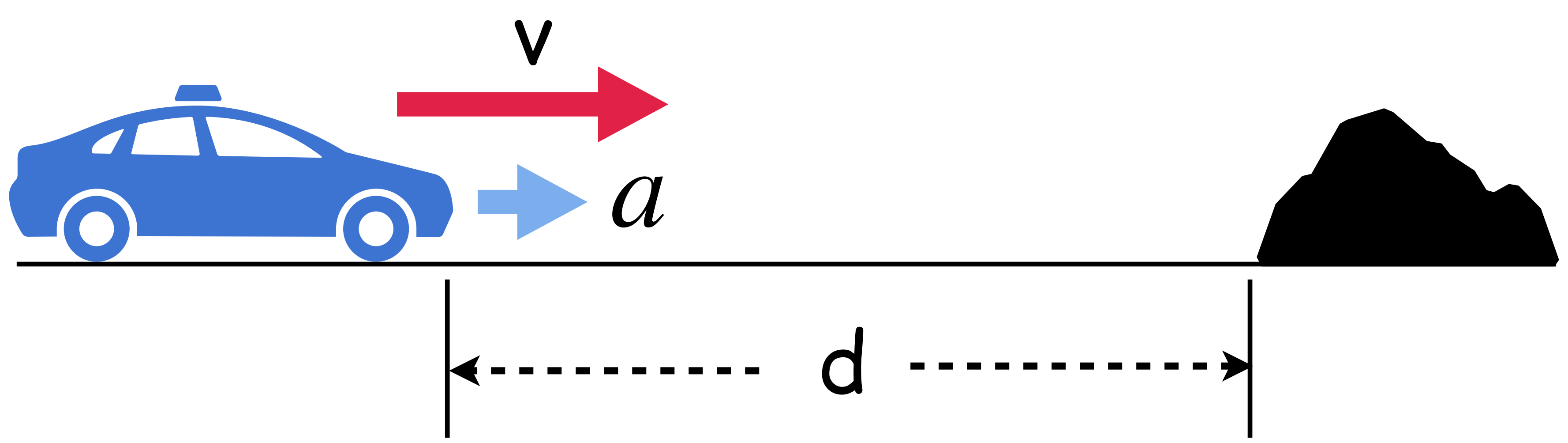

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$$(A \geq 0 \land B \geq 0 \land 2Bd > v^2) \rightarrow [(\text{ctrl};\text{plant})^*]\ (d > 0)$$

Example with Sensing

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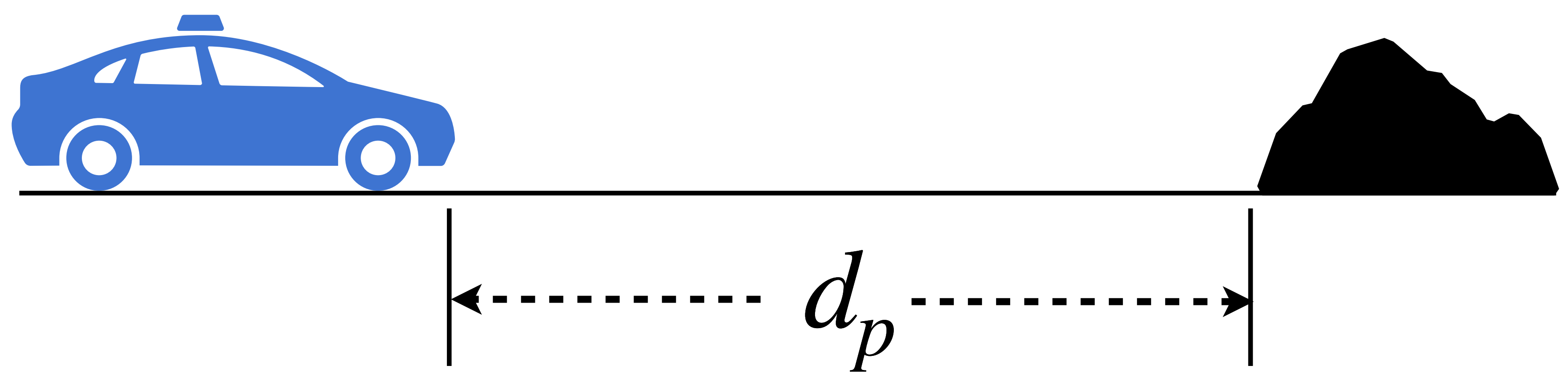
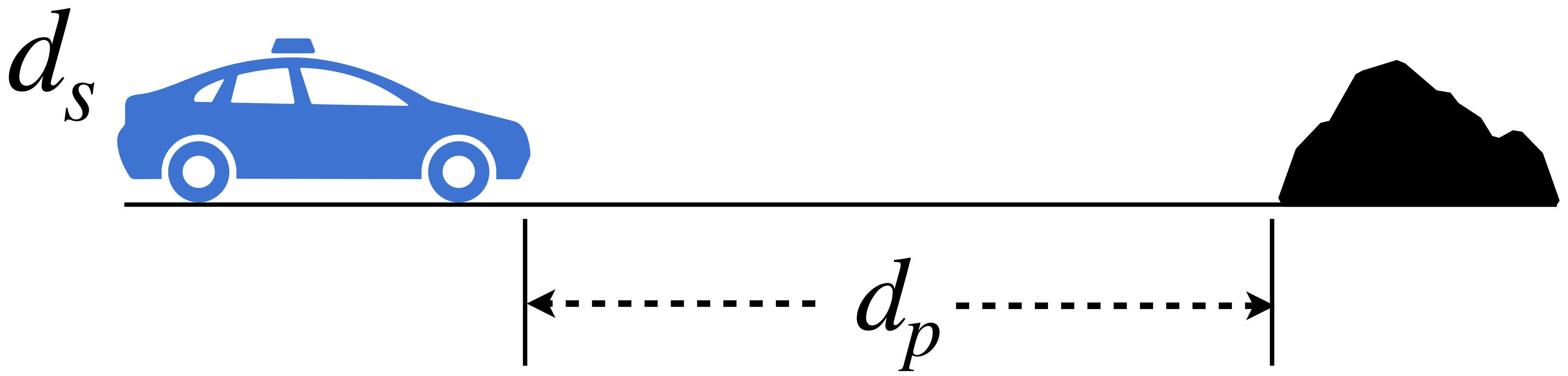


d_p : distance (physical)

Example with Sensing



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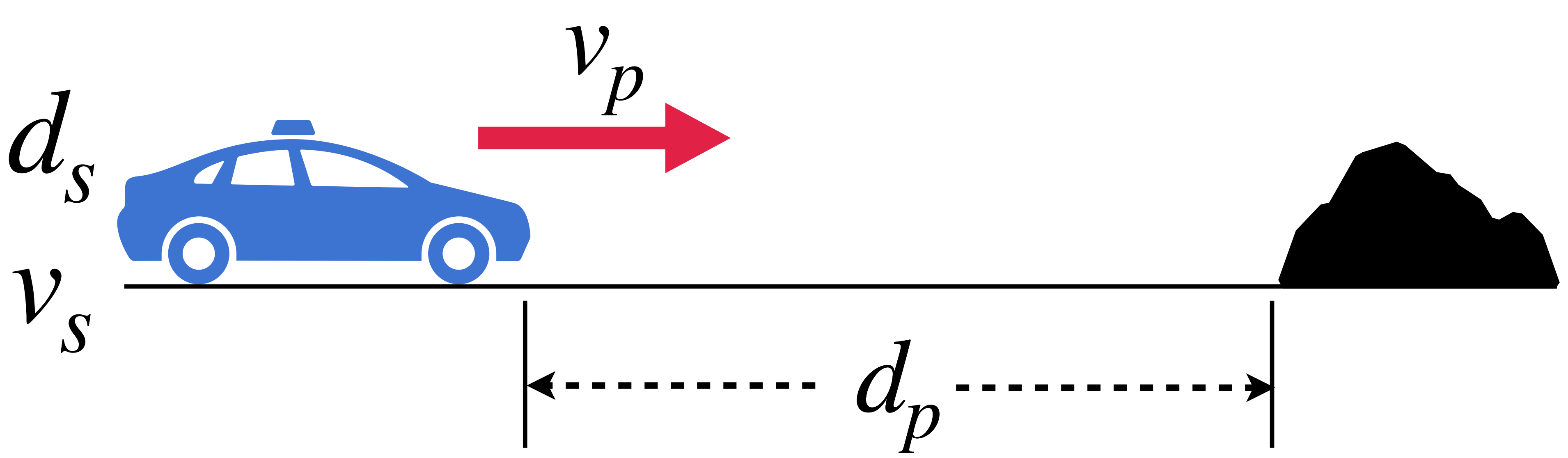

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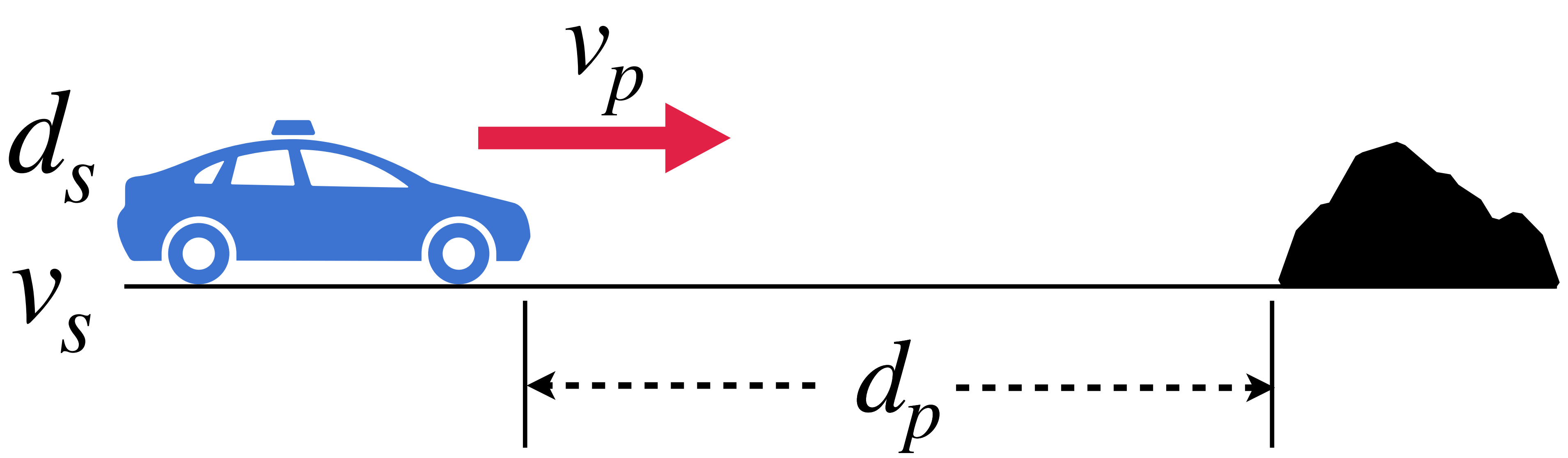

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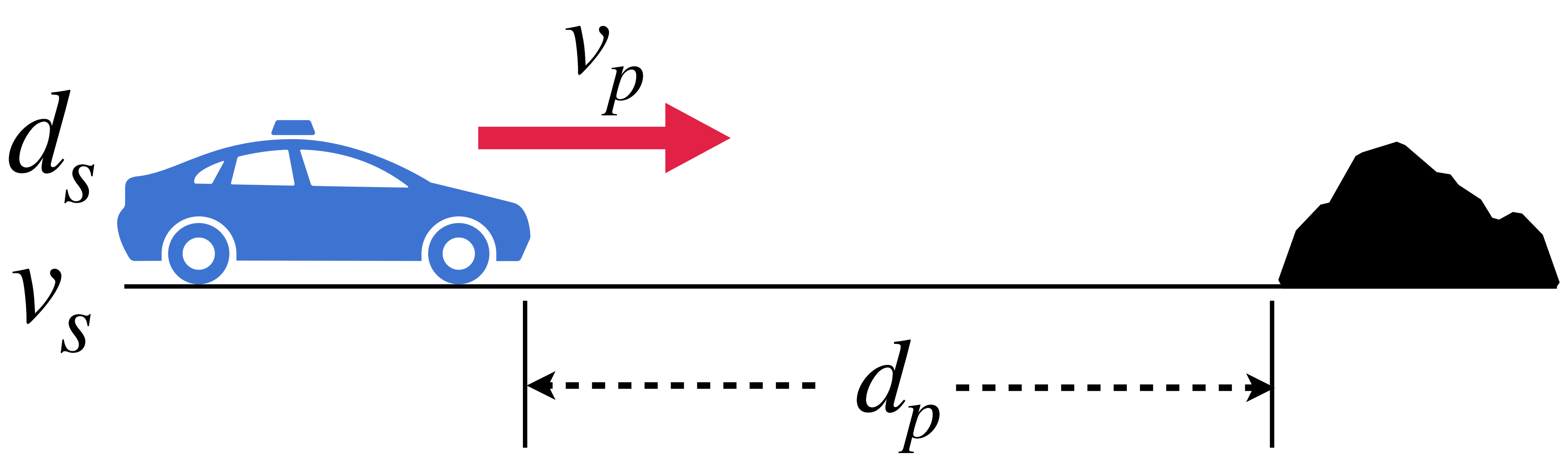

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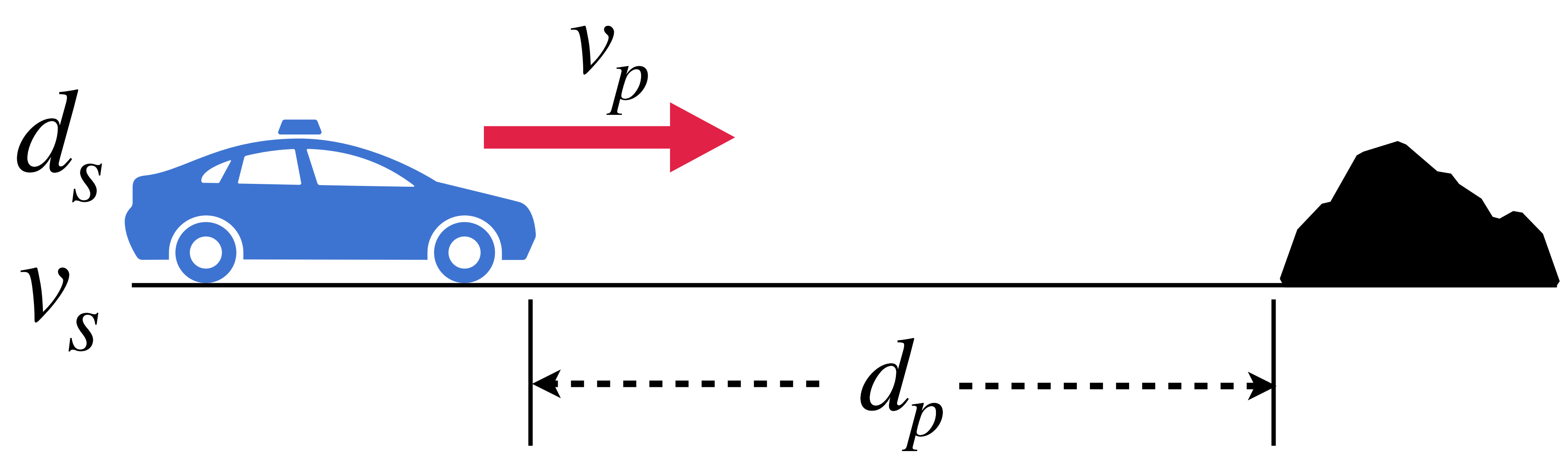

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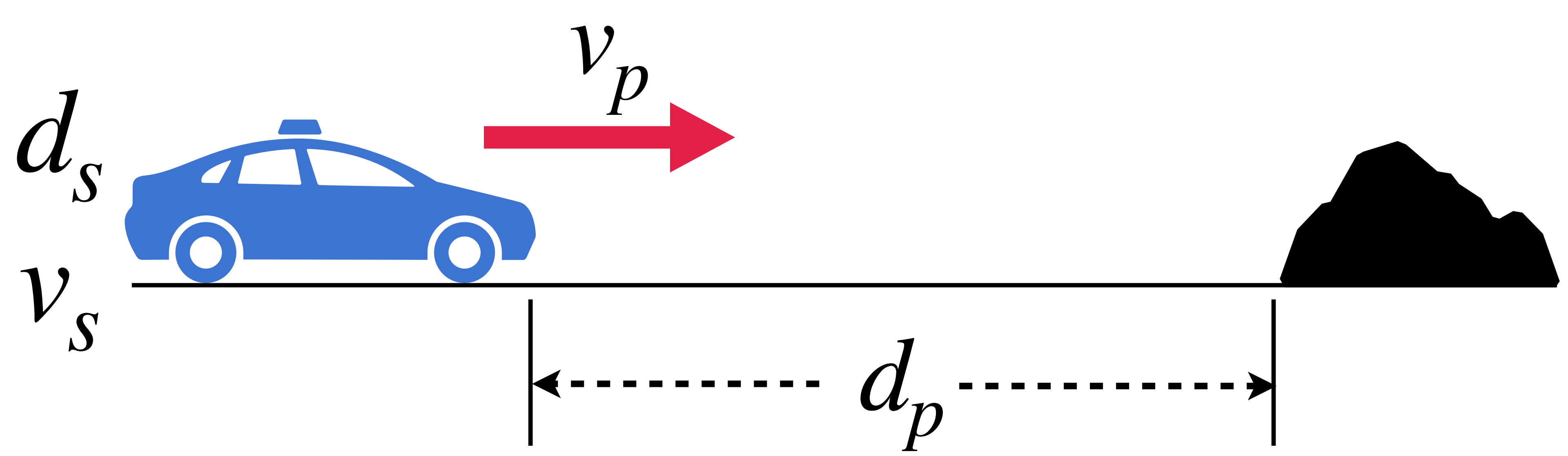

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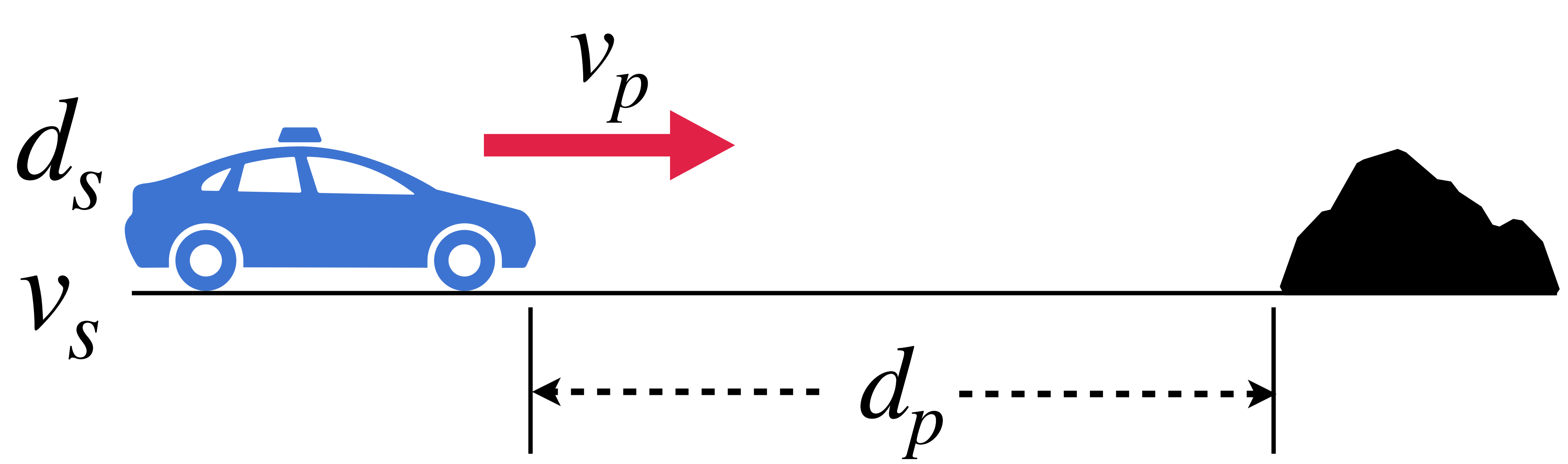

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$$(A \geq 0 \land B \geq 0 \land 2Bd_p > v_p^2) \quad \rightarrow [(\text{ctrl};\text{plant})^*]\ (d_p > 0)$$

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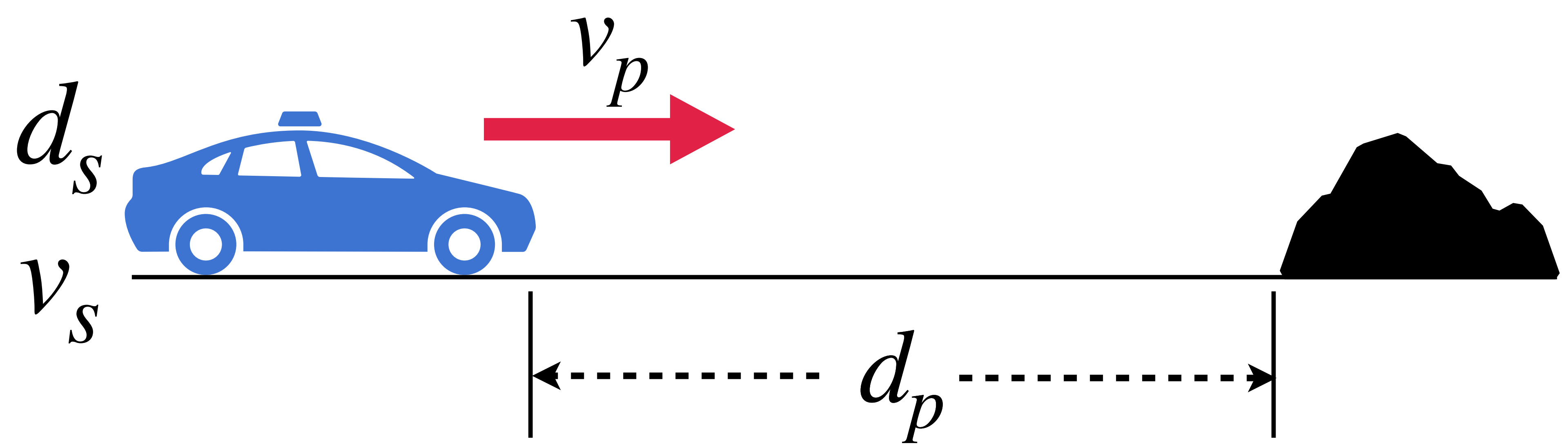

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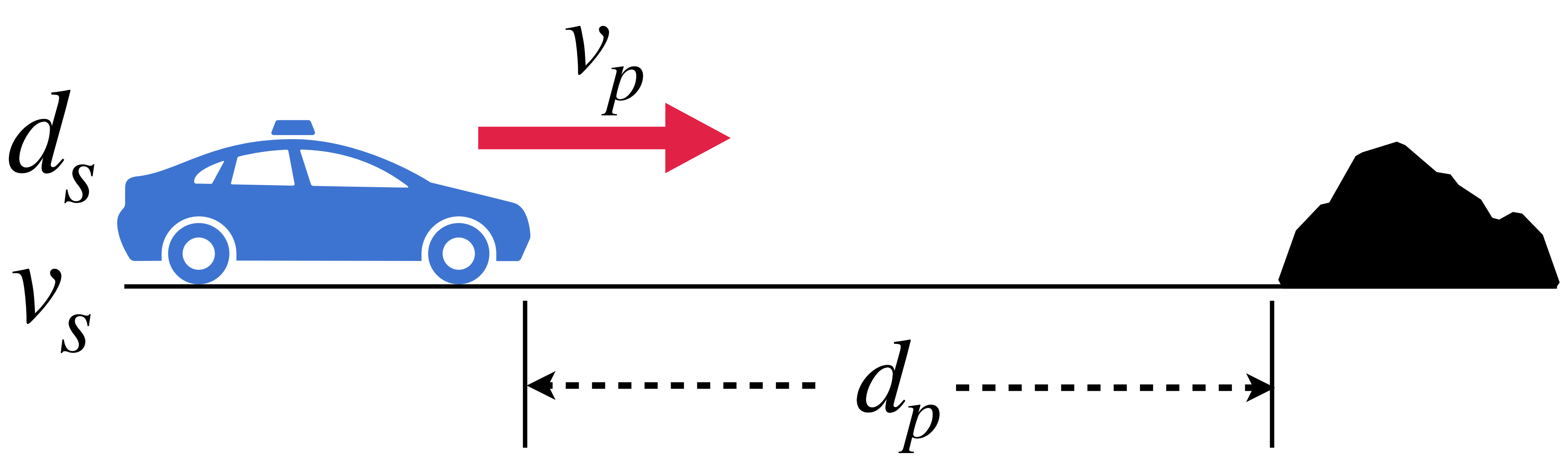

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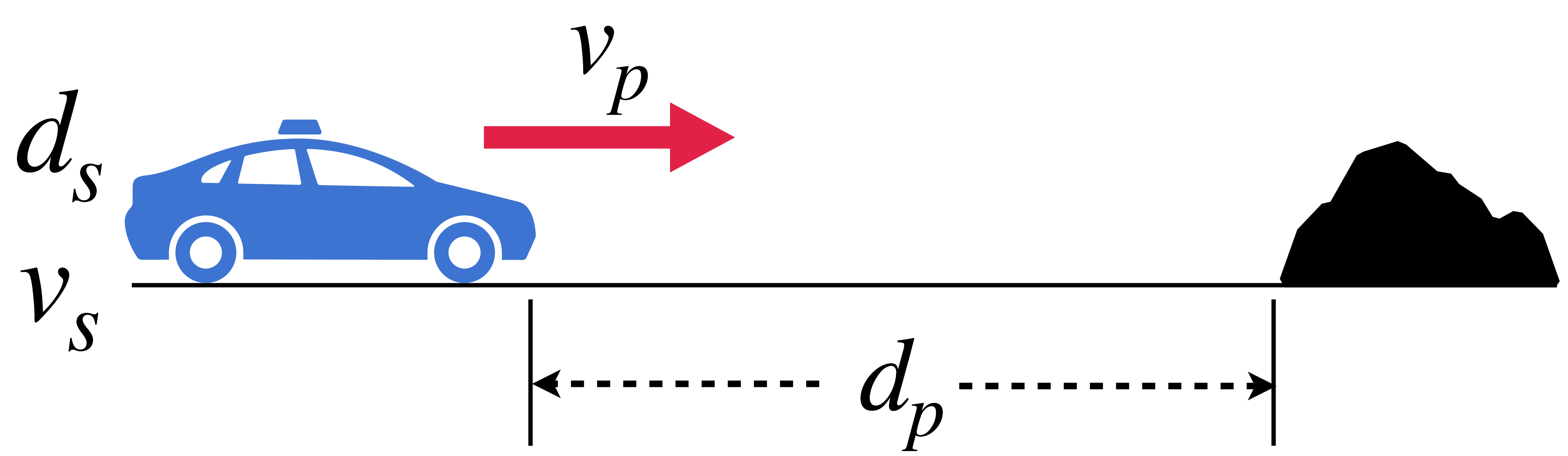

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Two Robustness Properties

P

$P_{Attacked}$

Two Robustness Properties

P

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Robustness of high-integrity state

H-Equivalence of Two Programs

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$$\begin{tikzcd}
\omega_0 \arrow[d, "\alpha"'] & \nu_0 \arrow[d, bend left, "\beta"] \\
\omega_1 \arrow[d, "\alpha"'] & \nu_1 \arrow[d, bend left, "\beta"] \\
\omega_2 \arrow[d, "\alpha"'] & \nu_2 \arrow[d, bend left, "\beta"] \\
\omega_3 & \nu_3 \\
\vdots & \vdots
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\omega_3 & \approx_H & \nu_3 \\
\vdots & & \vdots
\end{tikzcd}
$$

$\omega_i \approx_H \nu_i$ Two states agree on values of all variables in set H

Proving H-equivalence: Self-Composition

Proving: $P \approx_H P_{Attacked}$

Challenges:

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1. Non-determinism,
   e.g., $accel \cup brake$

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Proving: $P \approx_H P_{Attacked}$

Challenges:

1. Non-determinism, e.g., $accel \cup brake$

2. Duration of continuous evolution, e.g., $d' = -v, v' = a \,\&\, (v \geq 0 \wedge t \leq \epsilon)$

Main Results & Future Work

1. A formal threat model
2. Two robustness properties
3. An equivalence relation for reasoning robustness
4. Two proof techniques
5. Three case studies

Future Work:

We are working on a more expressive relational logic

Thank you!