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A Quantale of Information

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Information Flow



= Dependency



w depends on y and z

Conjunctive Dependency



w depends on y and z

Conjunctive Dependency



w depends on y and z

Fine-Grained Dependency



w depends on y and the first 5 minutes of z

Conditional Dependency



The Lattice of Information

Various semantic models of information flow use equivalence relations to model information





Disjunctive Information Flow

if x then w = y else w = z

w depends on x and y or x and z Disjunctive Information Flow What is it Good For?

Brewer & Nash IEEE Securty & Privacy 1989

THE CHINESE WALL SECURITY POLICY

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ABSTRACT

Everyone who has seen the movie Wall Street will have seen a commercial security policy in action. The recent work of Clark and Wilson and the WIPCIS initiative (the Workshop on Integrity Policy for Computer Information Systems) has drawn attention to the existence of a wide range of commercial security policies which are both significantly different from each other and quite alien to current "military" thinking as implemented in products for the security market place.

This paper presents a basic mathematical theory which implements one such policy, the Chinese Wall, and shows that it cannot be correctly represented by a Bell-LaPadula model.

The Chinese Wall policy combines commercial discretion with legally enforceable mandatory controls. It is required in the operation of many financial services organizations and is, therefore, perhaps as significant to the financial world as Bell-LaPadula's policies are to the military.

INTRODUCTION

Until recently, military security policy thinking has

However, the analyst is free to advise corporations which are not in competition with each other, and also to draw on general market information. Many other instances of Chinese Walls are found in the financial world.

Unlike Bell and LaPadula, access to data is not constrained by attributes of the data in question but by what data the subject already holds access rights to. Essentially, datasets are grouped into "conflict of interest classes" and by mandatory ruling all subjects are allowed access to at most one dataset belonging to each such conflict of interest class; the actual choice of dataset is totally unrestrained provided that this mandatory rule is satisfied. We assert that such policies cannot be correctly modelled by Bell-LaPadula.

It should be noted that in the United Kingdom the Chinese Wall requirements of the UK Stock Exchange [6] have the authority of law [7] and thus represent a mandatory security policy whether implemented by manual or automated means.

Furthermore, correct implementation of this policy is important to English Financial Institutions since it provides a legitimate defence against certain penal classes of offence under their law.





Contributions

A semantic model for disjunctive information flow generalising the lattice of information

• Disjunctive policies (ethical wall)

Model enjoys properties useful for reasoning about programs

- Disjunctive completion of IF lattices
- Compositional reasoning principles

The Lattice of Information

Equivalence Relations as Partitions

Assume (e.g.) a data domain $D = \{0,...,3\}$



Equivalence Relations as Partitions

Assume (e.g.) a data domain $D = \{0,...,3\}$



Equivalence Relations as Partitions

Assume (e.g.) a data domain $D = \{0, \dots, 3\}$



The Lattice of Partitions



The lattice of Information



A sublattice of LoI(D)

Information Flow Properties

A computation modelled as a function



Information Flow Properties

What can be observed by a given observer



Information Flow Properties

What may be learned by the observer What can be observed by a given observer



"P maps R-equivalent things to S-equivalent things"

Expressing disjunctive policies?

LoI can express a *specific* instance of a disjunctive flow:

"VW data flows to Consultant if they speak German, otherwise Volvo data flows to consultant"



But such conditions may be complex, unknown, or irrelevant

Lattice of Information

	Lattice of Information
Elements	Equivalence relations 02 1 3
Conjunction of information	
Disjunctive of information	

Lattice of Information

	Lattice of Information
Elements	Equivalence relations 02 1 3
Conjunction of information	Lattice least upper bound \sqcup $ \begin{bmatrix} 0 \\ 2 \\ 1 \\ 3 \end{bmatrix} \sqcup \begin{bmatrix} 0 \\ 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 3 \end{bmatrix} $
Disjunctive of information	

Lattice of Information

	Lattice of Information
Elements	Equivalence relations 02 1 3
Conjunction of information	Lattice least upper bound \sqcup $ \begin{array}{c} 0 \\ 2 \\ 1 \\ 3 \end{array} \sqcup \begin{array}{c} 0 \\ 2 \\ 1 \\ 3 \end{array} = \begin{array}{c} 0 \\ 2 \\ 1 \\ 3 \end{array} $
Disjunctive of information	×



	Lattice of Information	Quantale of Information
Elements	Equivalence relations $\begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$	Sets of Equivalence relations
Conjunction of information	Lattice least upper bound \sqcup $ \begin{array}{c} 0\\2\\1\\3 \end{array} \sqcup \begin{array}{c} 0\\2\\1\\3 \end{array} = \begin{array}{c} 0\\2\\1\\3 \end{array} $	
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Disjunctive of information		

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Disjunctive of information		Lattice least upper bound = set union

	Lattice of Information	Quantale of Information
Elements	Equivalence relations $\begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$	Sets of Equivalence relations (*)
Conjunction of information	Lattice least upper bound \sqcup $ \begin{bmatrix} 0 \\ 2 \\ 1 \\ 3 \end{bmatrix} \sqcup \begin{bmatrix} 0 \\ 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 3 \end{bmatrix} $	Tensor operator \otimes $P \otimes Q = \{ p \sqcup q \mid p \in P, q \in Q \}$ (*)
Disjunctive of information		Lattice least upper bound = set union (*)

(*) Sets closed under a special "mixing" operations on sets of equivalence relations called tiling

Tiling closure

Equivalence classes = observations

Tiling closure of a set of partitions = all relations that can be built by mixing and matching observations







Conclusion

A Quantale of Information: a strict generalisation of the Lattice of Information

More in the paper, including:

- Capture the essence of ethical wall policies in a precise sense
- Nice compositional properties which make reasoning easier:

$$\frac{f_1: \mathbb{P} \Rightarrow \mathbb{Q} \quad \mathbb{Q} \sqsupseteq \mathbb{Q}' \quad f_2: \mathbb{Q}' \Rightarrow \mathbb{R}}{f_1; f_2: \mathbb{P} \Rightarrow \mathbb{R}}$$