Subterm-based proof techniques for improving the automation and scope of security protocol analysis

Cas Cremers ⧺ Charlie Jacomme ⧺ **Philip Lukert**
∃t. Cispa ⊏ Saarbrücken ⊏ Germany

Subterm-based p
techni
th scope of
s protocol analysis

Subterms for Tamarin

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Protocol Analysis

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Protocol Analysis → **Data-structures**

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Protocol Analysis → **Data-structures** → **Subterms**

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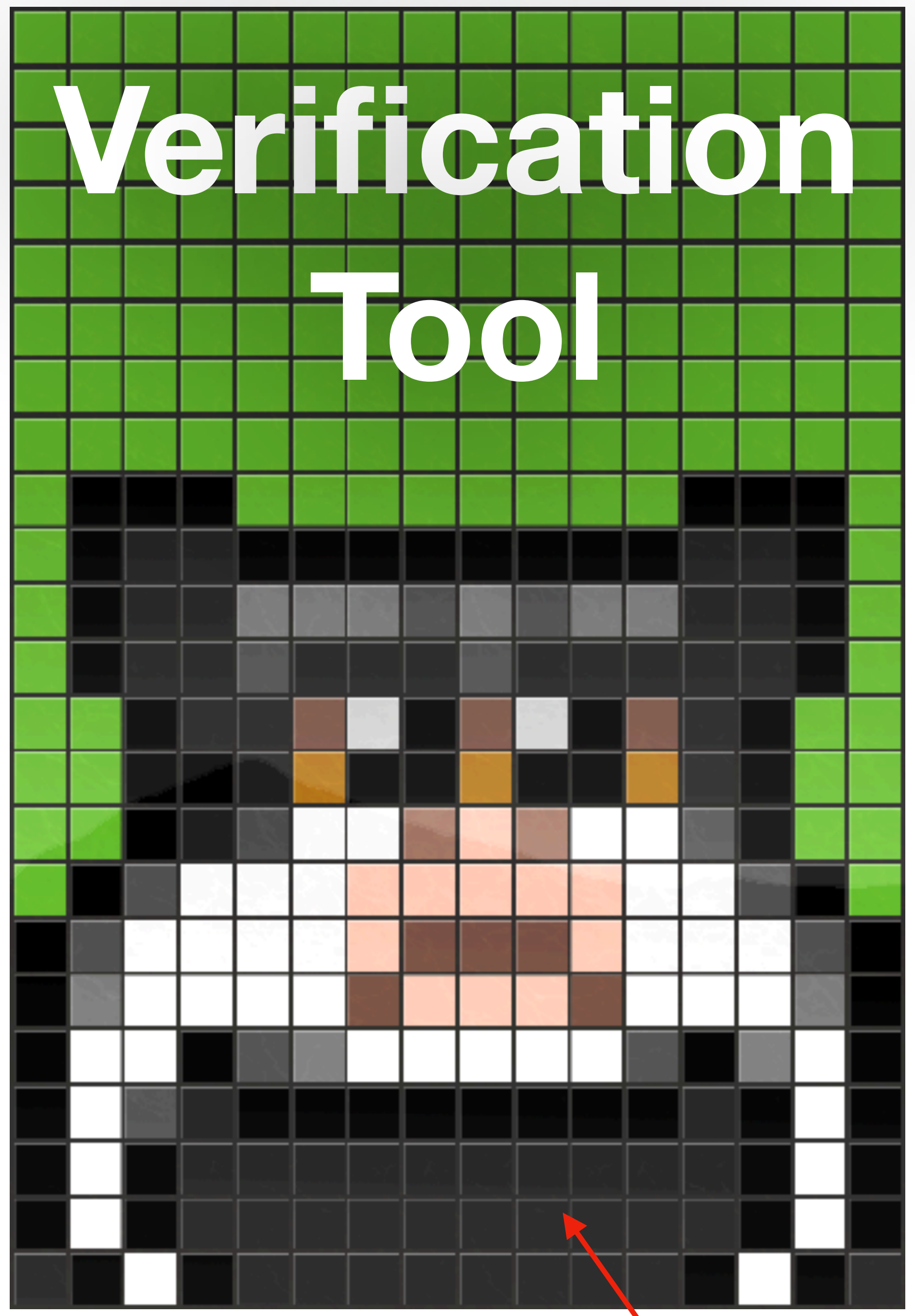
Protocol Analysis → **Data-structures** → **Subterms** → **Great Proofs**

Protocol Analysis



it's a Tamarin

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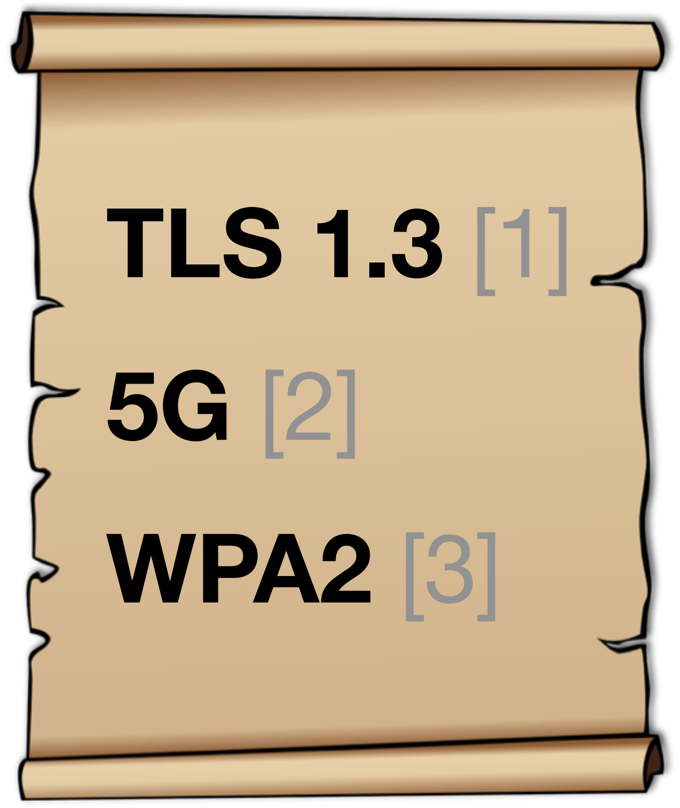

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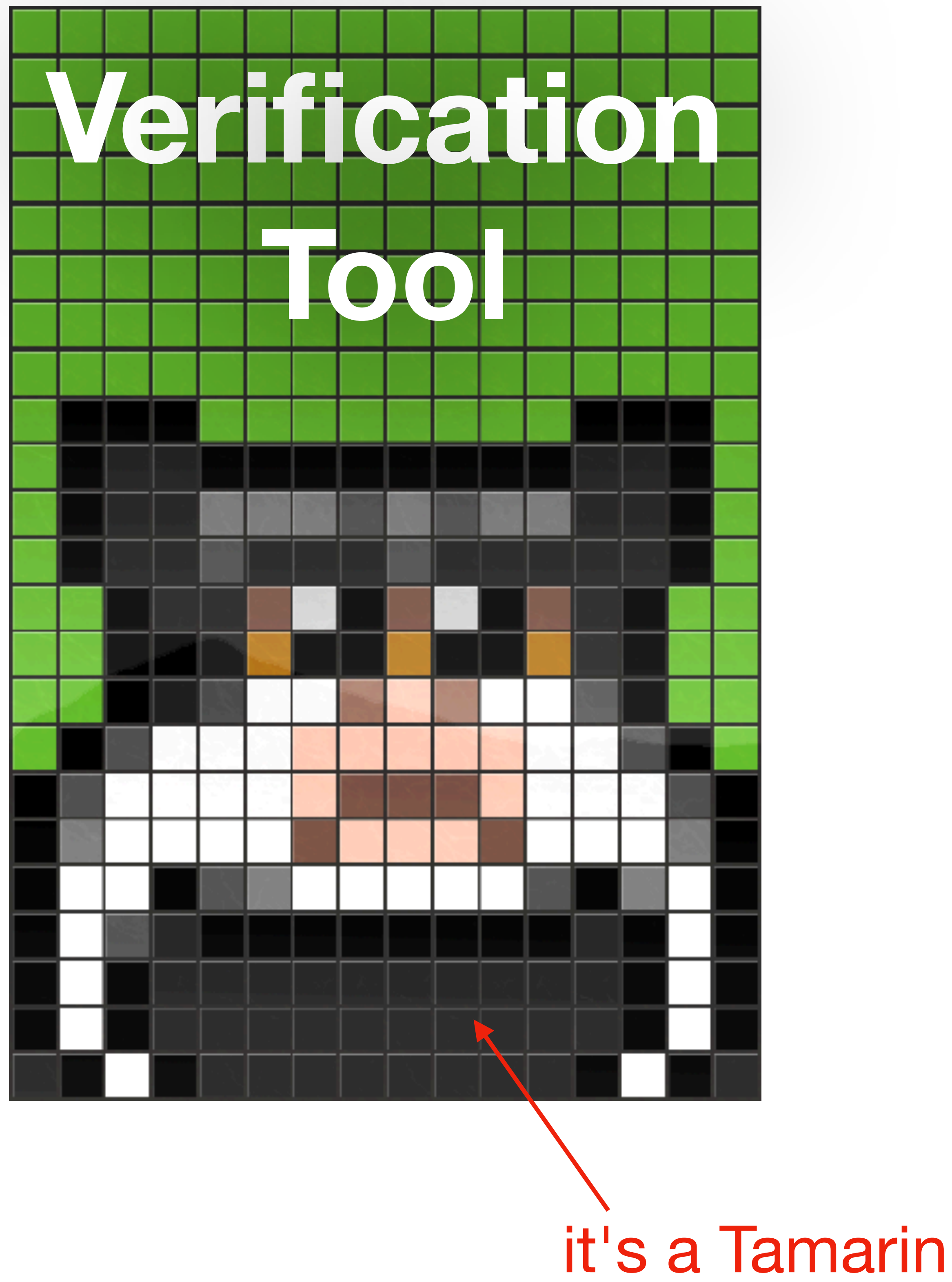

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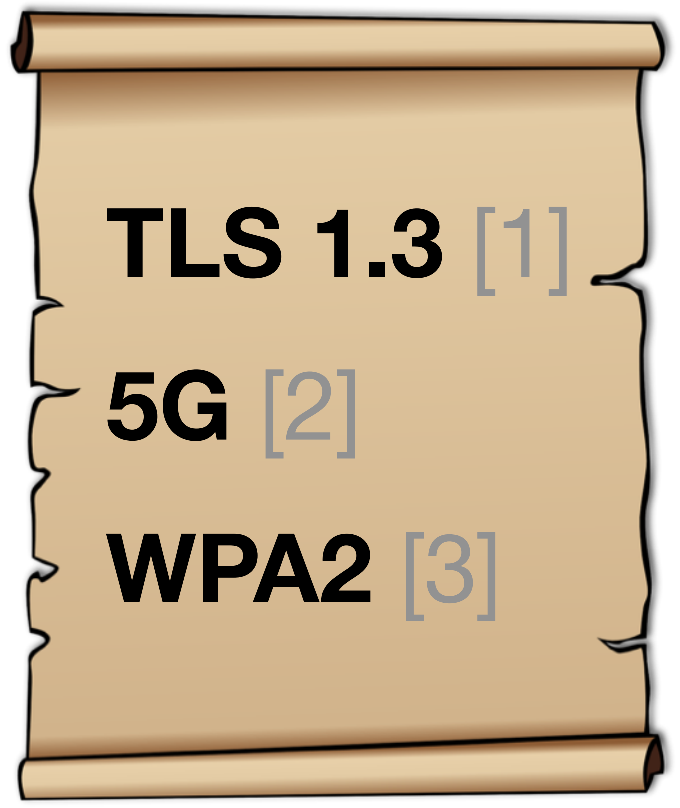

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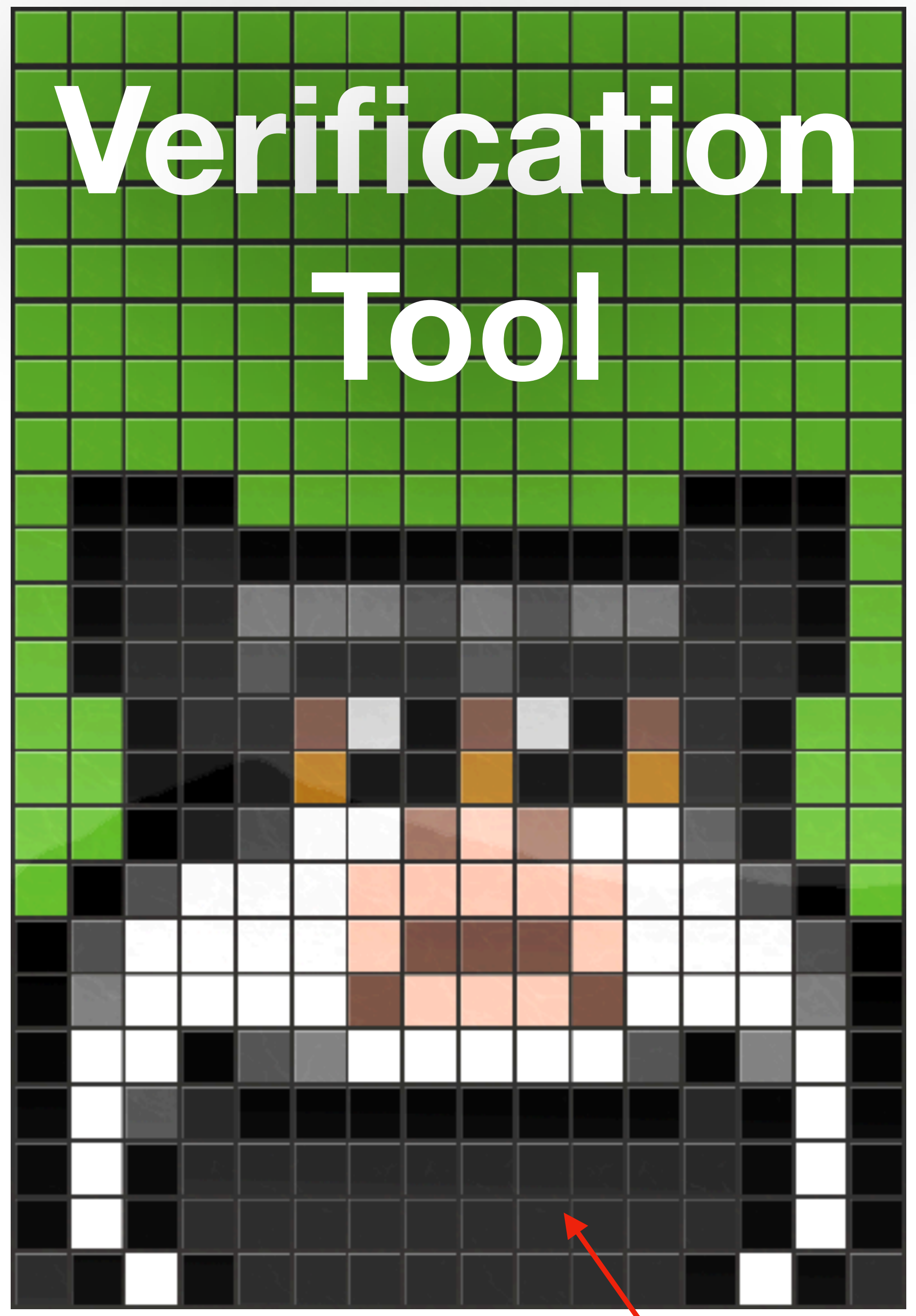

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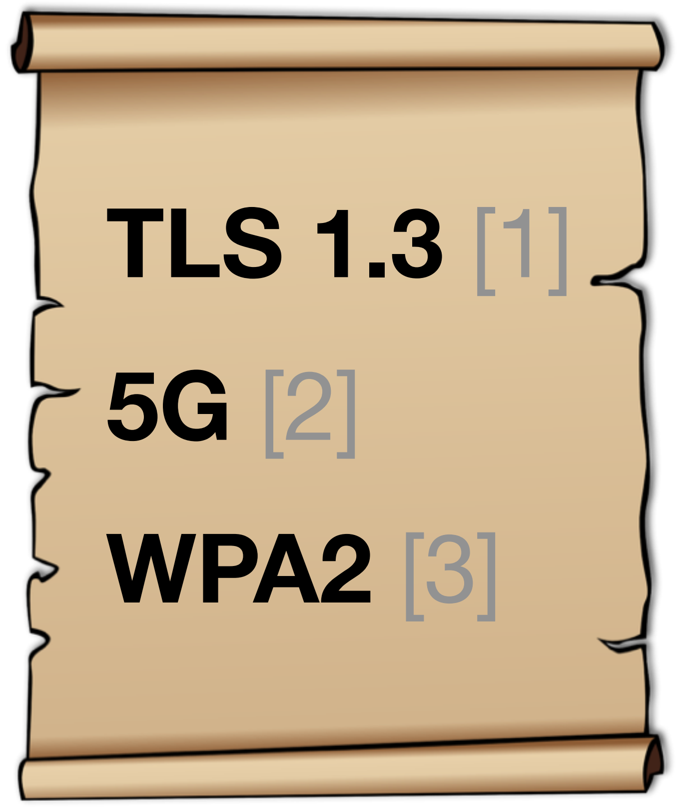

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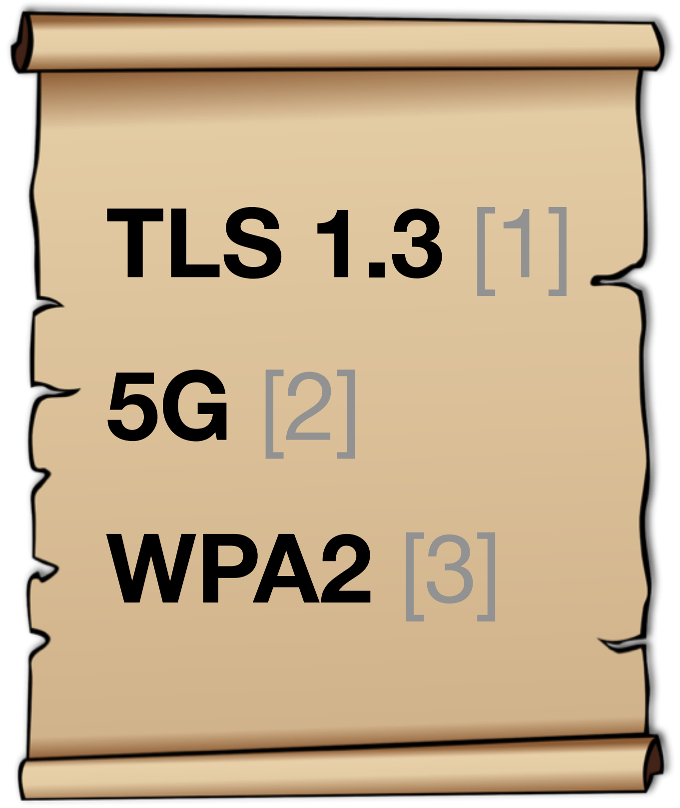

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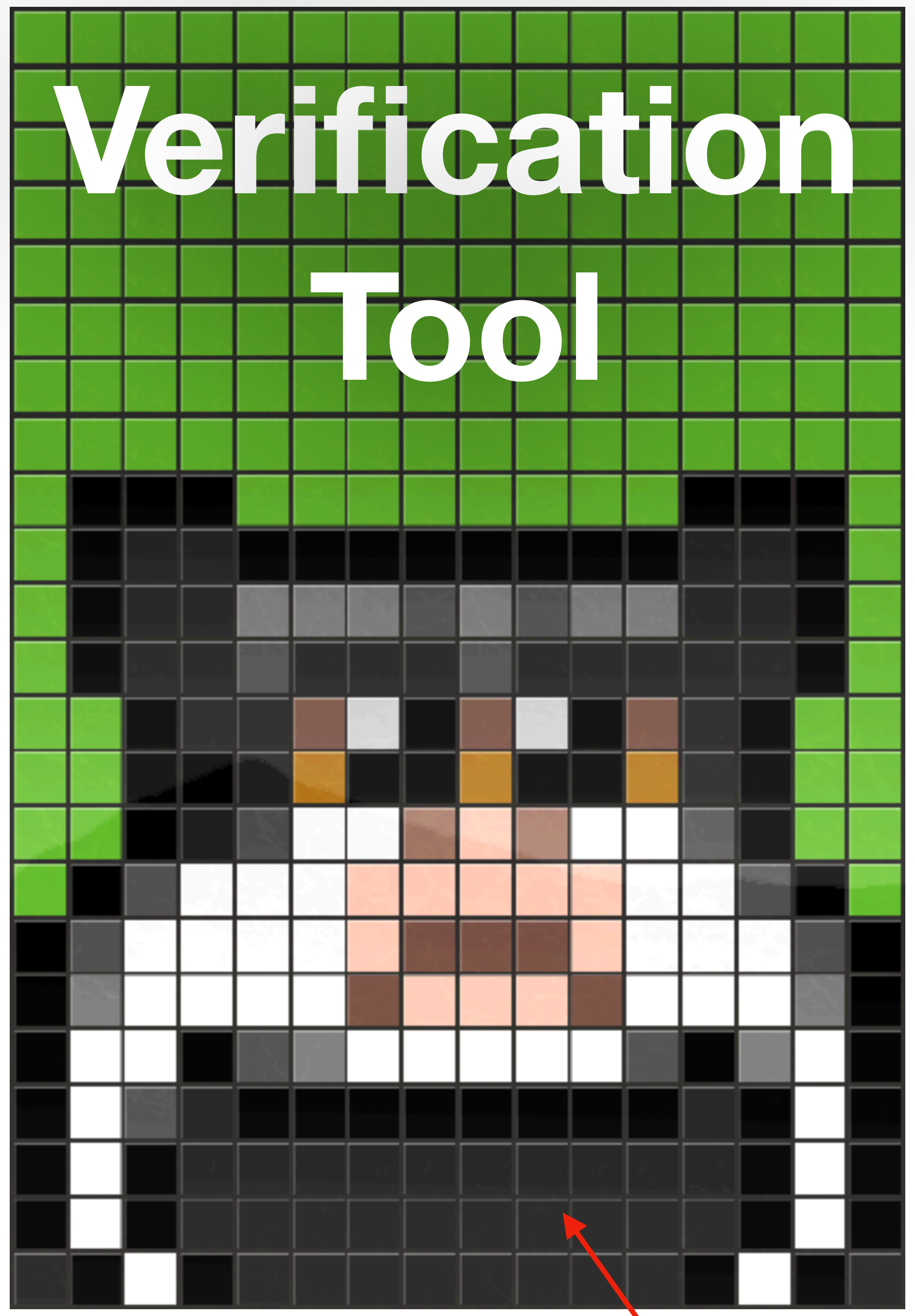

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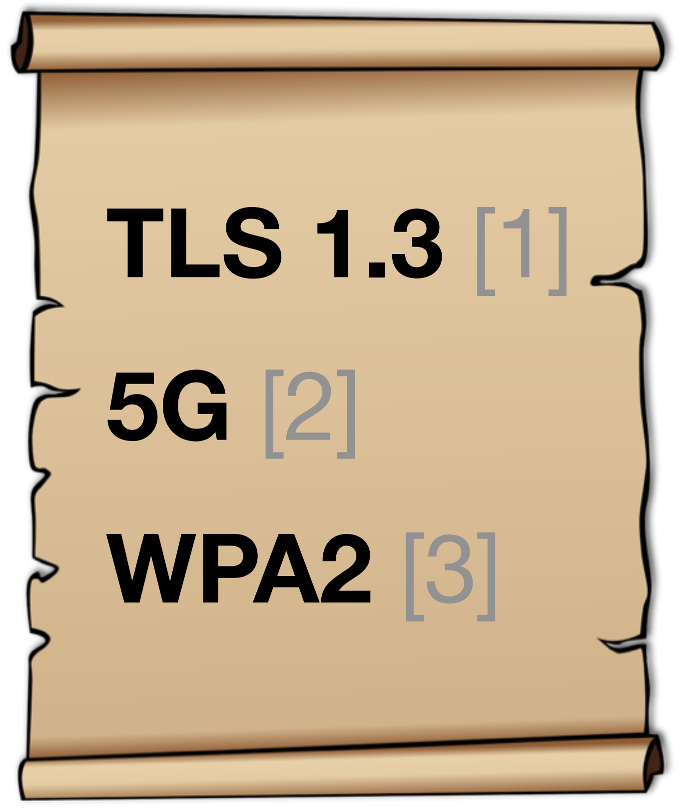

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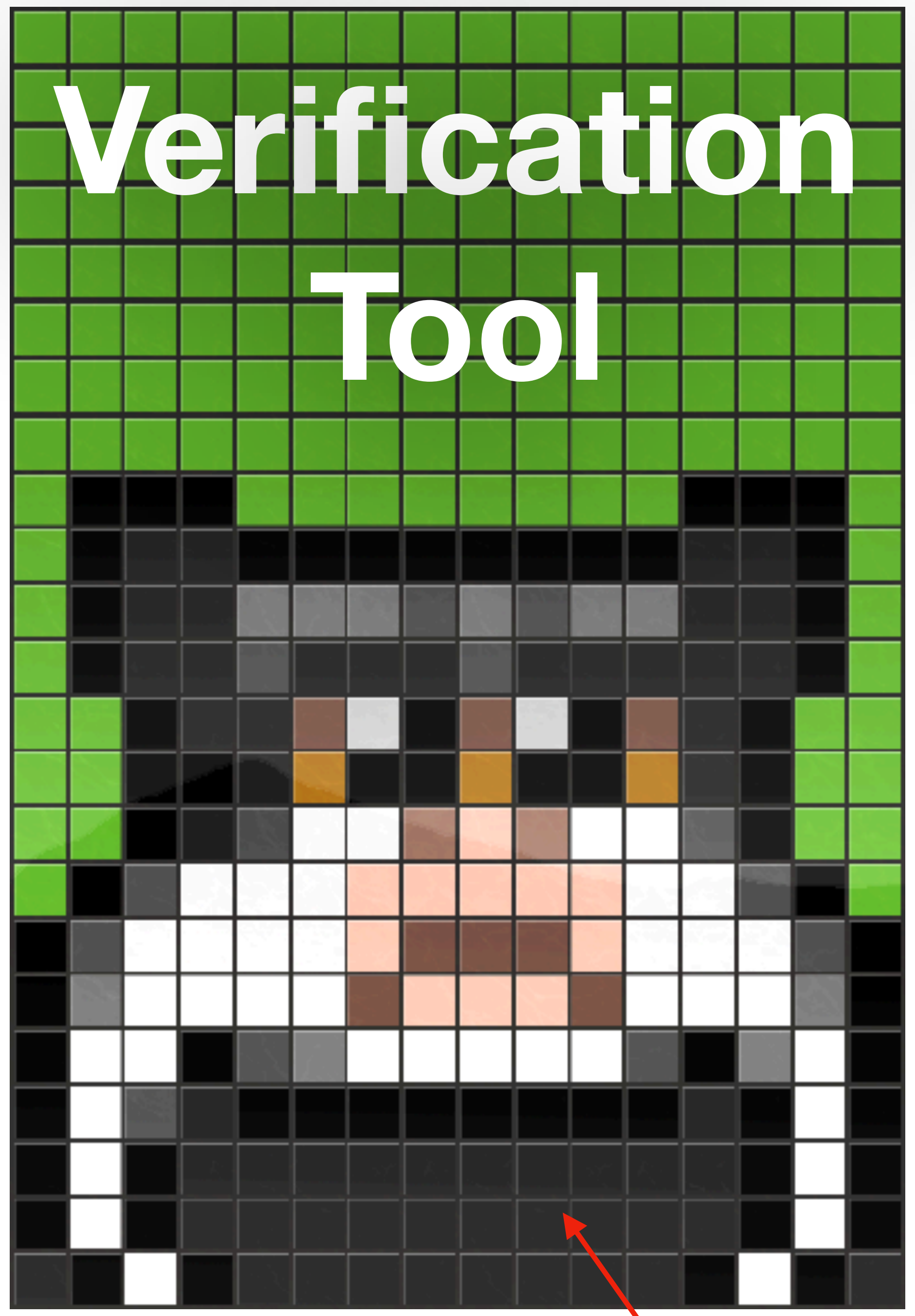

Proof

Attack

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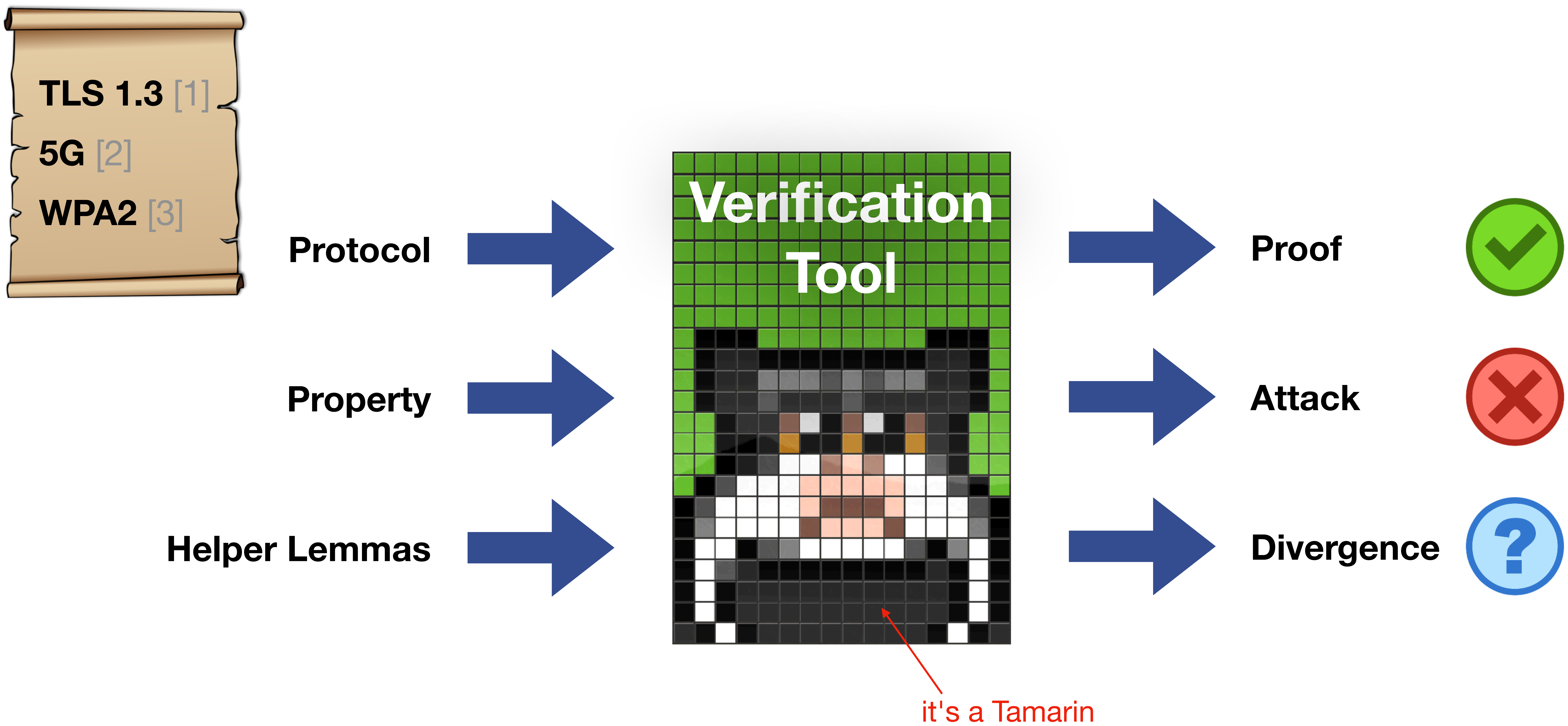

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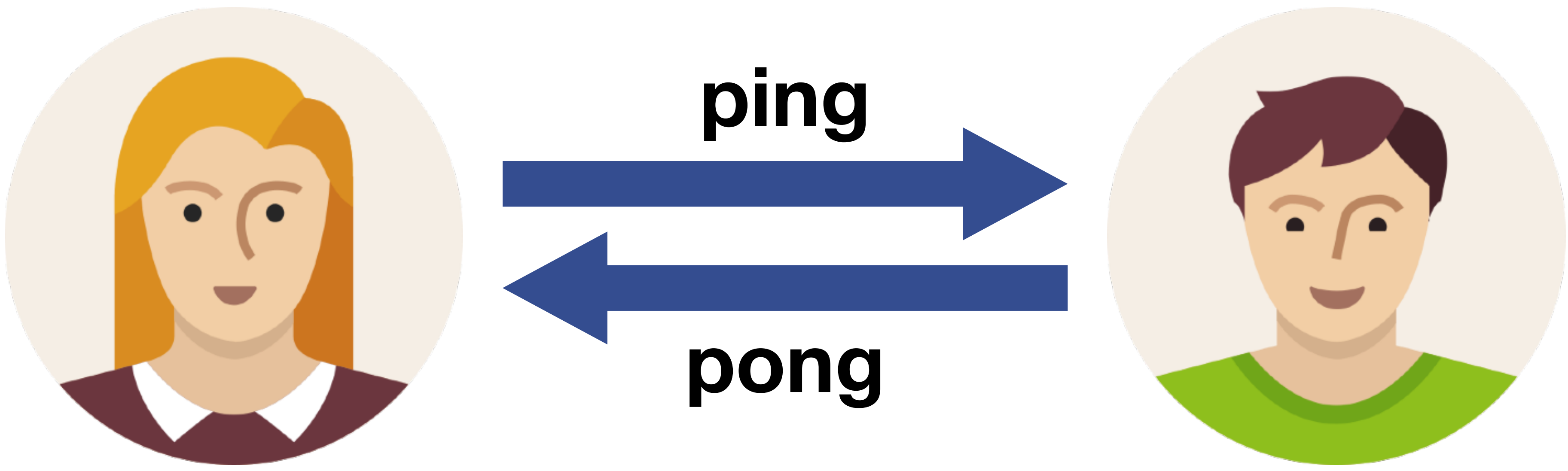

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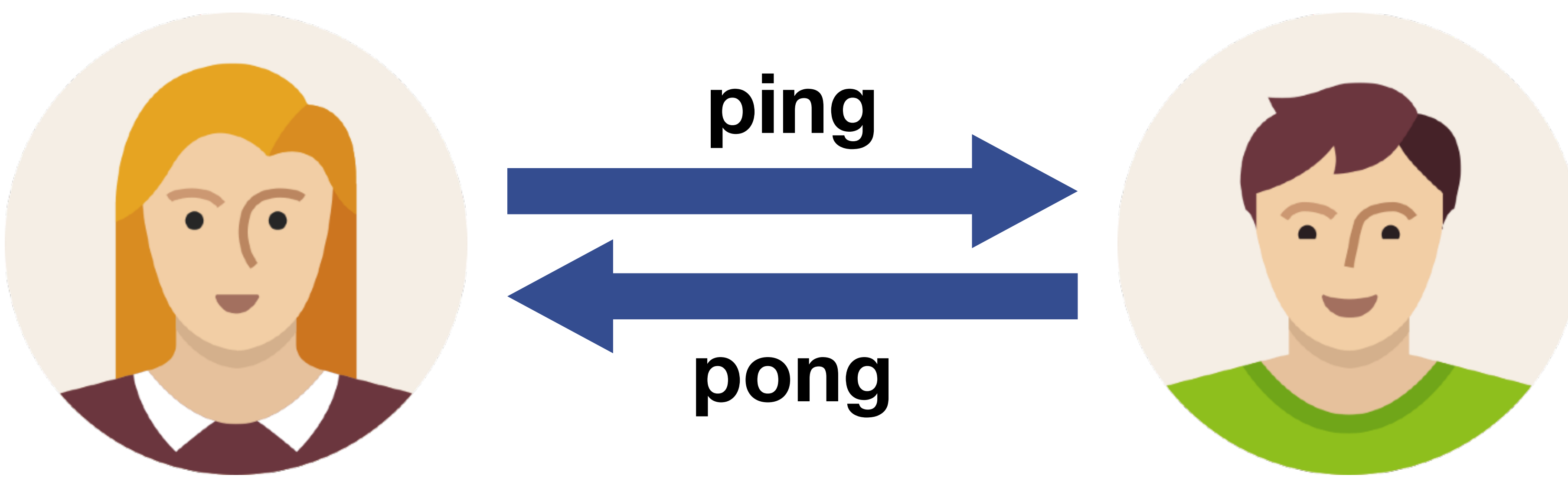


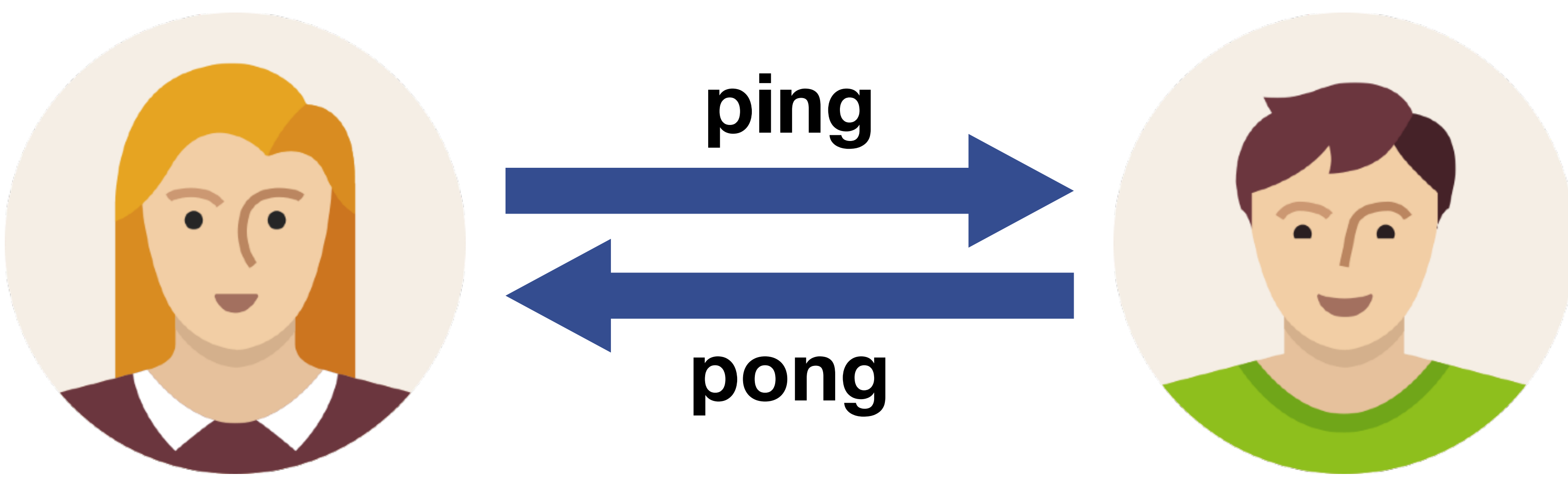
required state facts

action

produced state facts

[ ] --[Start]--> [Out("ping"), State()]

Tamarin



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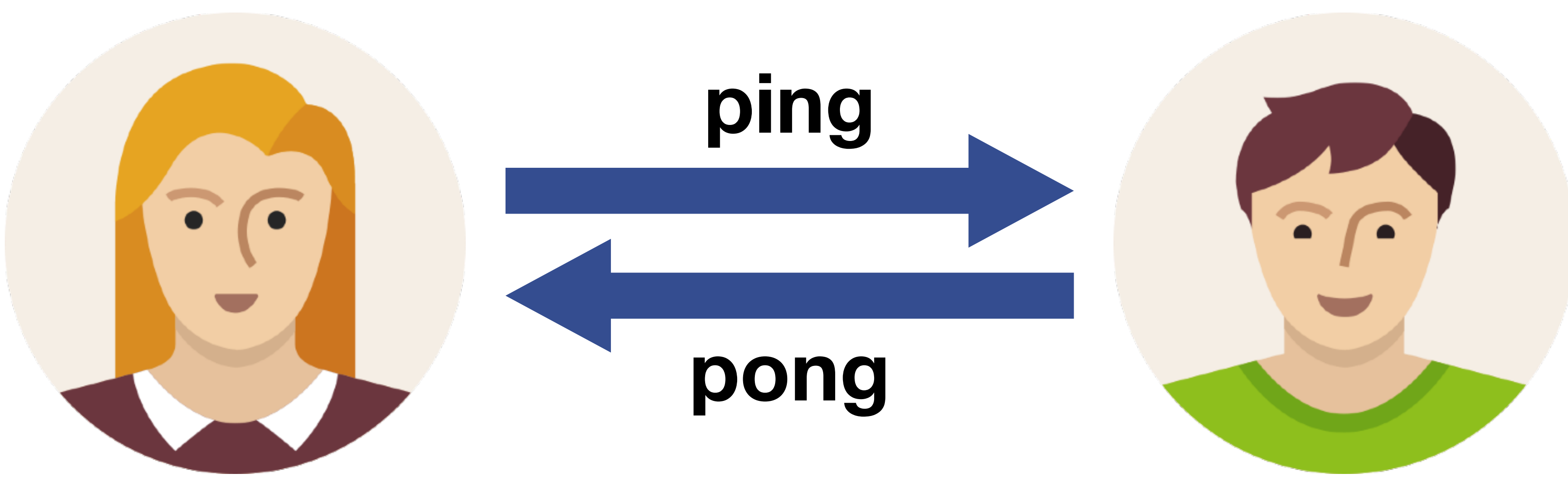

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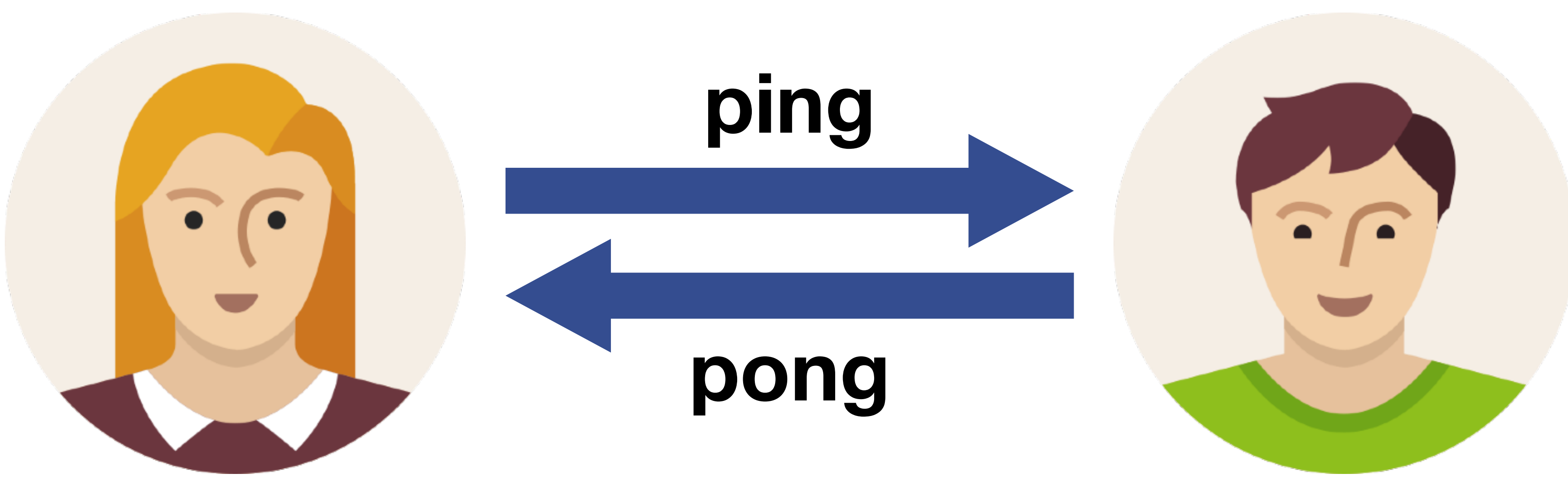

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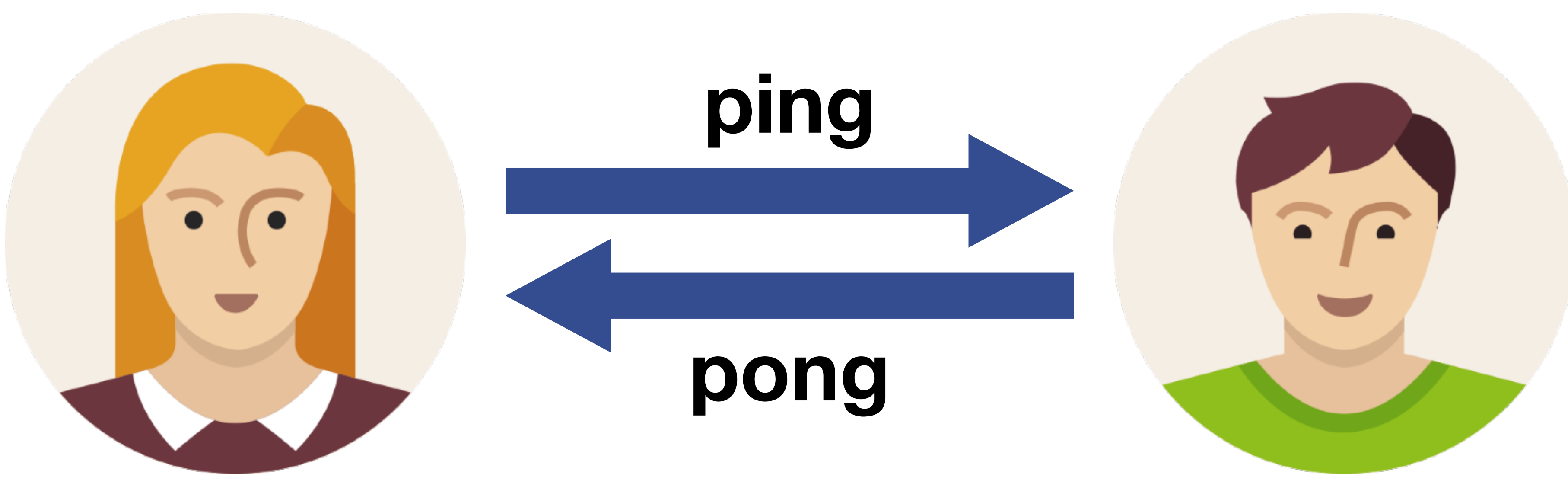

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Start → Answer → Finish

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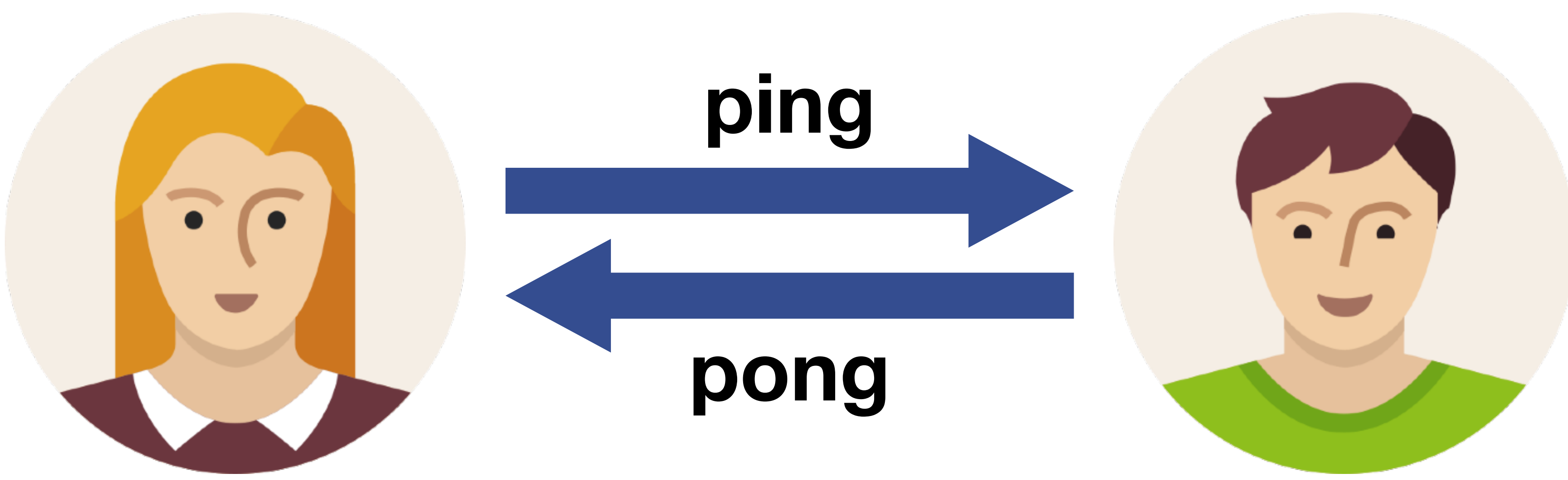

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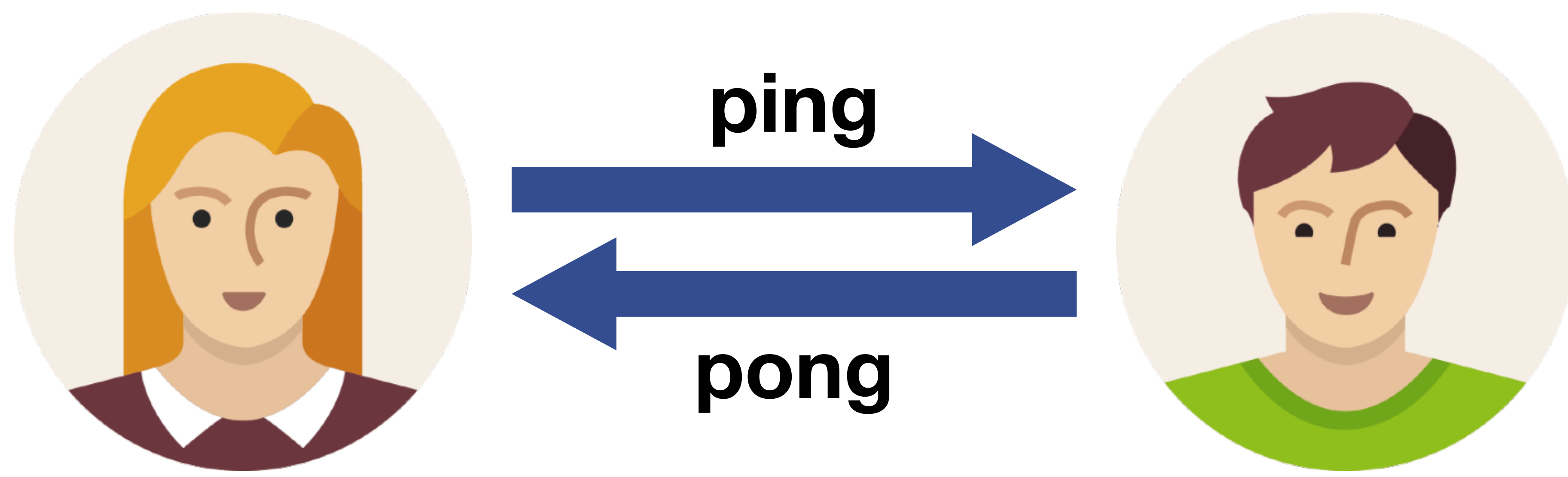

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$\forall$ Finish $\Rightarrow$ $\exists$ Answer

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∀ Finish ⇒ ∃ Answer

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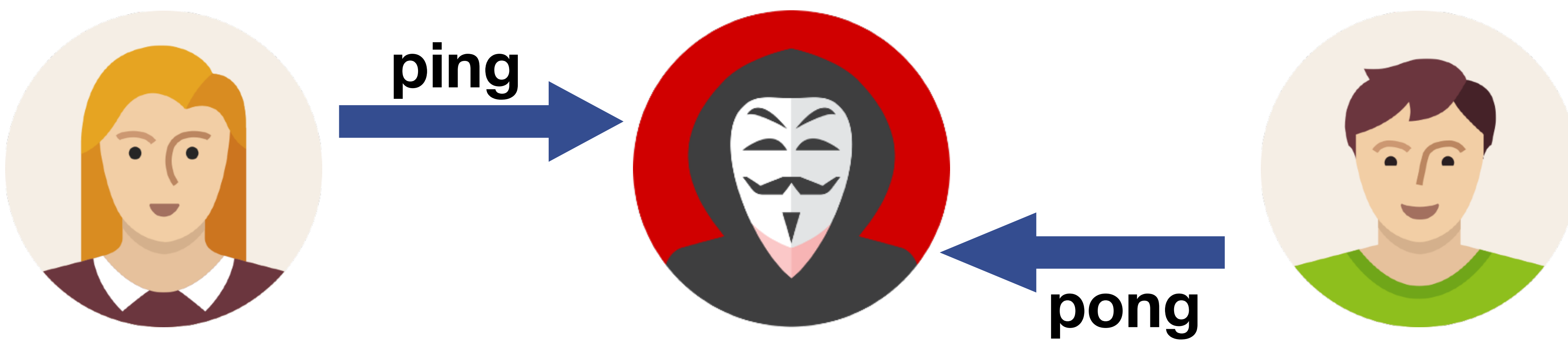

Symbolic Attacker
- observe all Out(...)

required state facts | action | produced state facts

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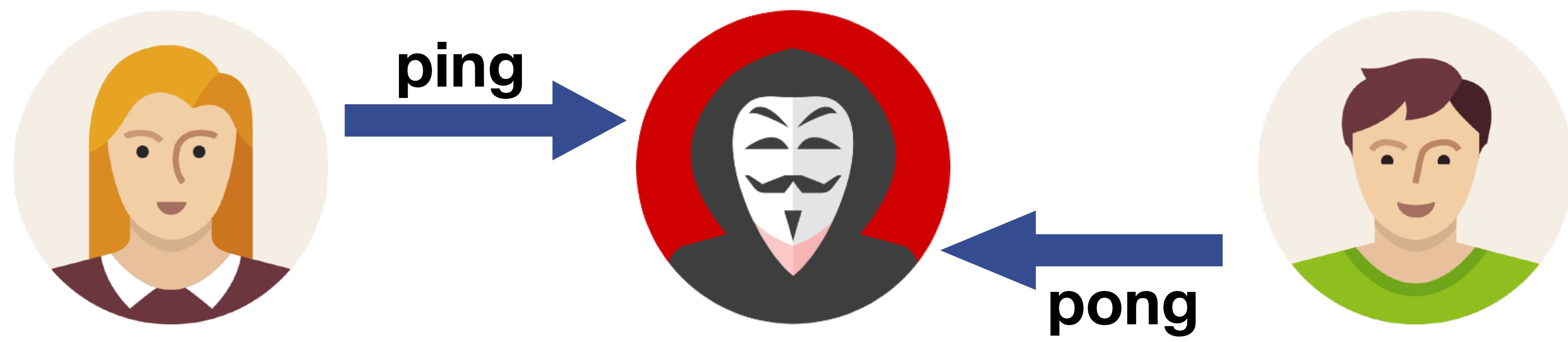

Symbolic Attacker
- observe all Out(...)
- controls all In(...)

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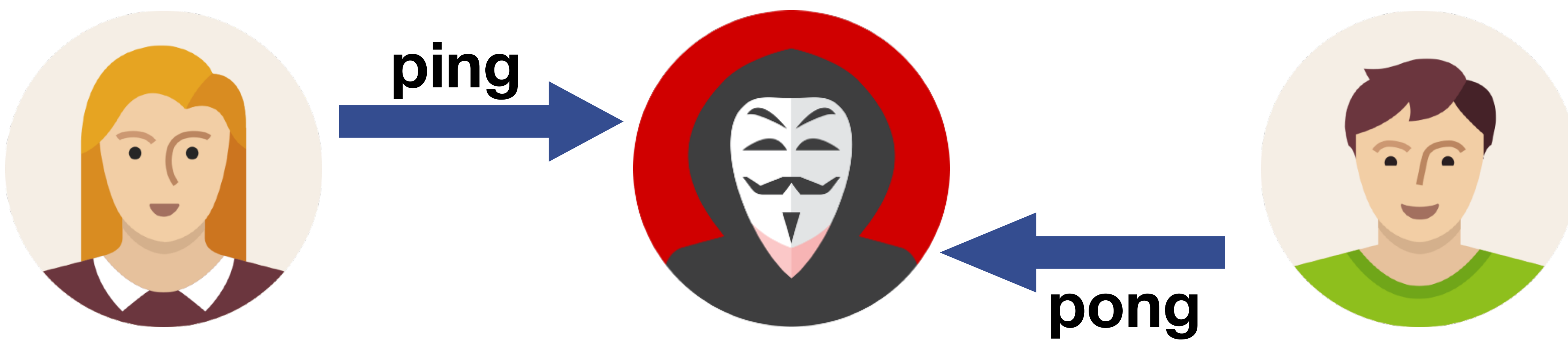

Symbolic Attacker
- observe all Out(...)
- controls all In(...)
  - drop messages

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Symbolic Attacker

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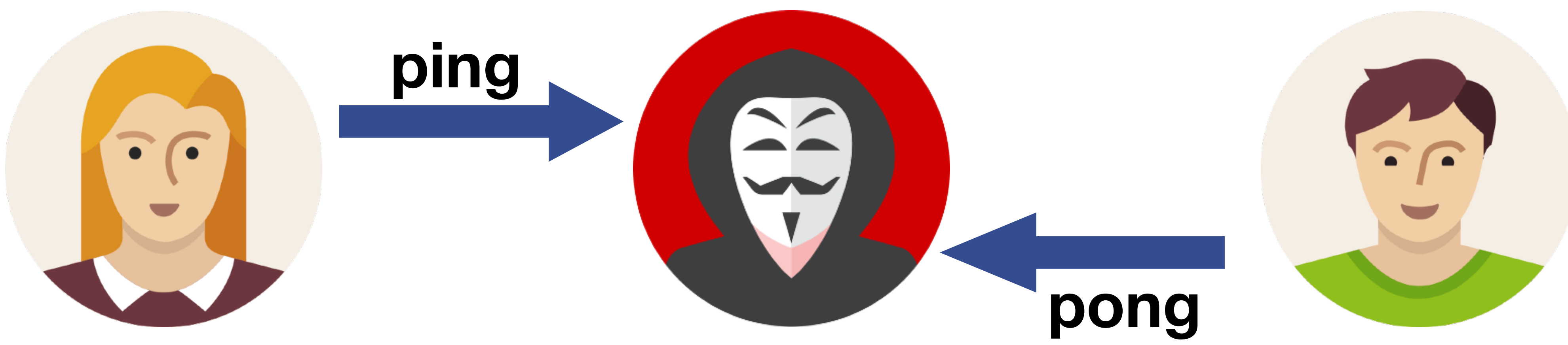

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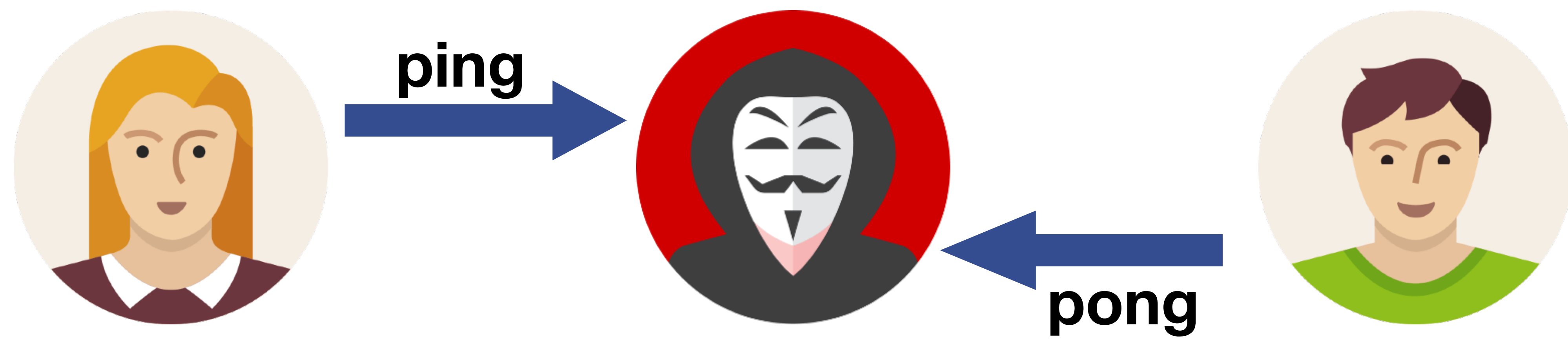
Start → Finish

attacker sends "pong"



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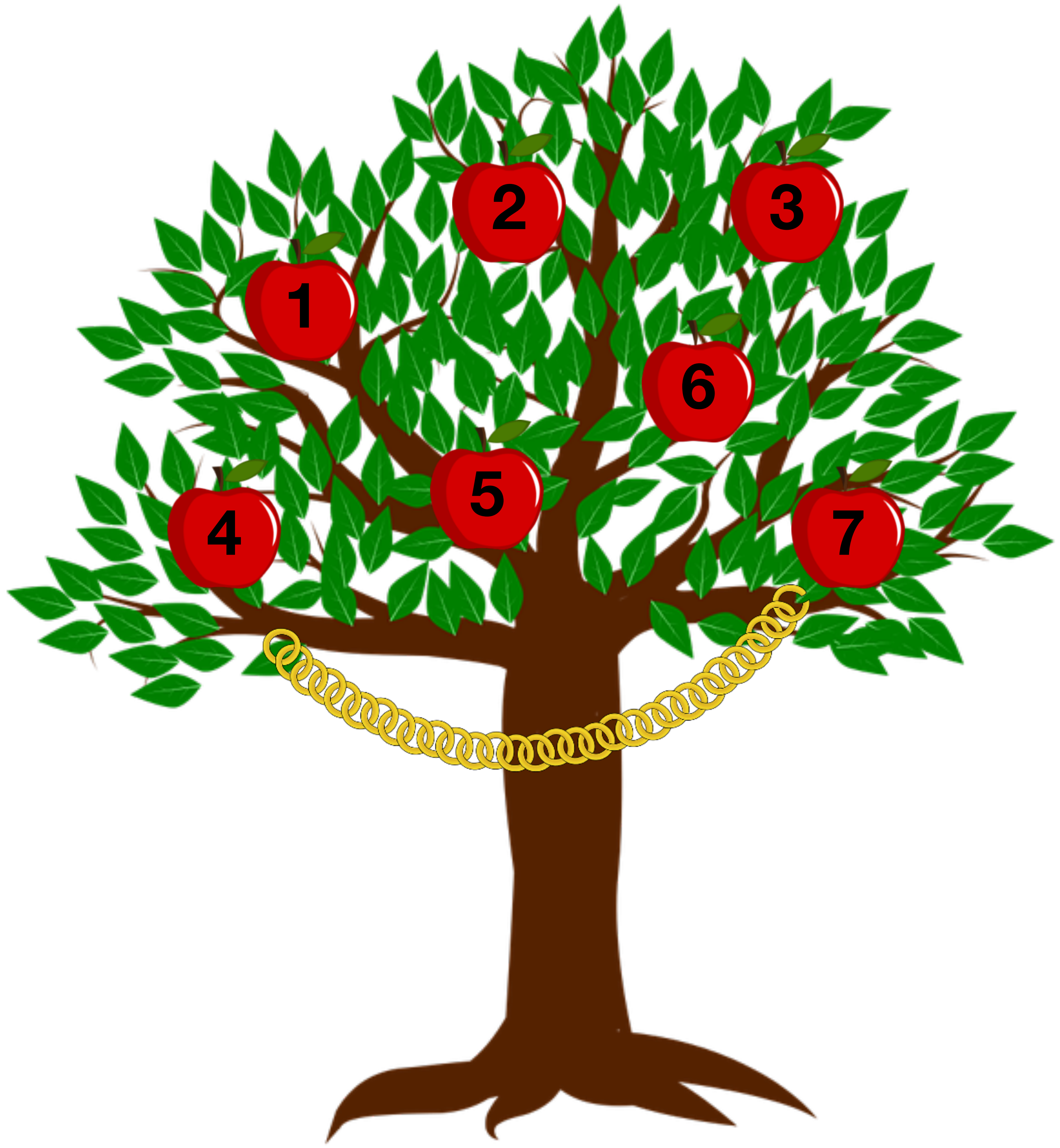

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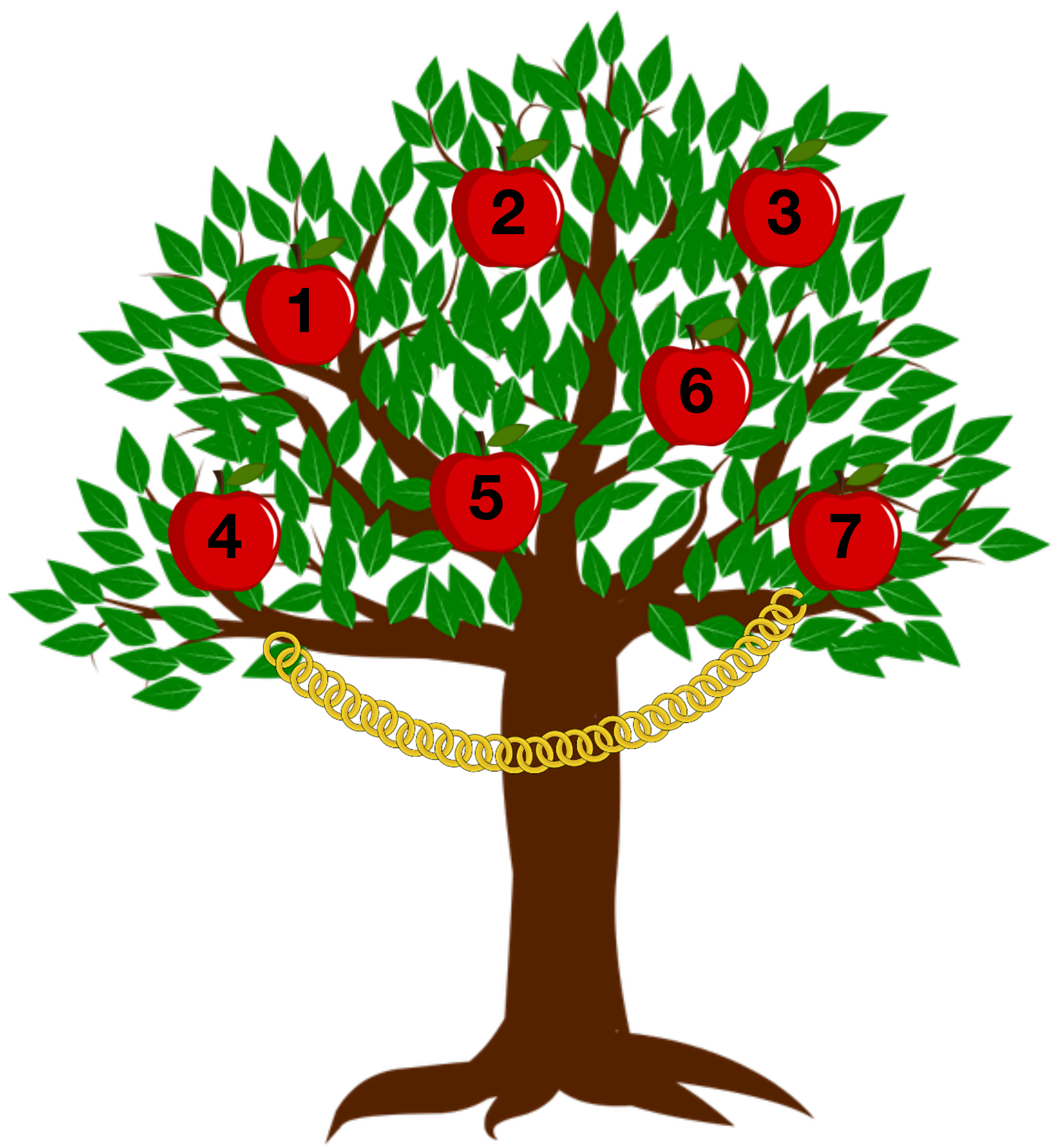

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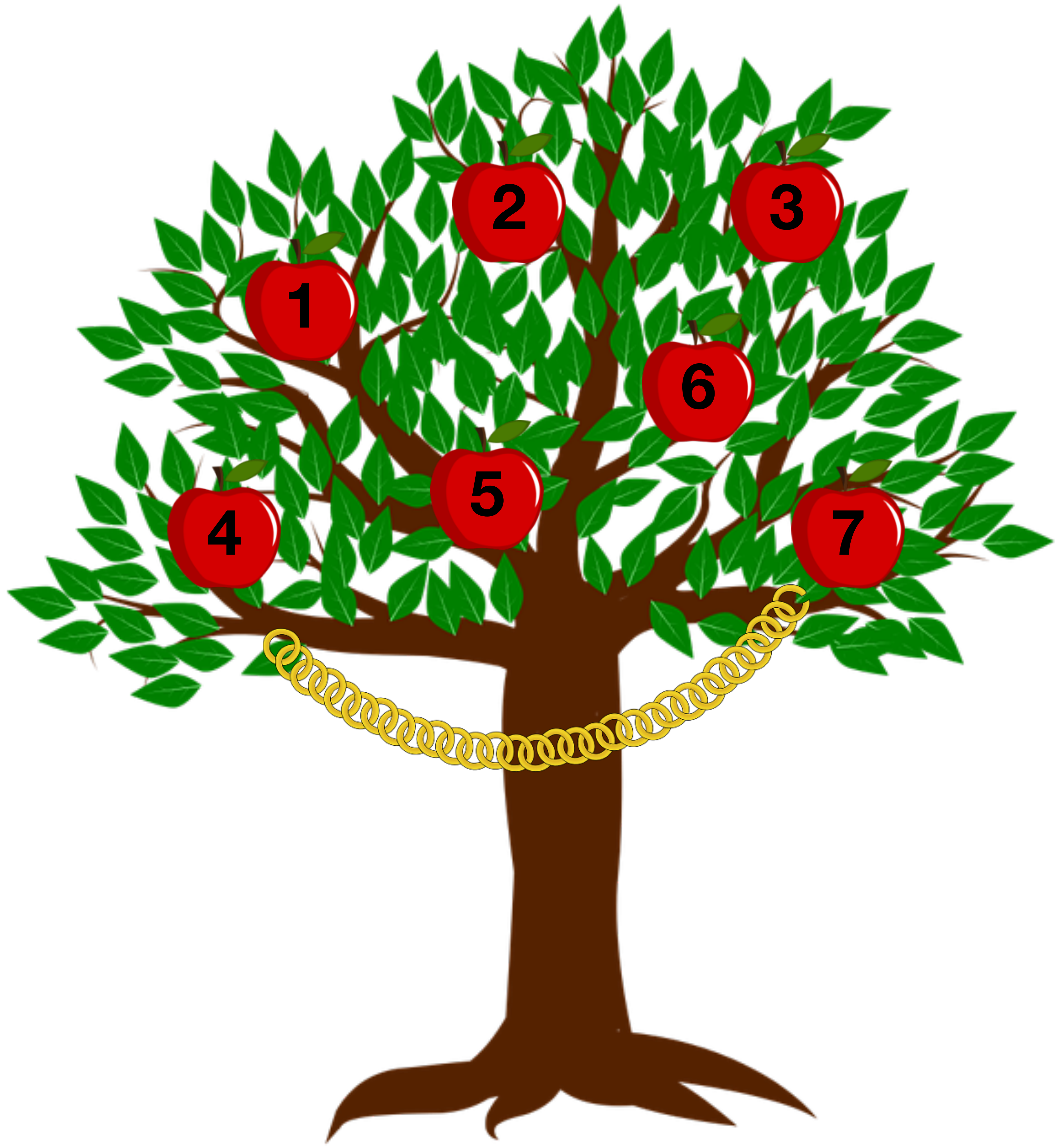

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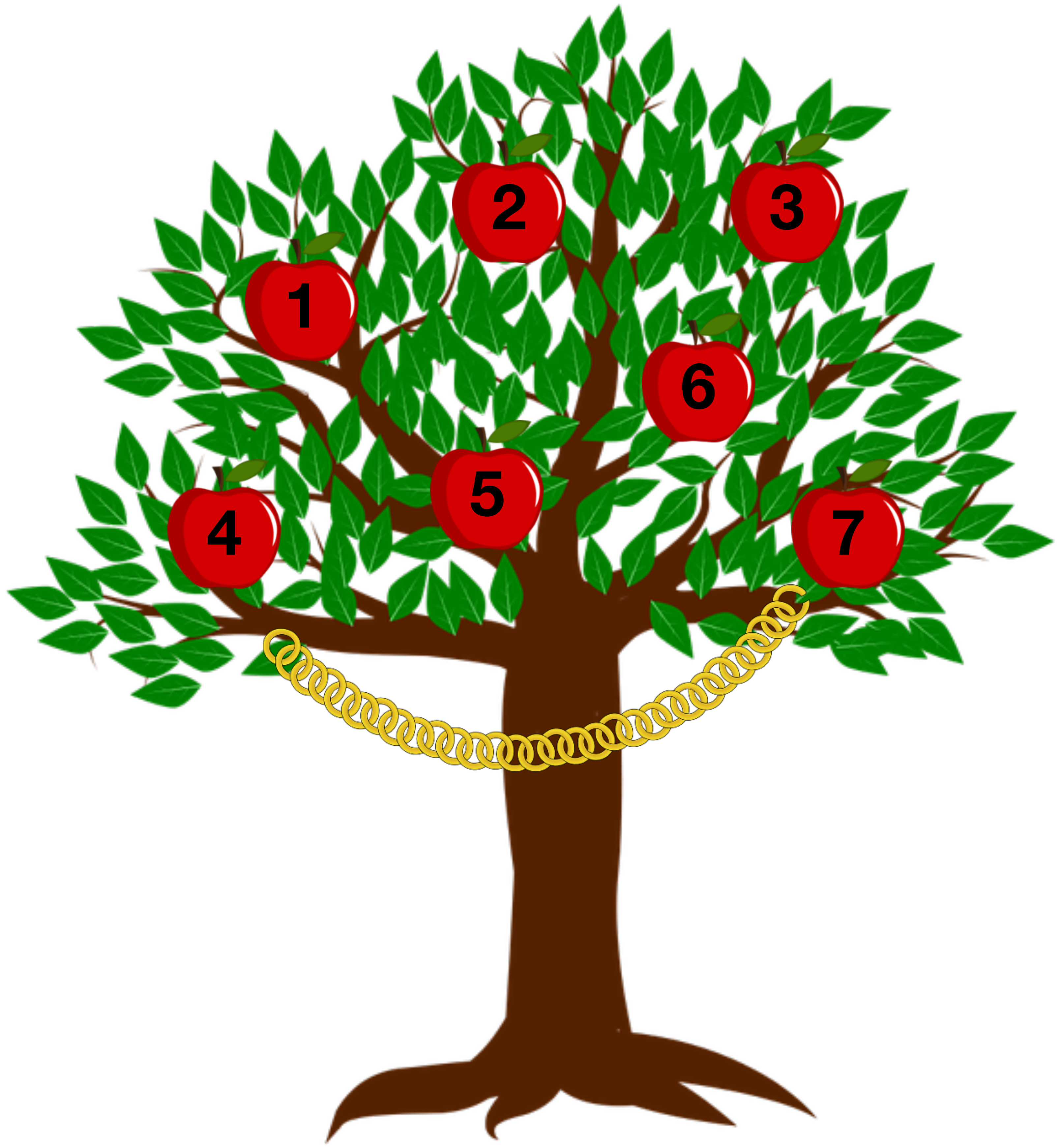
→ **Divergence**



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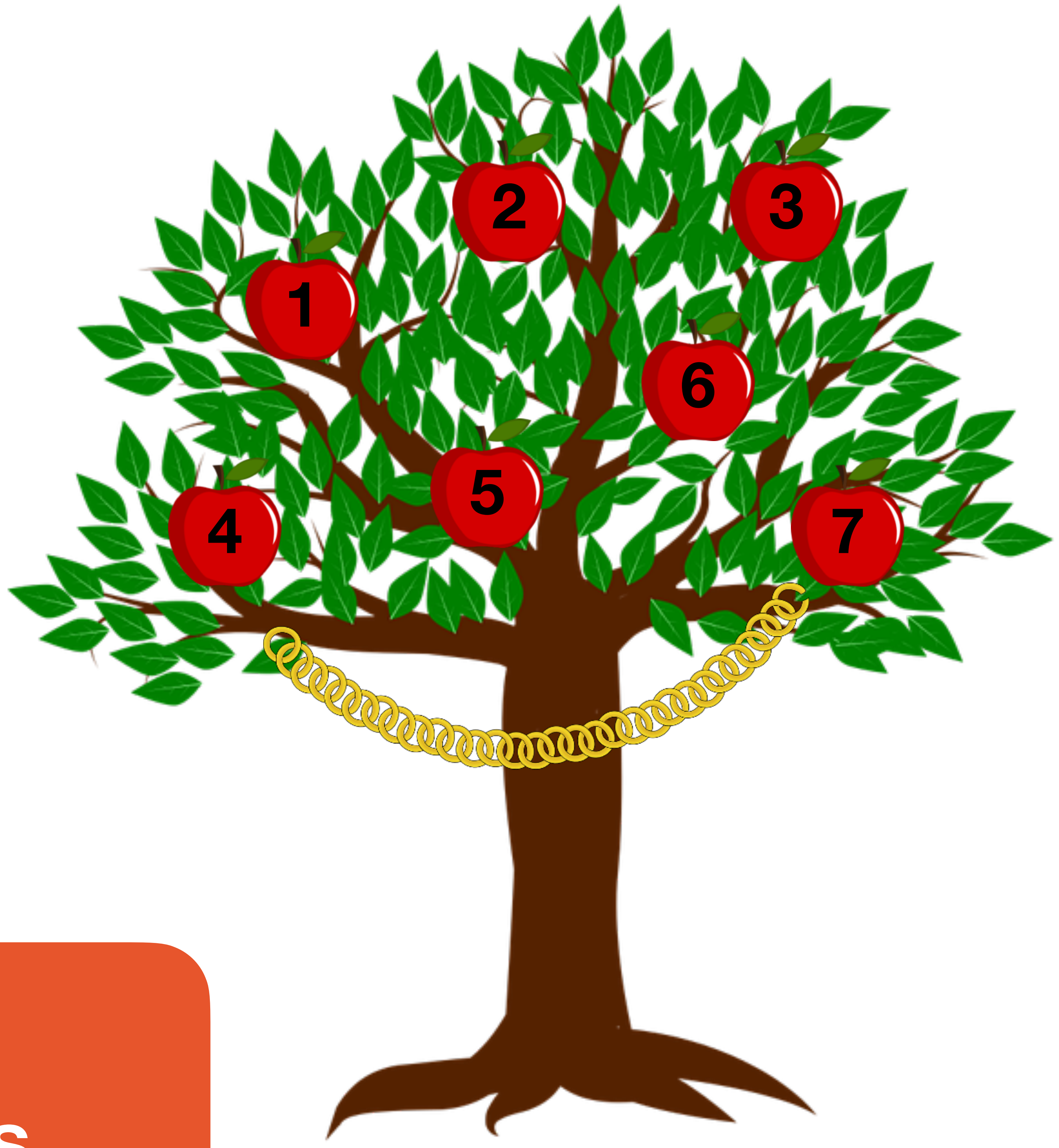

Solution: Subterms

Our Contribution

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Modeling

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$\sqsubset$

- Subterm-Predicate "$\sqsubset$"
  - we can now state $x \sqsubset h(h(x))$

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"under the hood"

- Algorithm for Numbers
- Monotonicity
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Case Studies

- New Proofs
- Application to Old Proofs

(small) Numbers

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$= a \mathbin{+\!\!\!+} (b \mathbin{+\!\!\!+} c)$

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  - comperator: $a < b \quad \Leftrightarrow \quad \exists\, x.\ a + x = b$

(small) Numbers

$(a \mathbin{+\!\!+} b) \mathbin{+\!\!+} c$
$= a \mathbin{+\!\!+} (b \mathbin{+\!\!+} c)$

$a \mathbin{+\!\!+} b$
$= b \mathbin{+\!\!+} a$

- well studied: associative and commutative (AC) operator $\mathbin{+\!\!+}$
- used as multiset: $a \mathbin{+\!\!+} b \mathbin{+\!\!+} b = \{a, b, b\}$
- used for counting: $\text{one} \mathbin{+\!\!+} \text{one} \mathbin{+\!\!+} \text{one} = 3$
- our improvement:
  - type system, dedicated operator $+$
  - comperator: $a < b \quad \Leftrightarrow \quad \exists\, x.\ a + x = b$
  - dedicated algorithm: $a < b < a{+}2 \quad \Rightarrow \quad b = a{+}1$

Dedicated Proof Techniques

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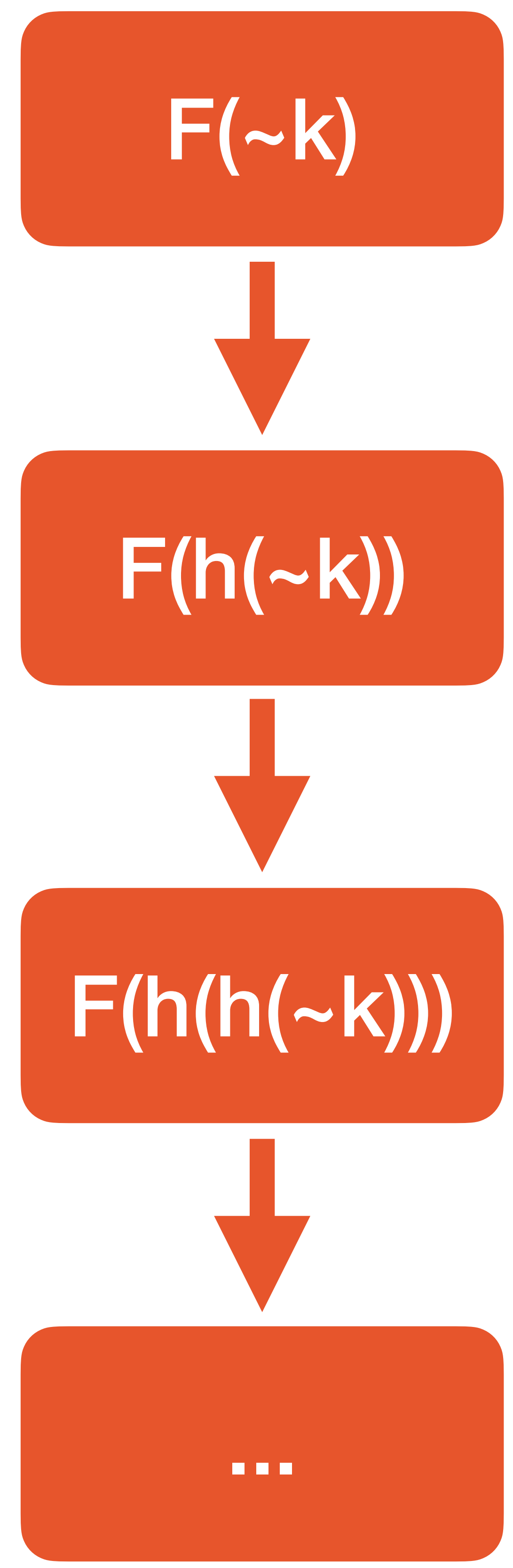
Monotonicity

F(~k) → F(h(~k)) → F(h(h(~k))) → ...

Dedicated Proof Techniques

Monotonicity

- F(s)@i,   F(t)@j



Dedicated Proof Techniques

Monotonicity

- F(s)@i,   F(t)@j
  - $s \sqsubset t \Rightarrow i < j$

$$F(\sim k) \rightarrow F(h(\sim k)) \rightarrow F(h(h(\sim k))) \rightarrow \ldots$$

Dedicated Proof Techniques

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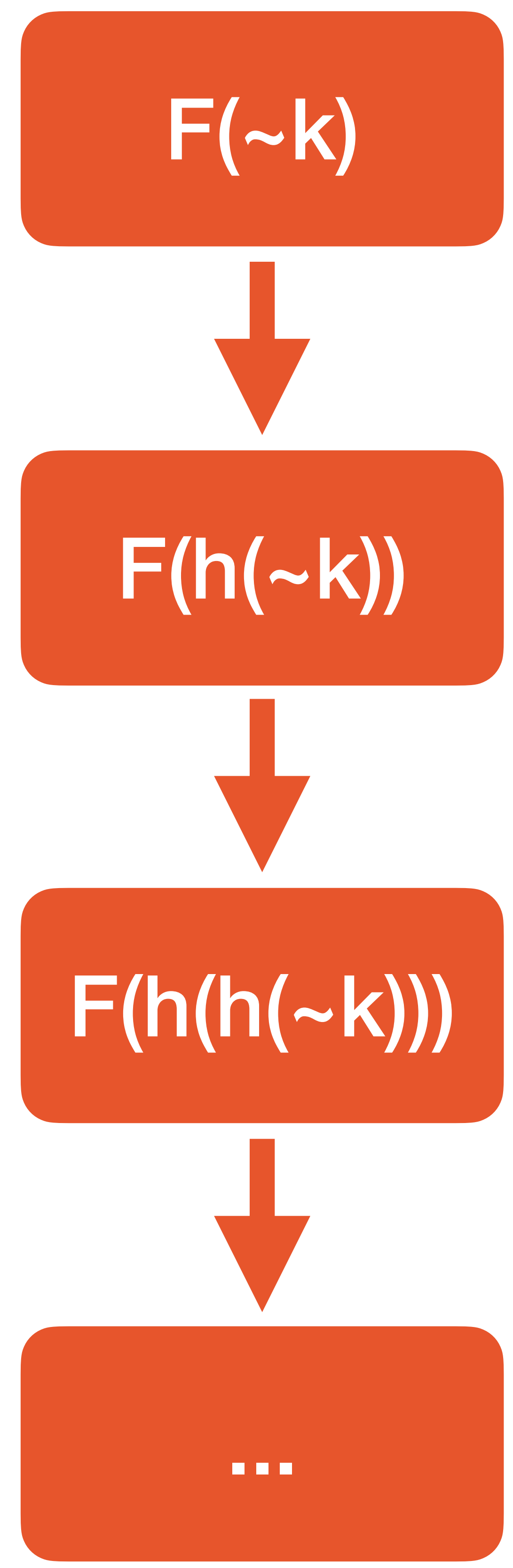

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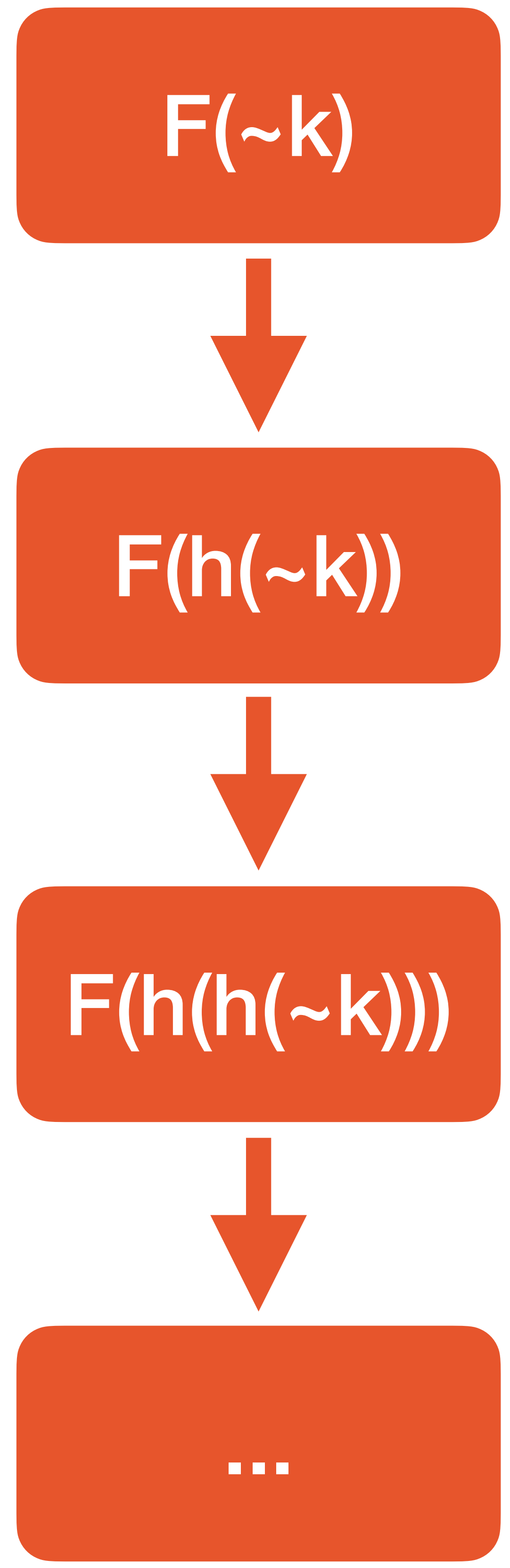

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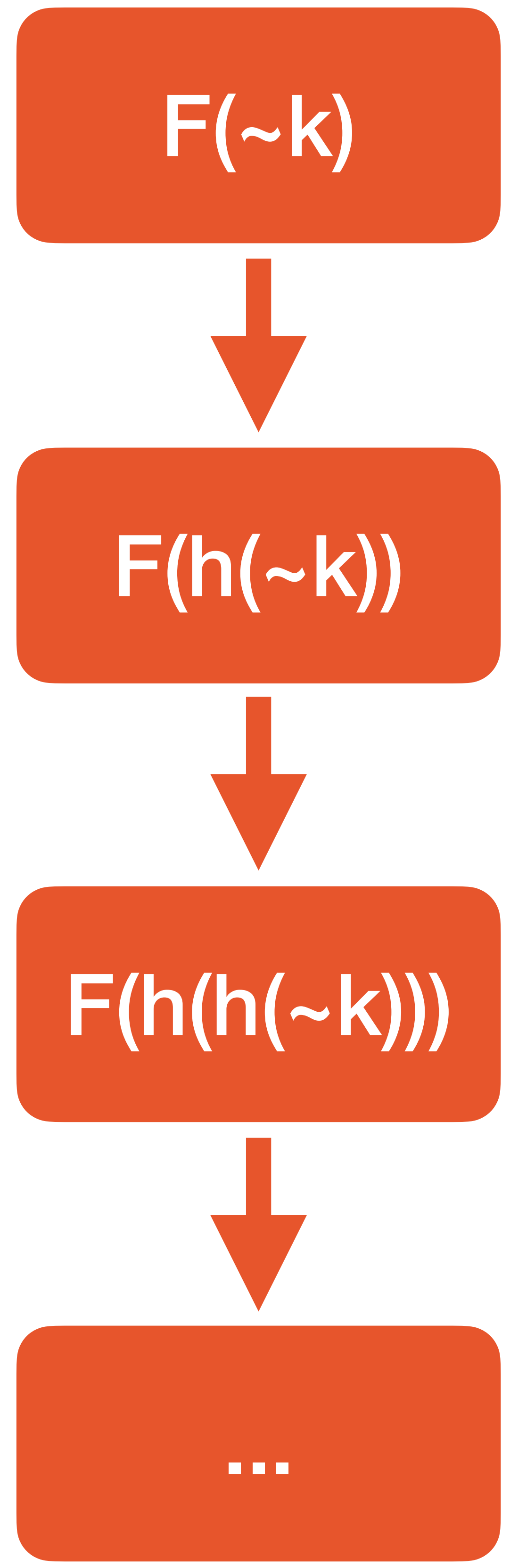

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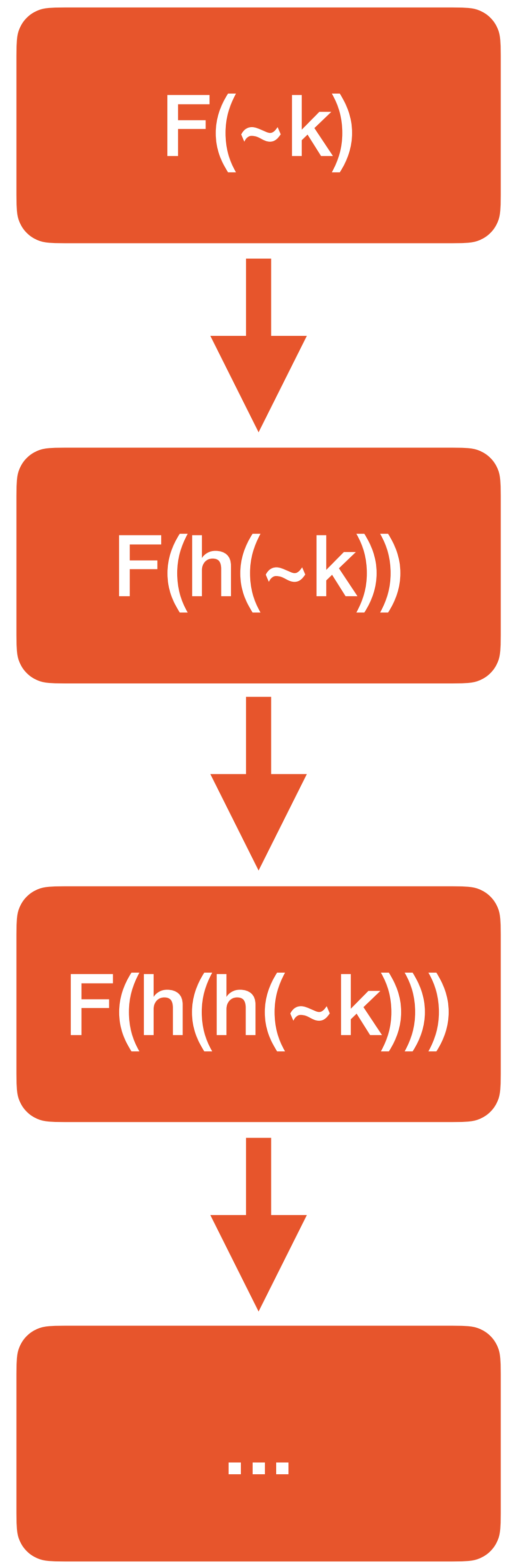

- 10x speed-up of WPA-2 proof

Dedicated Proof Techniques

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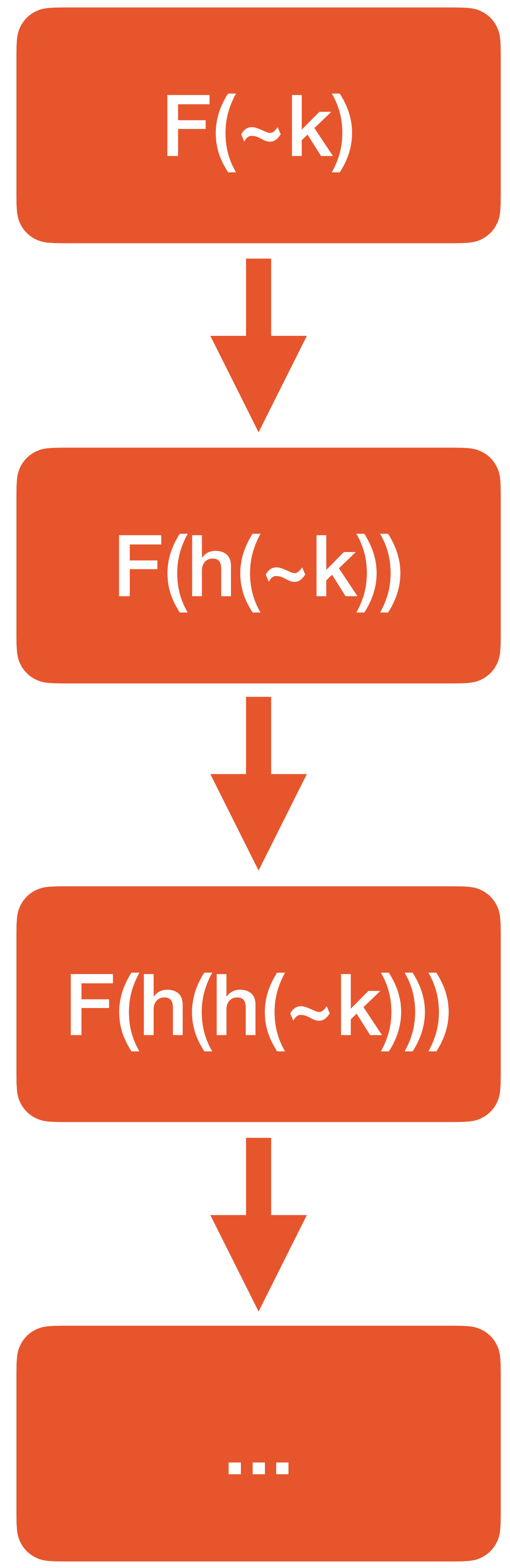

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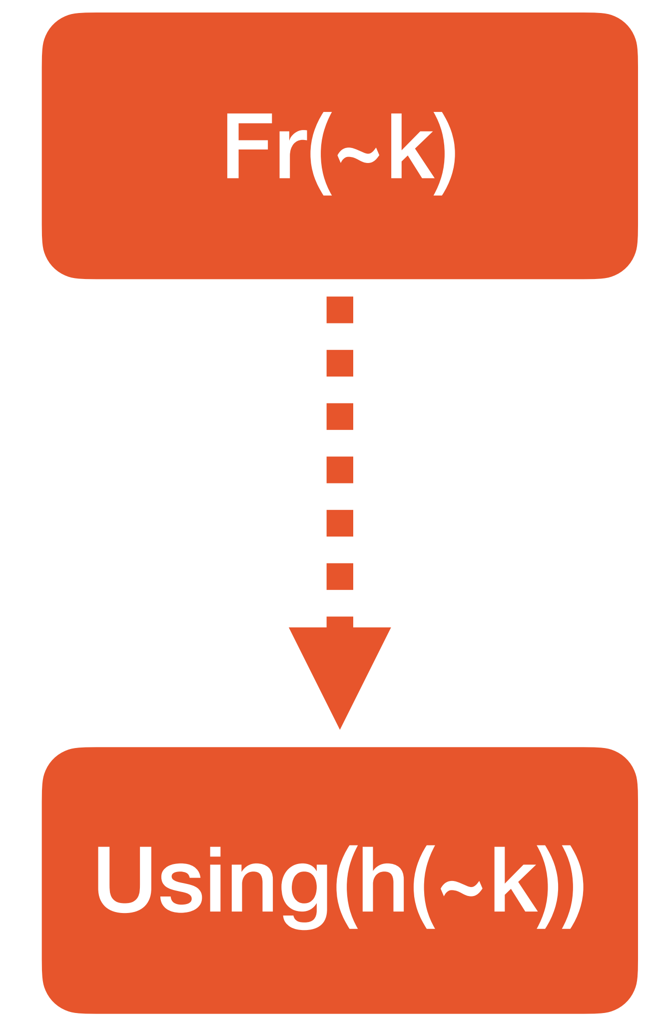
Fresh Order



Dedicated Proof Techniques

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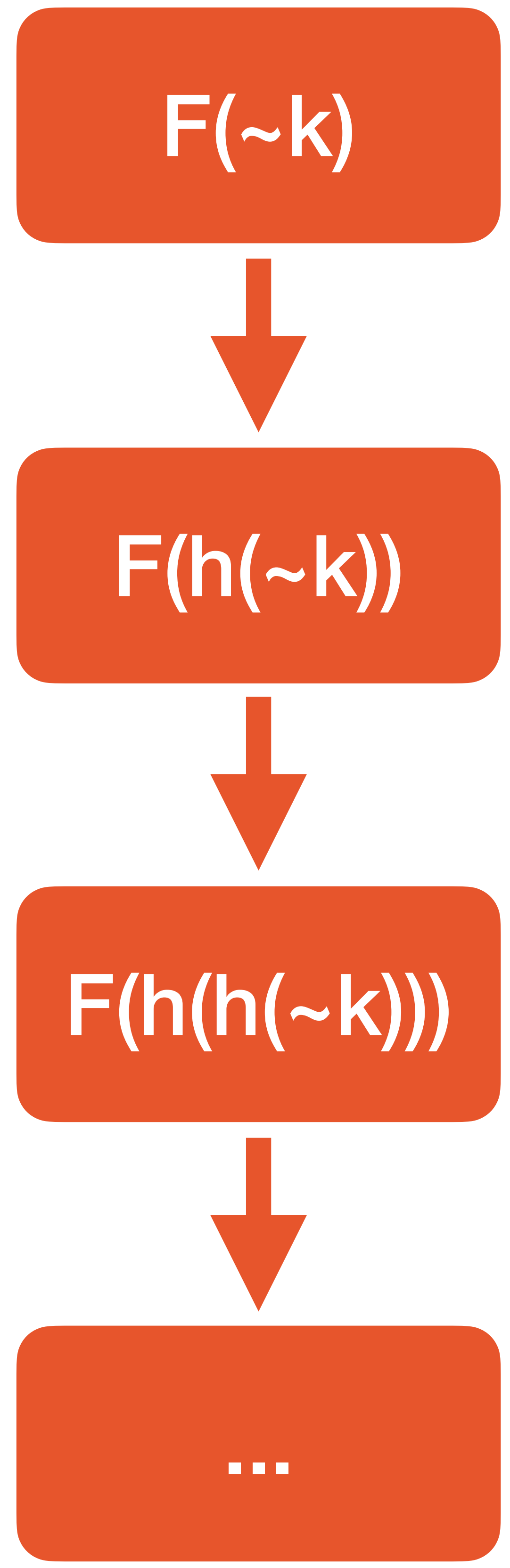

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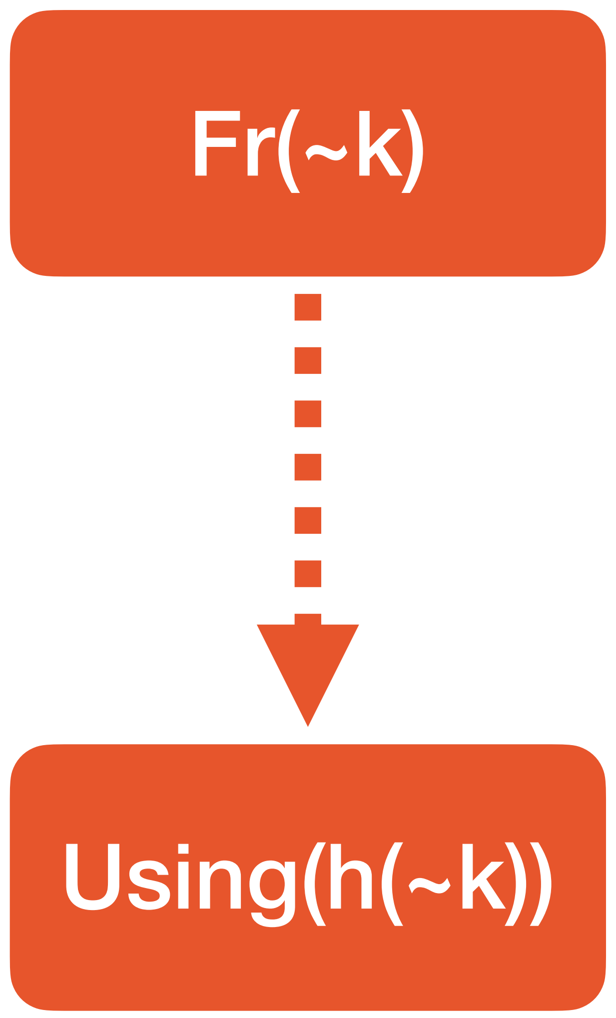

Dedicated Proof Techniques

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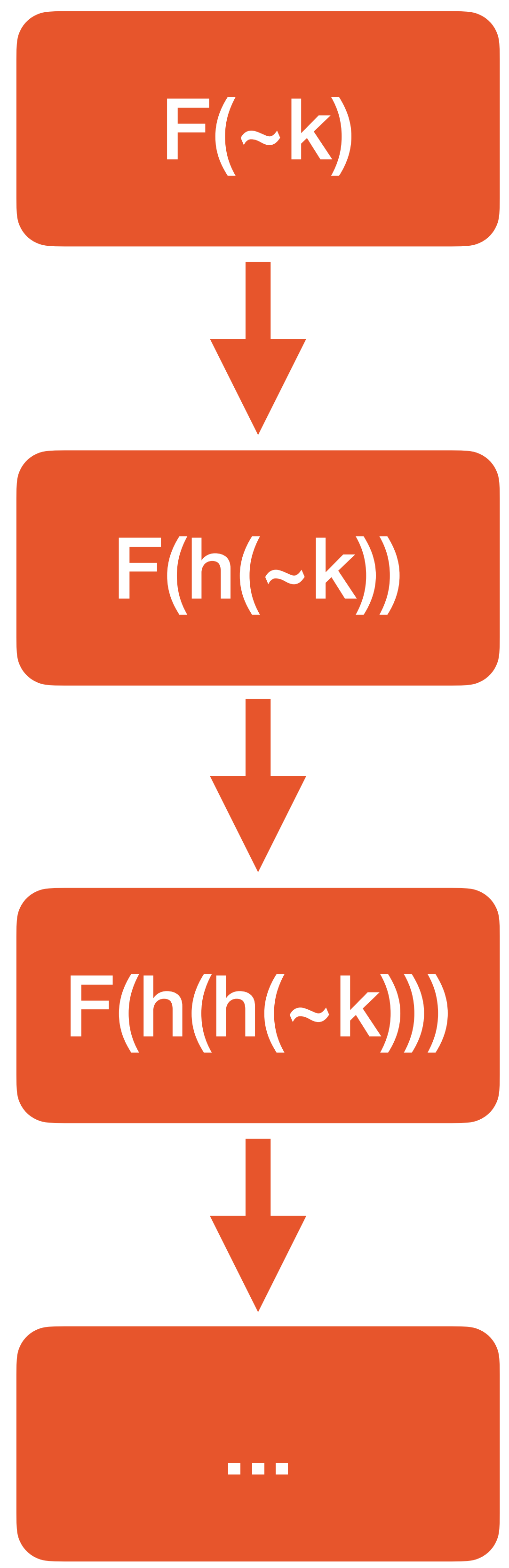

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Fresh Order

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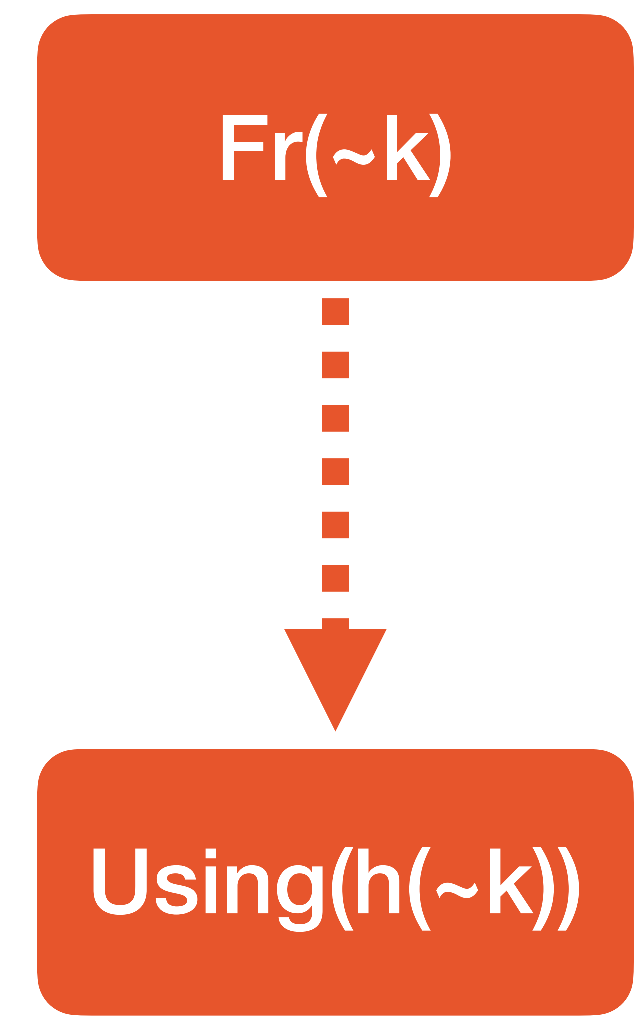
  Fr(~k) < Using(~k)
- great with $\sqsubset$



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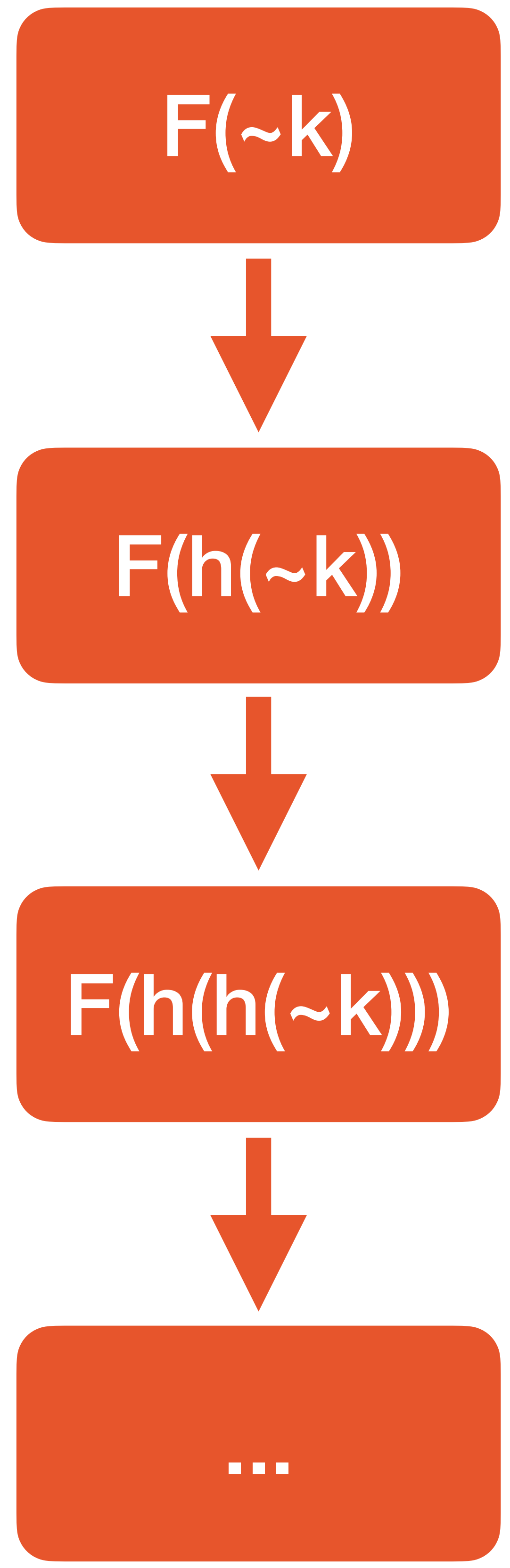

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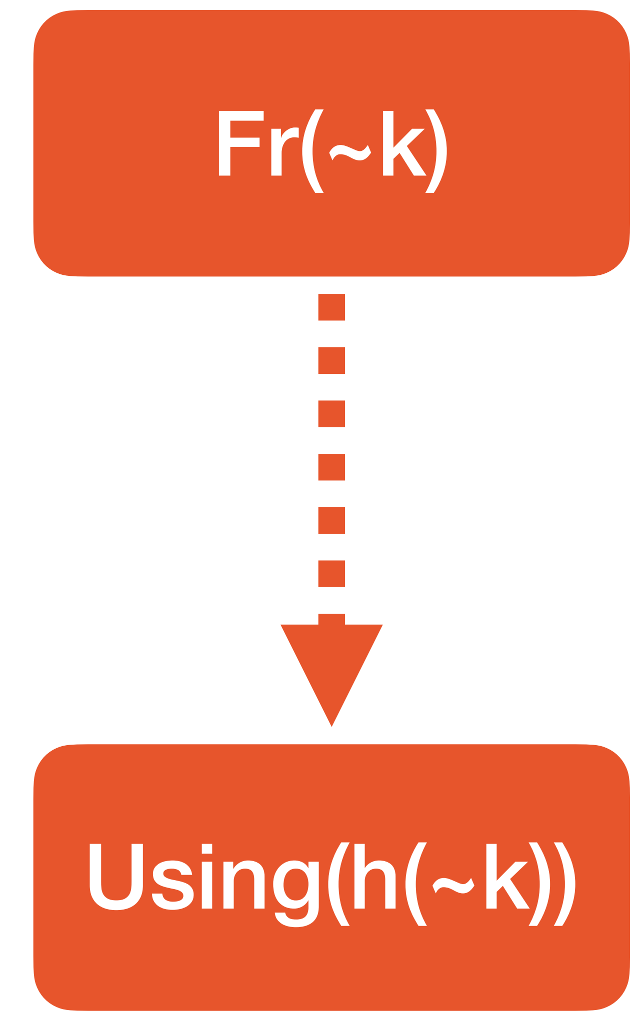

- 30x speed-up of CH'07 RFID proof

Applied to Existing Models

Protocol	Runtime	Helper-Lemmas	Why is it faster?
WPA-2	1:20h → 7min	74 → 73	monotonicity (of counters)
5G	8min → 2min	7 → 6	our number system
YubiKey	20s → 1s	4 → 3	our number system
PKCS#11	1min → 10s	4 → 0	each single improvement
CH'07 RFID	50min → 2min	0 → 0	fresh order

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https://www.ftsafe.com/products/OTP/Single_Button_OTP

Our Proofs

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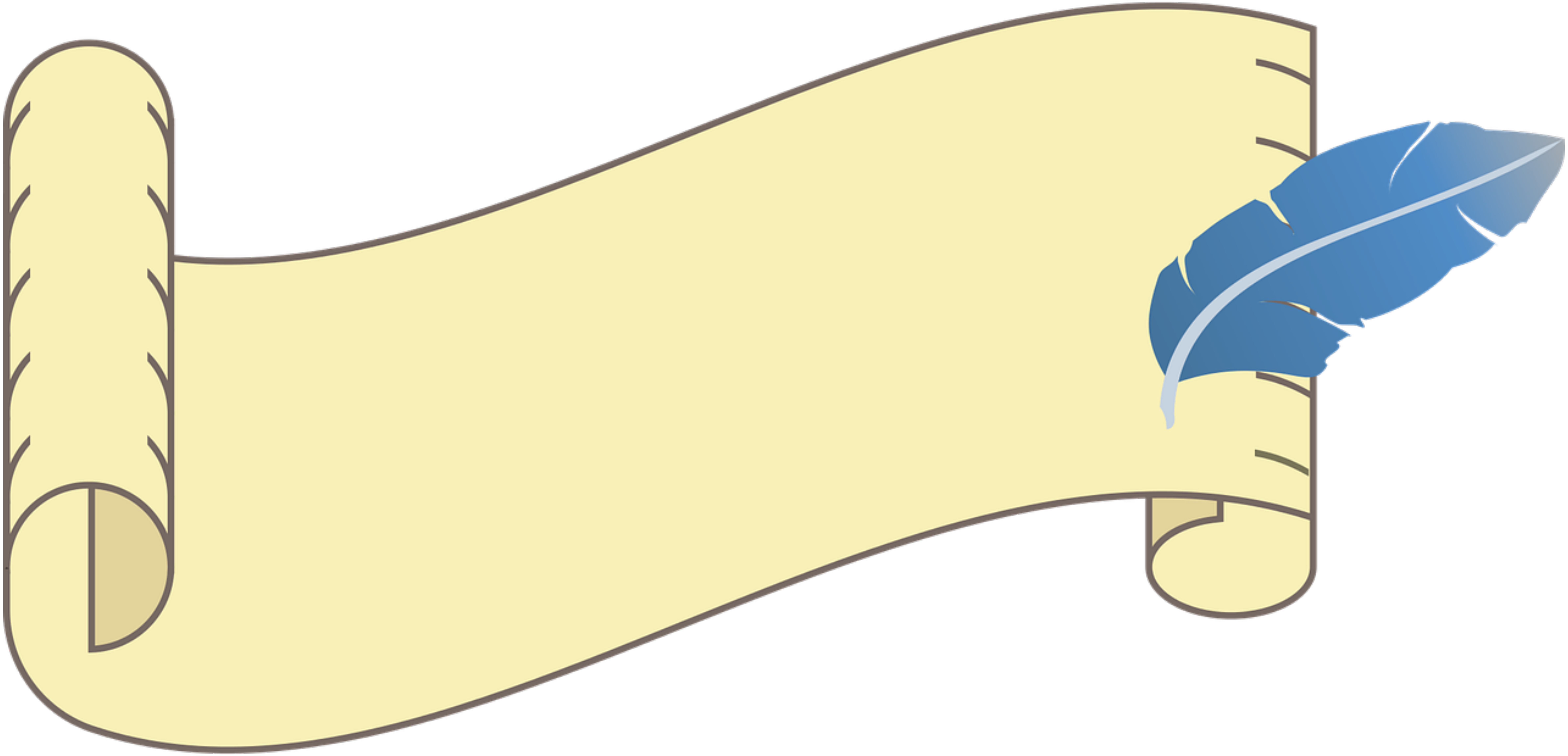
  - distributed tree
  - forward-secrecy
- S/Key
  - hash-chain
  - authentication
- Tesla Scheme 2
  - hash-chain like S/Key
  - authentication, secrecy
  - prev. example of Tamarins limits

https://www.ftsafe.com/products/OTP/Single_Button_OTP

Paper Summary



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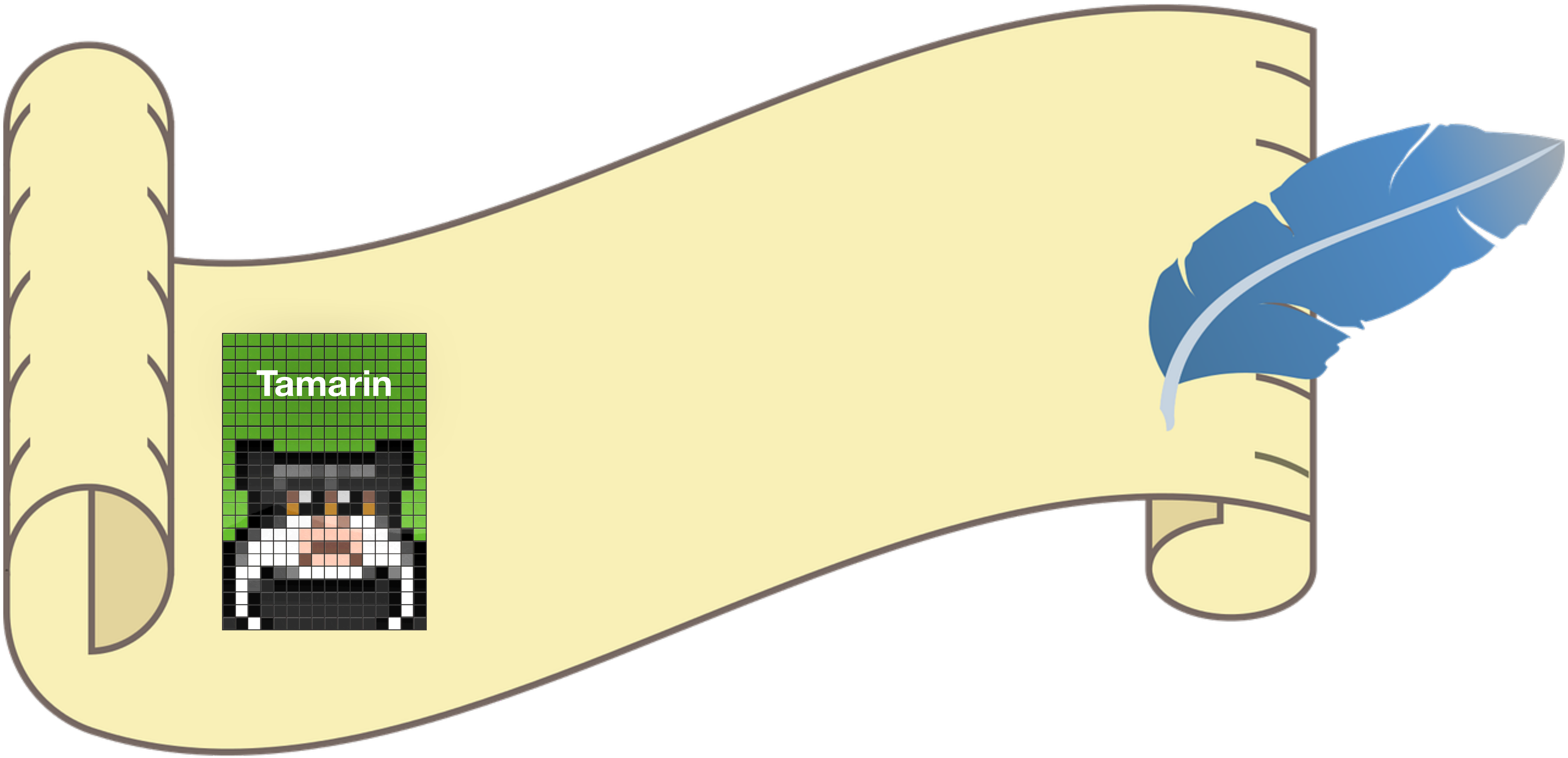

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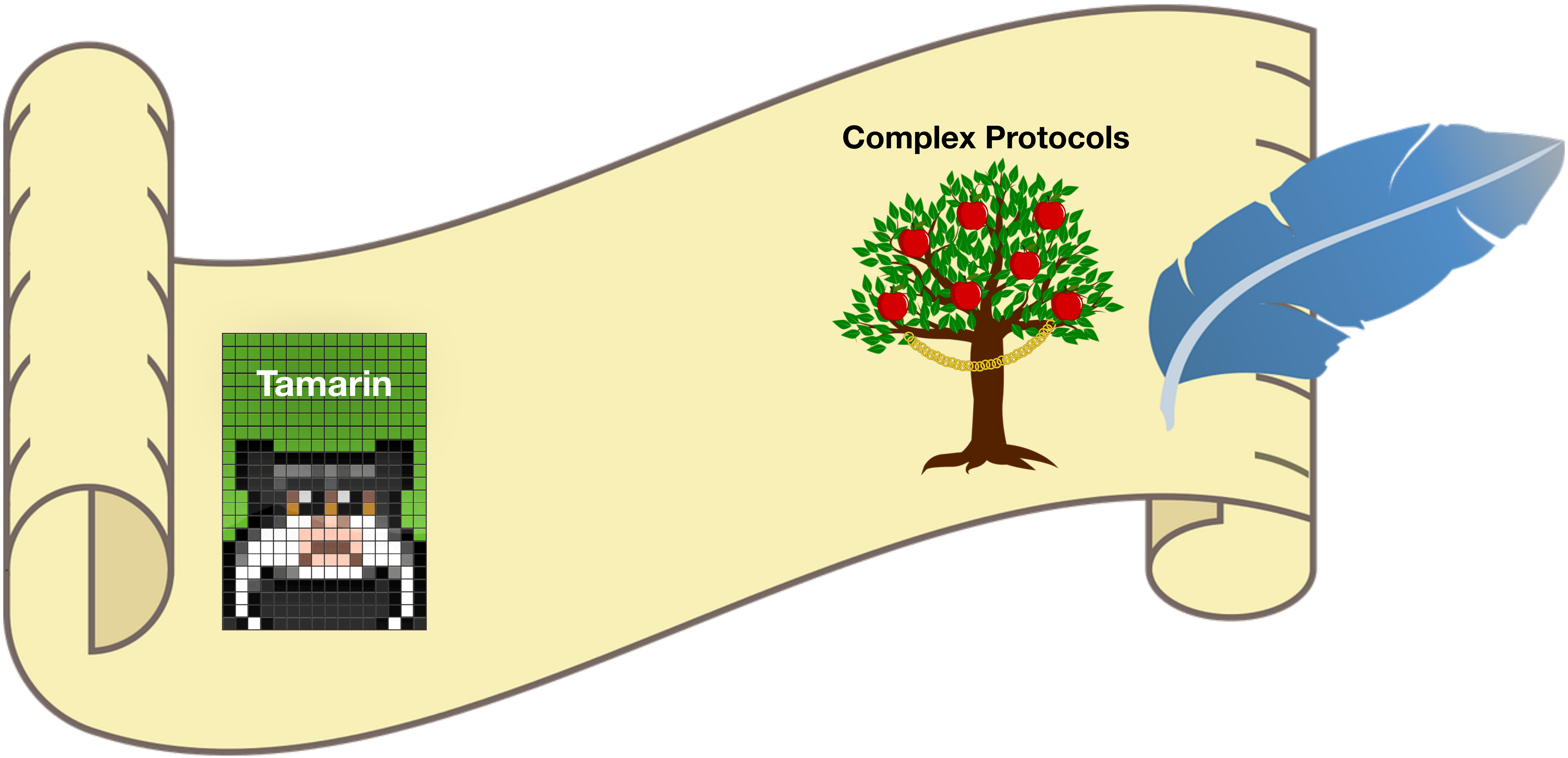

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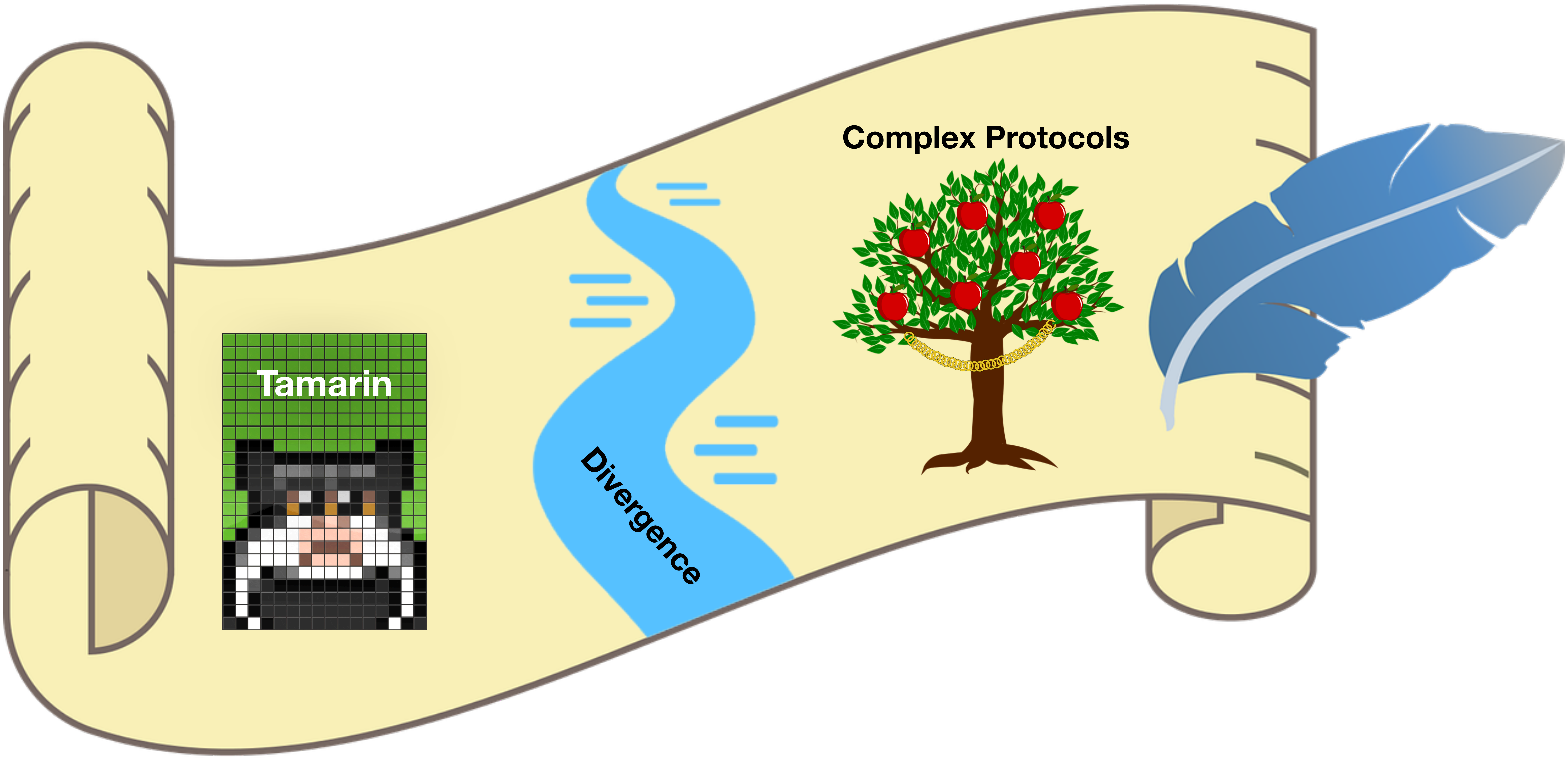

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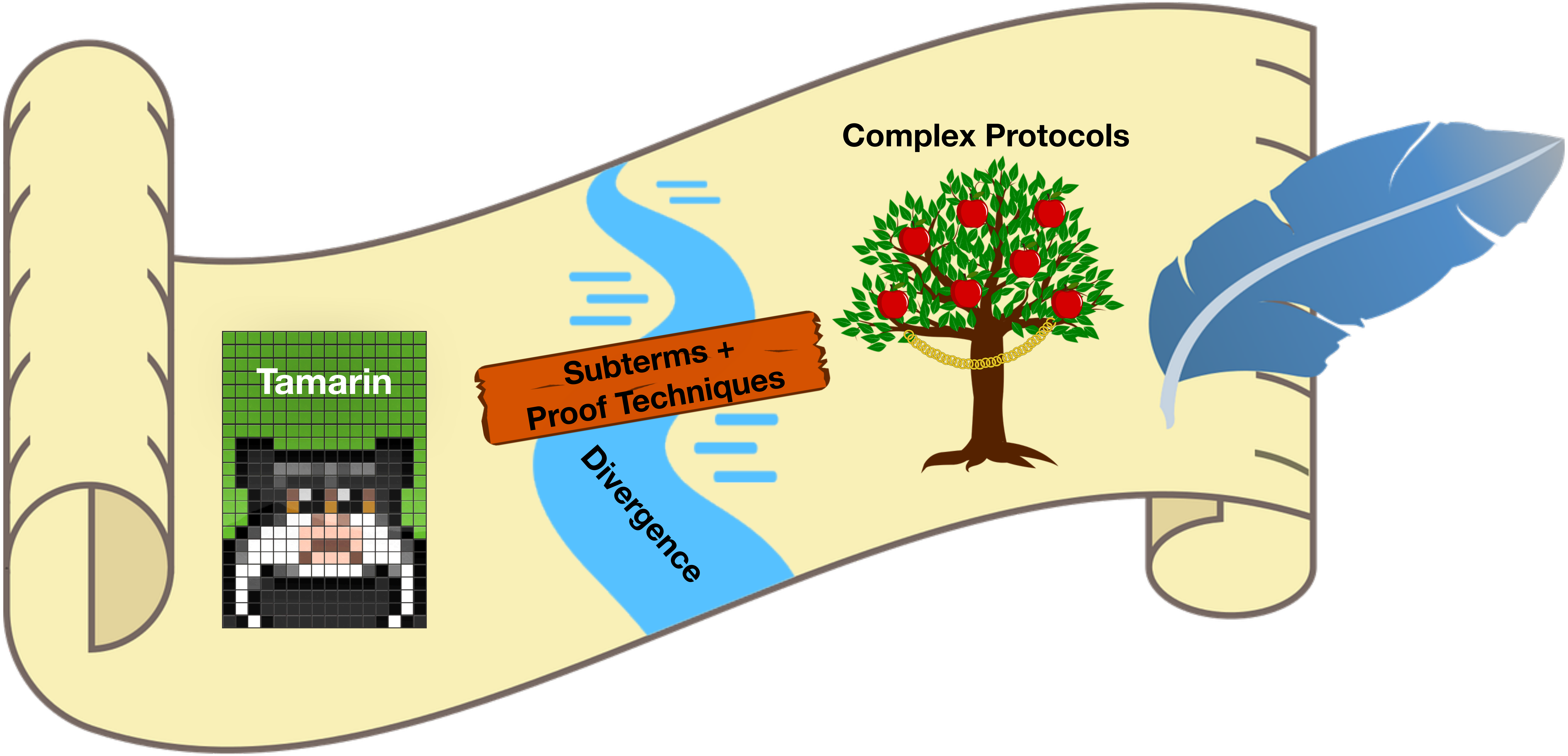

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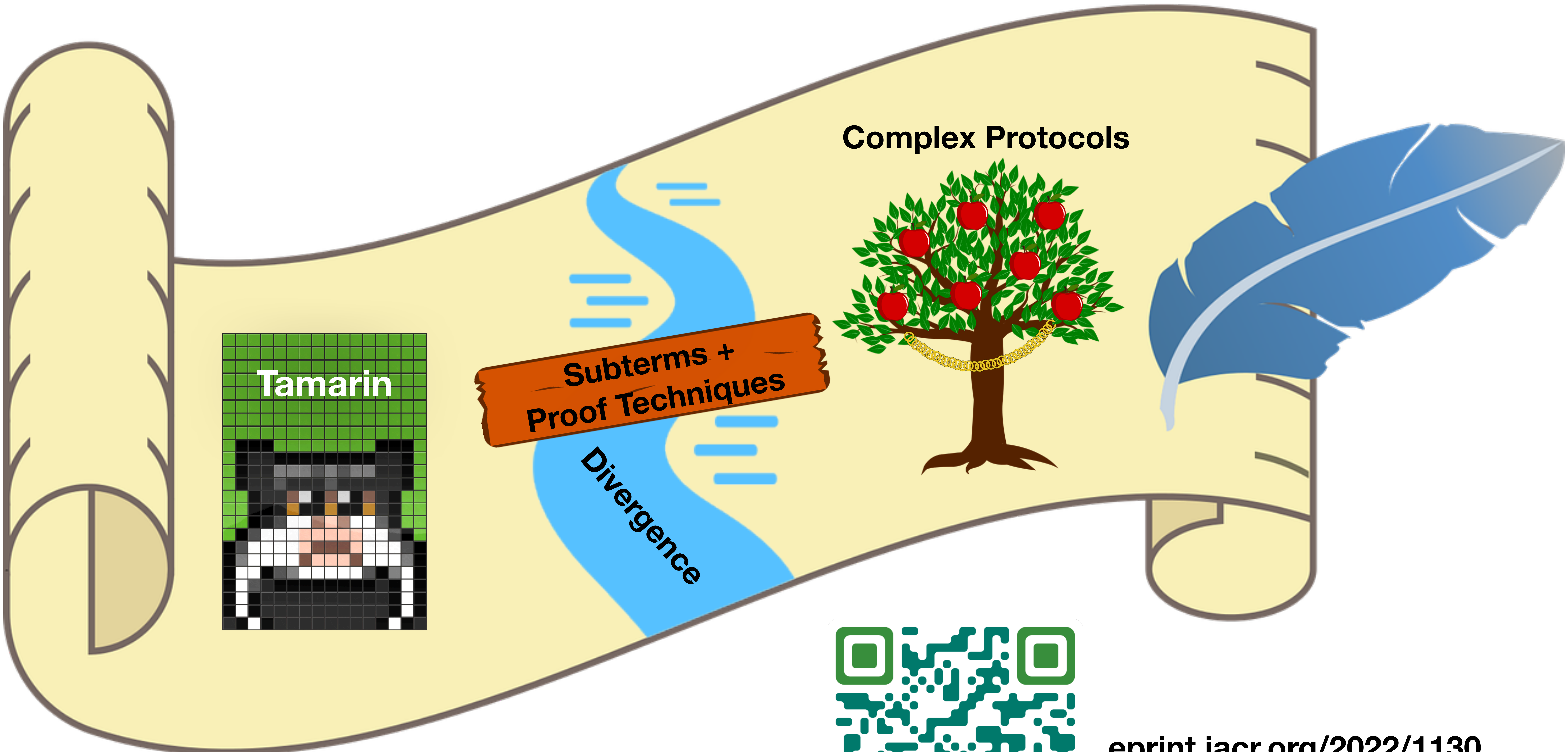

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