Cross Chain Swaps with Preferences

Eric Chan* Marek Chrobak Mohsen Lesani

University of California at Riverside, USA

CSF 2023

Cross Chain Swap

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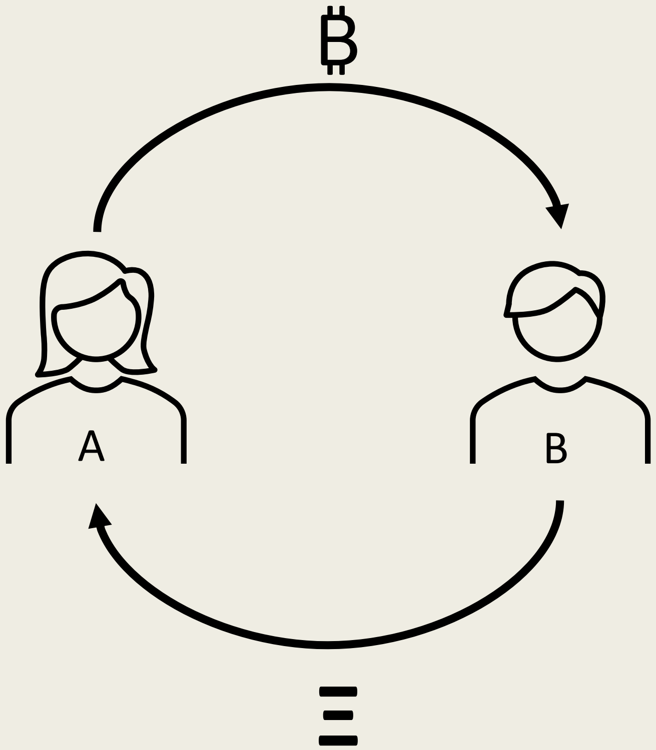
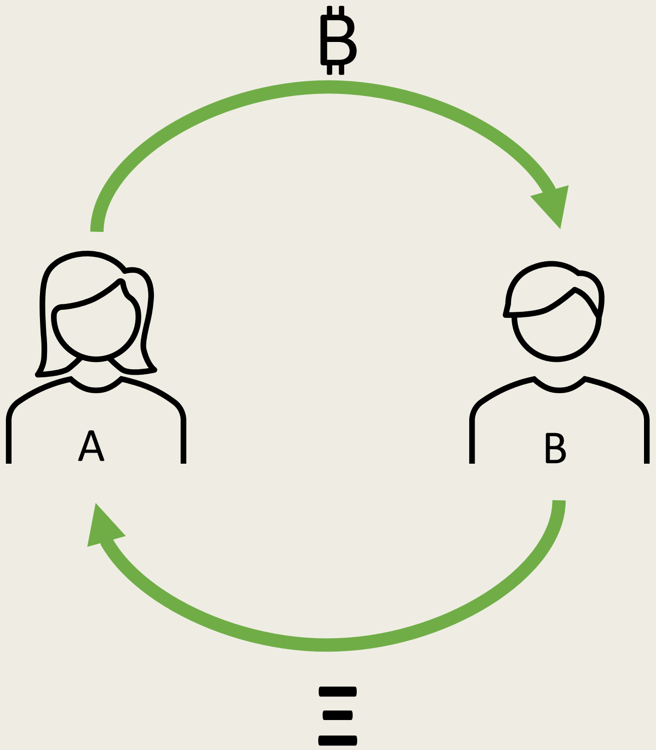
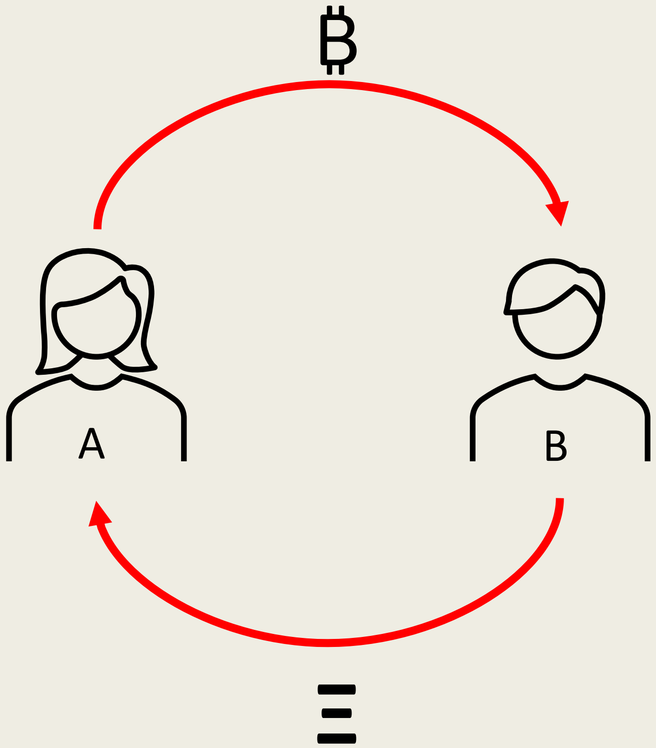


Cross Chain Swap – Fair Exchange





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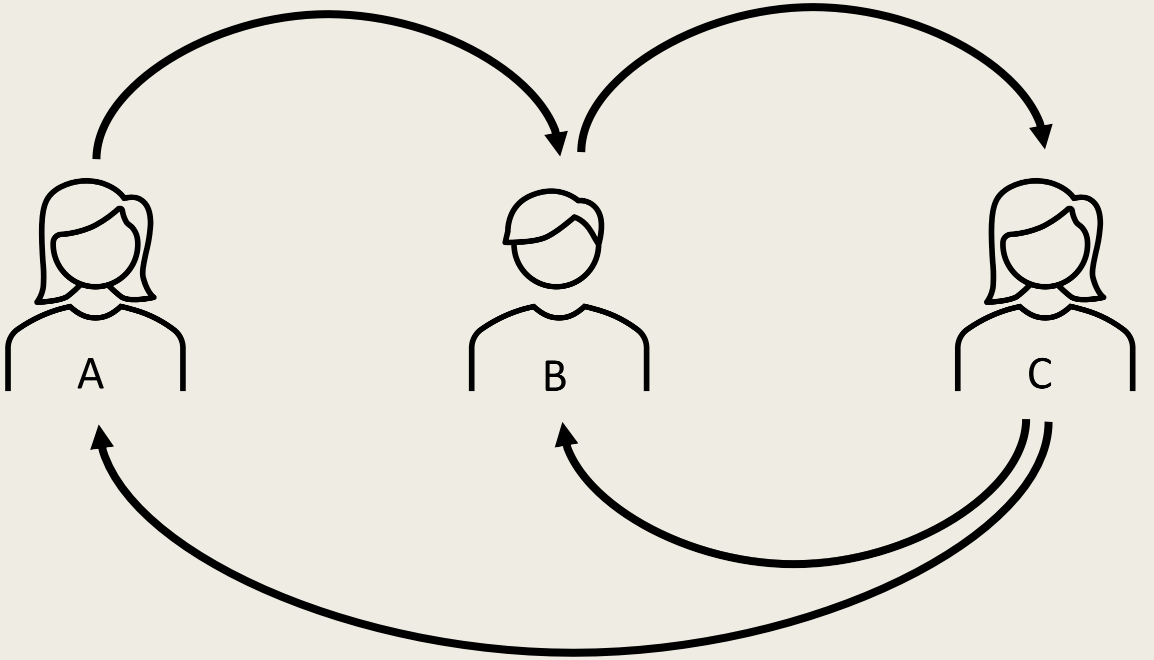
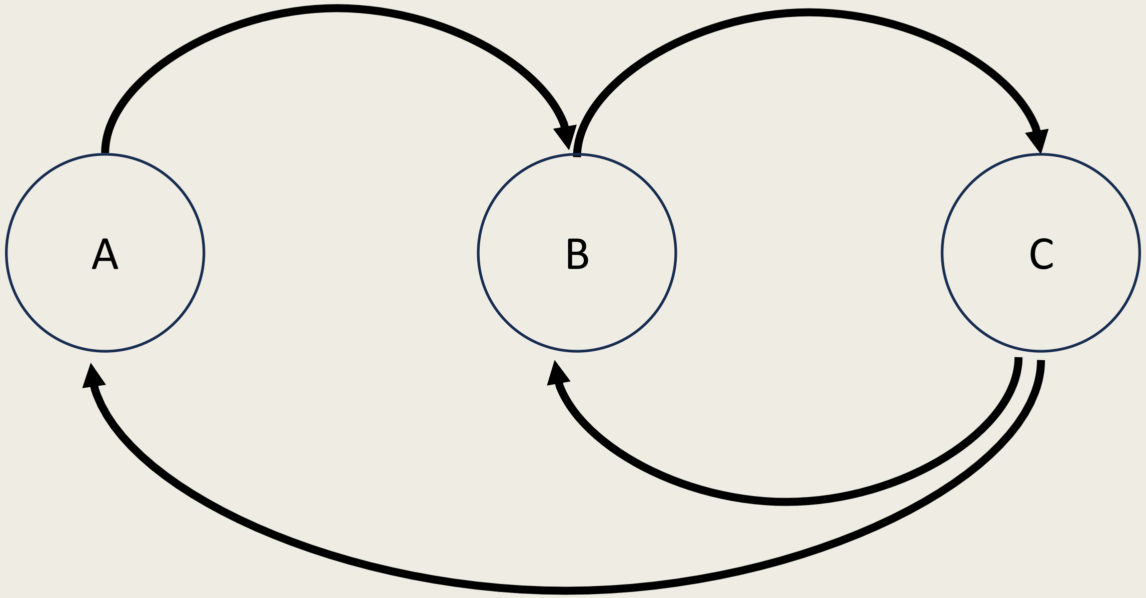
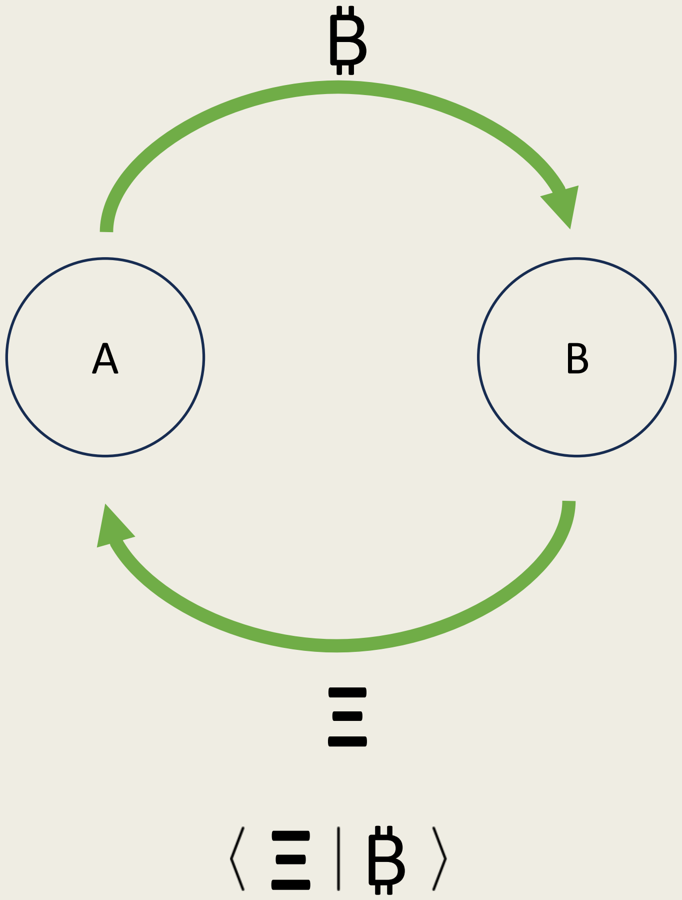


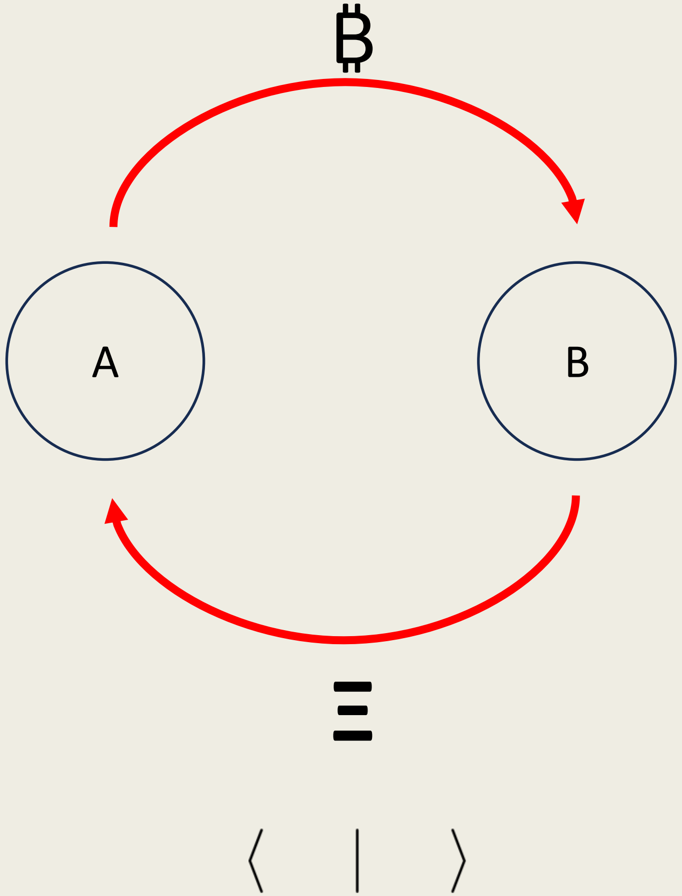
Formalization

Swap Digraph



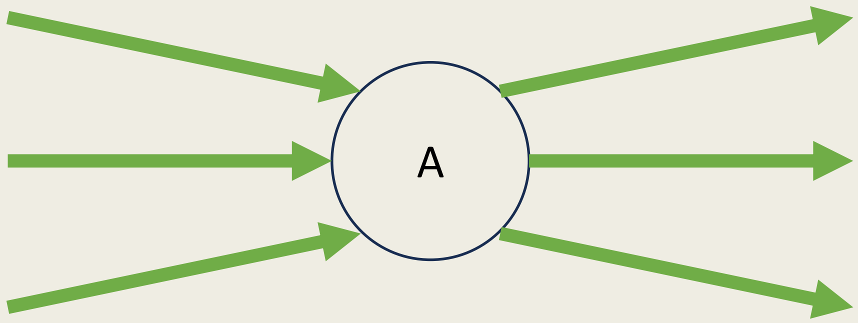
Outcomes





Outcomes

- DEAL: $\langle all \mid all \rangle$
- NODEAL: $\langle none \mid none \rangle$
- DISCOUNT: $\langle all \mid \neg all \rangle$
- FREERIDE: $\langle \neg none \mid none \rangle$
- UNDERWATER: $\langle \neg all \mid \neg none \rangle$ (everything else)



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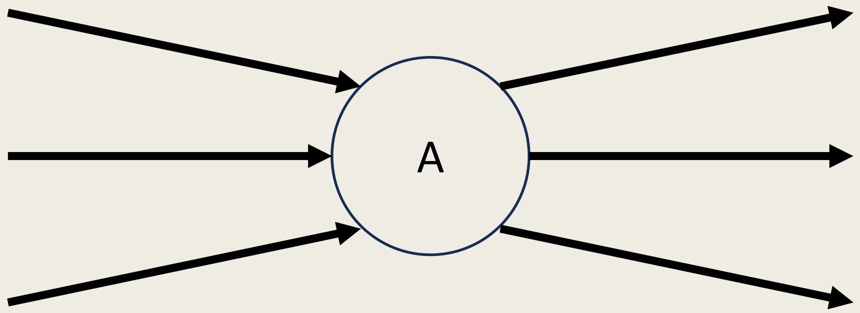

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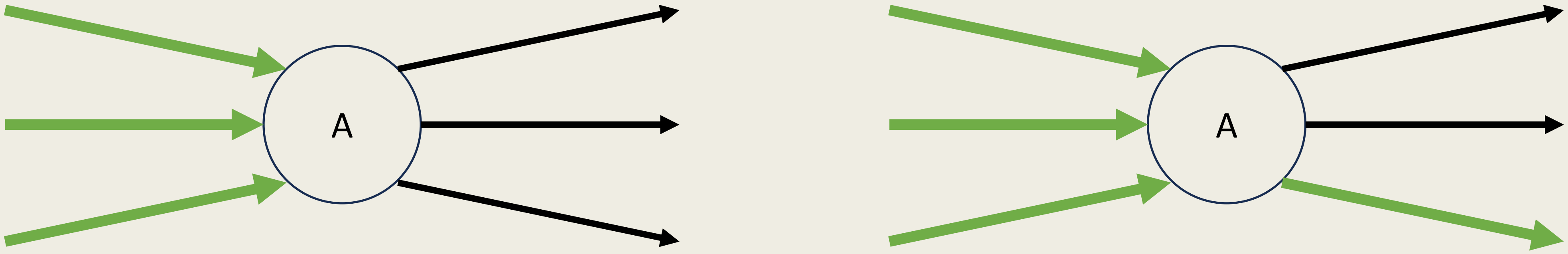

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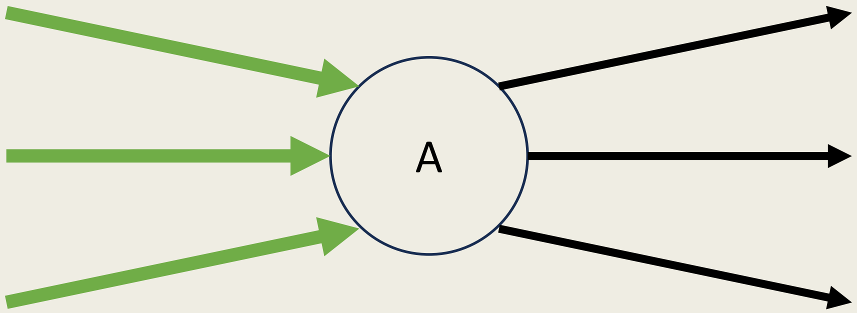
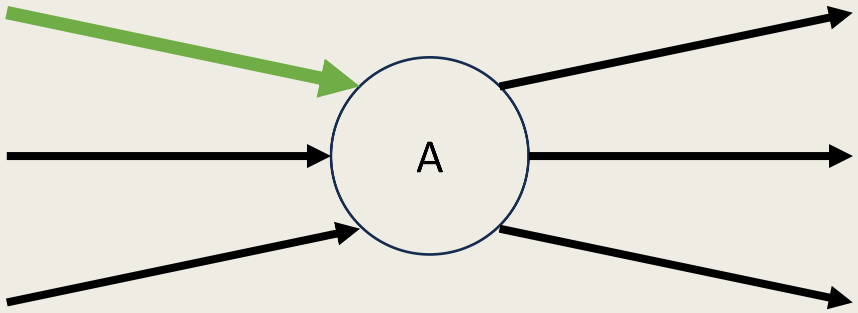
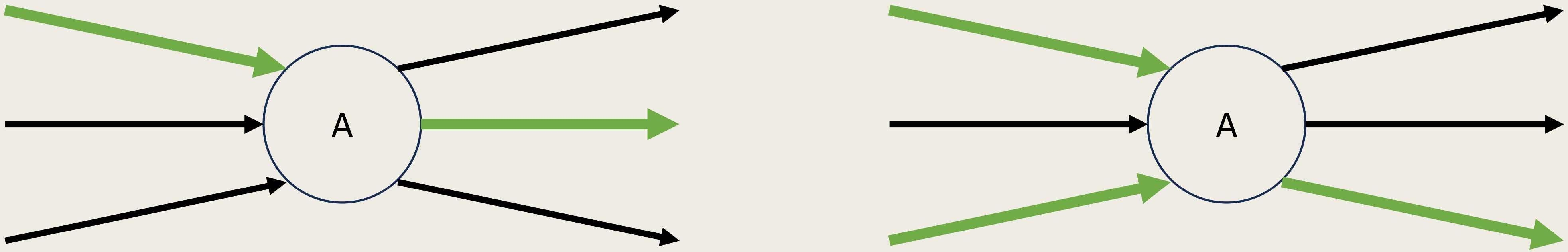




Outcomes

- $\textsc{Deal}$: $\langle all \mid all \rangle$
- $\textsc{NoDeal}$: $\langle none \mid none \rangle$
- $\textsc{Discount}$: $\langle all \mid \neg all \rangle$
- $\textsc{FreeRide}$: $\langle \neg none \mid none \rangle$
- $\textsc{Underwater}$: $\langle \neg all \mid \neg none \rangle$ (everything else)



Partial Ordering of Outcomes

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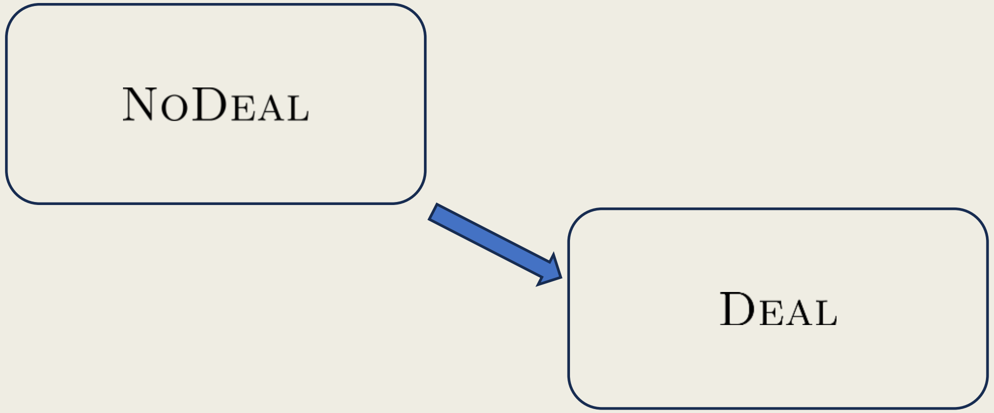

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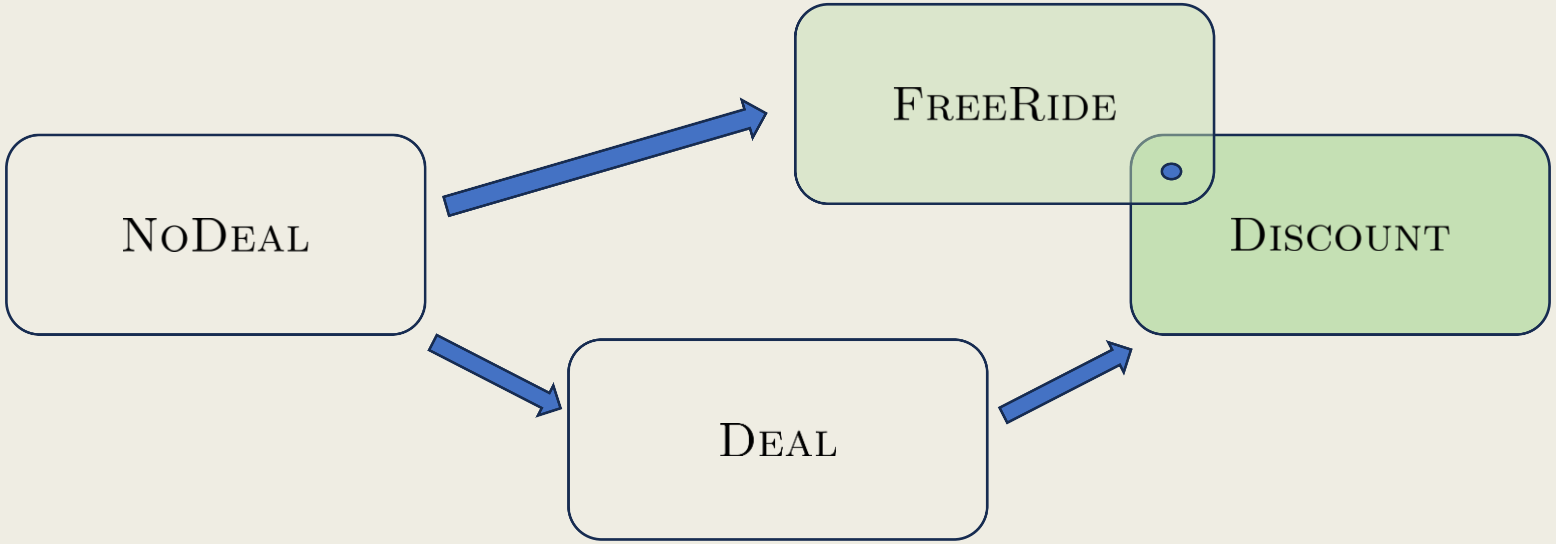

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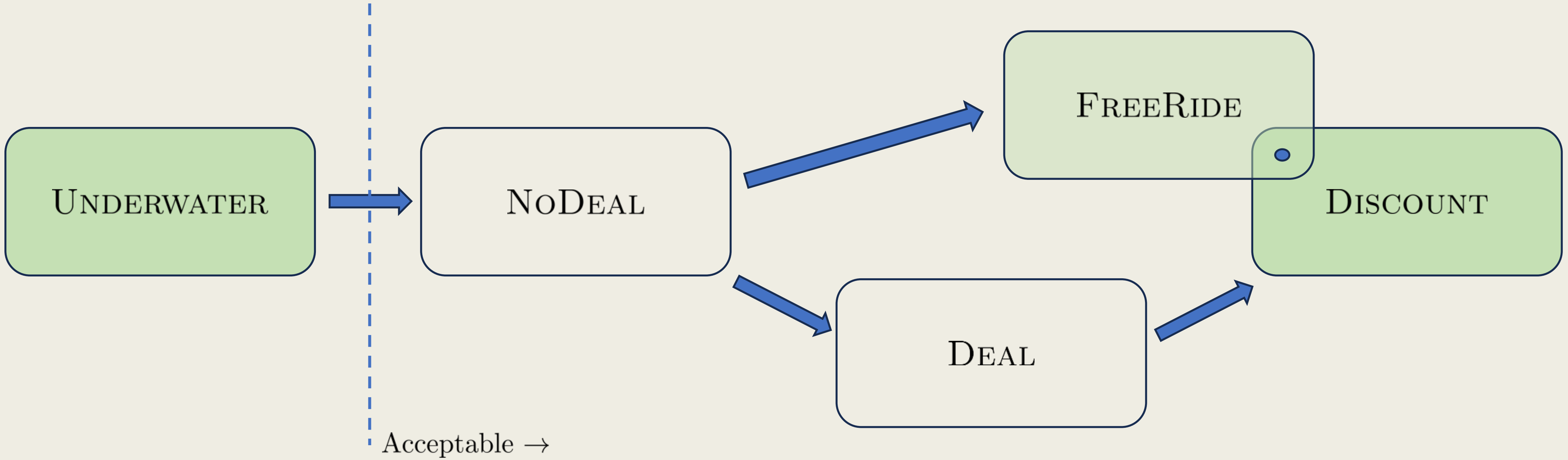
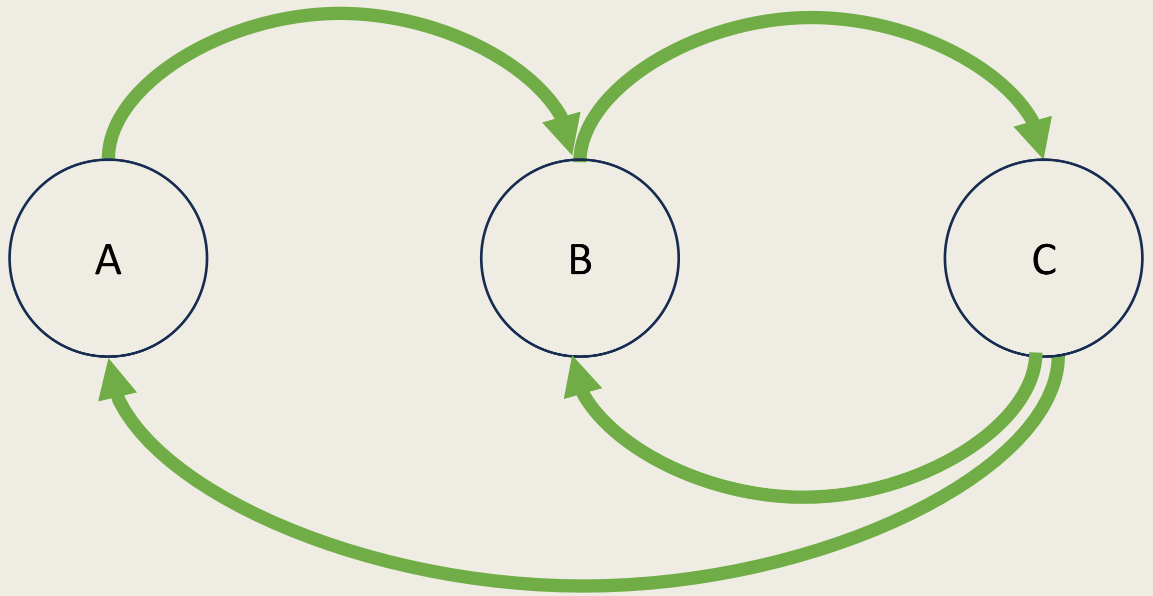


Protocol Properties

Atomic Protocol Properties

- *Liveness*: if every party follows $\mathbb{P}$, then every party finishes DEAL
- *Safety*: if a party follows $\mathbb{P}$, then it finishes in an acceptable outcome
- *Strong Nash Equilibria*: No coalition improves its payoff by deviating from $\mathbb{P}$



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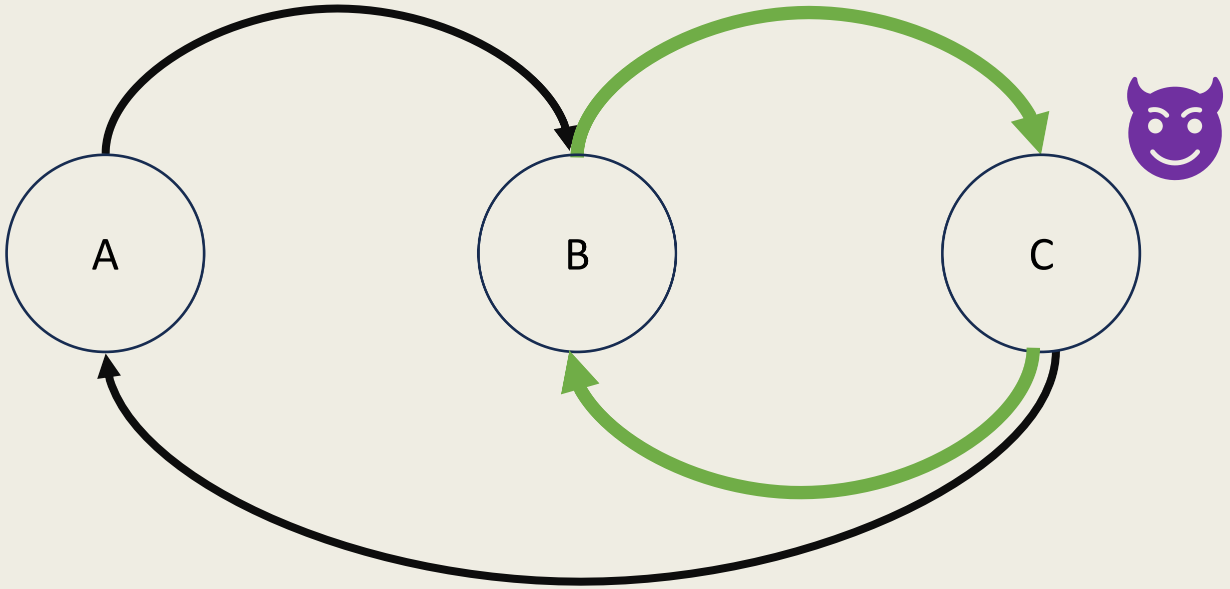

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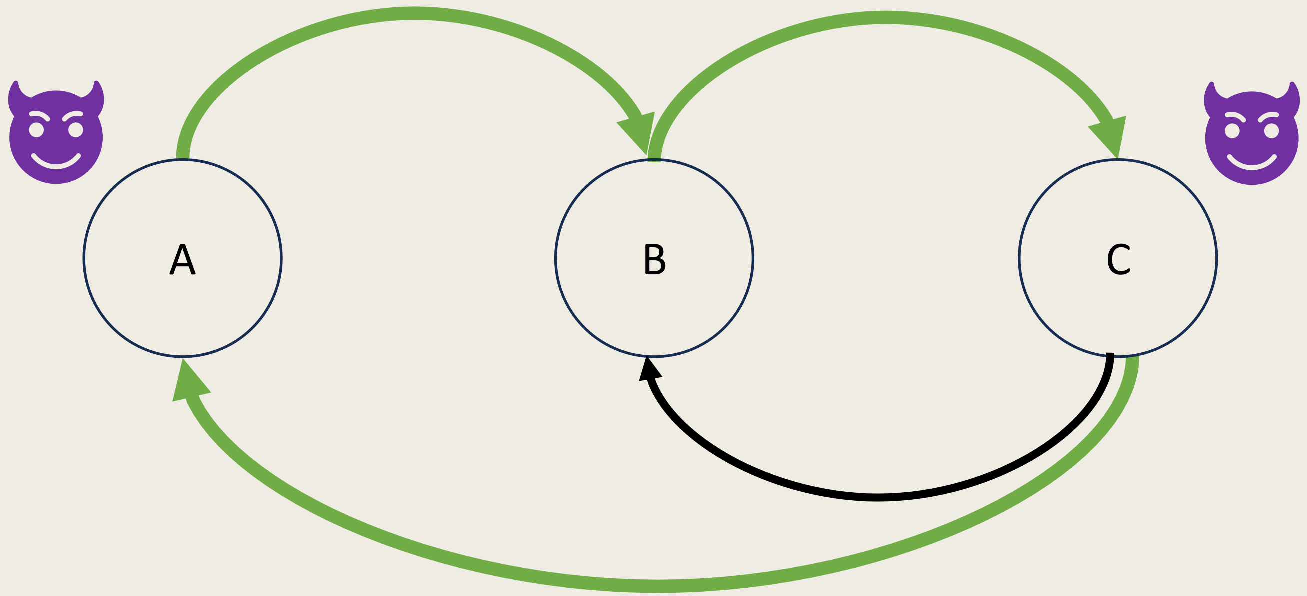
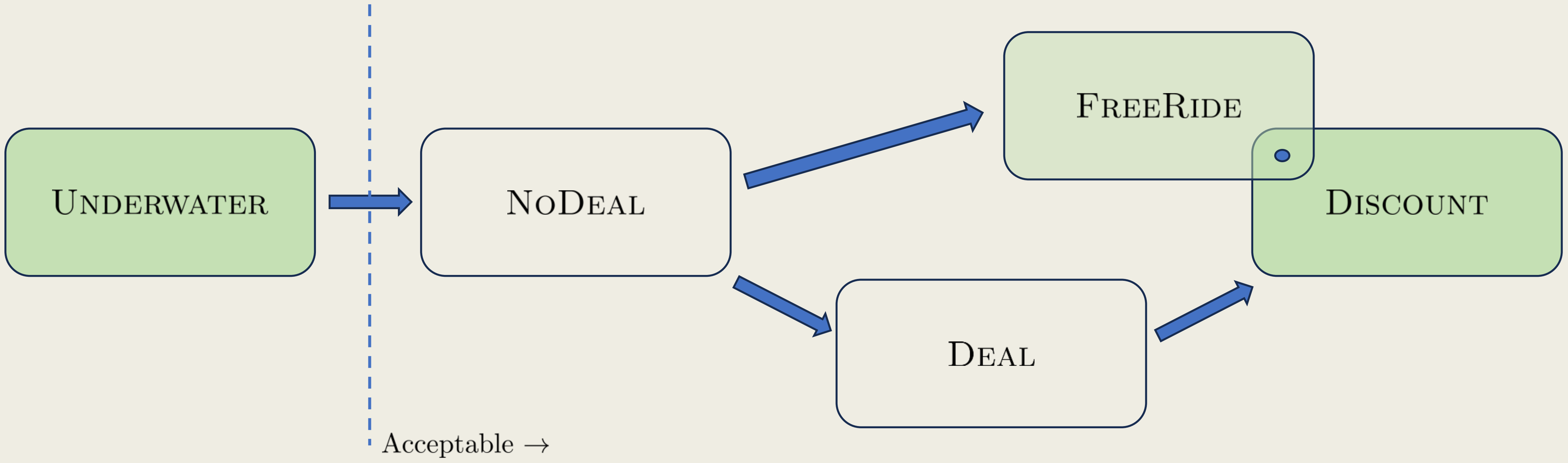


Herlihy's Protocol

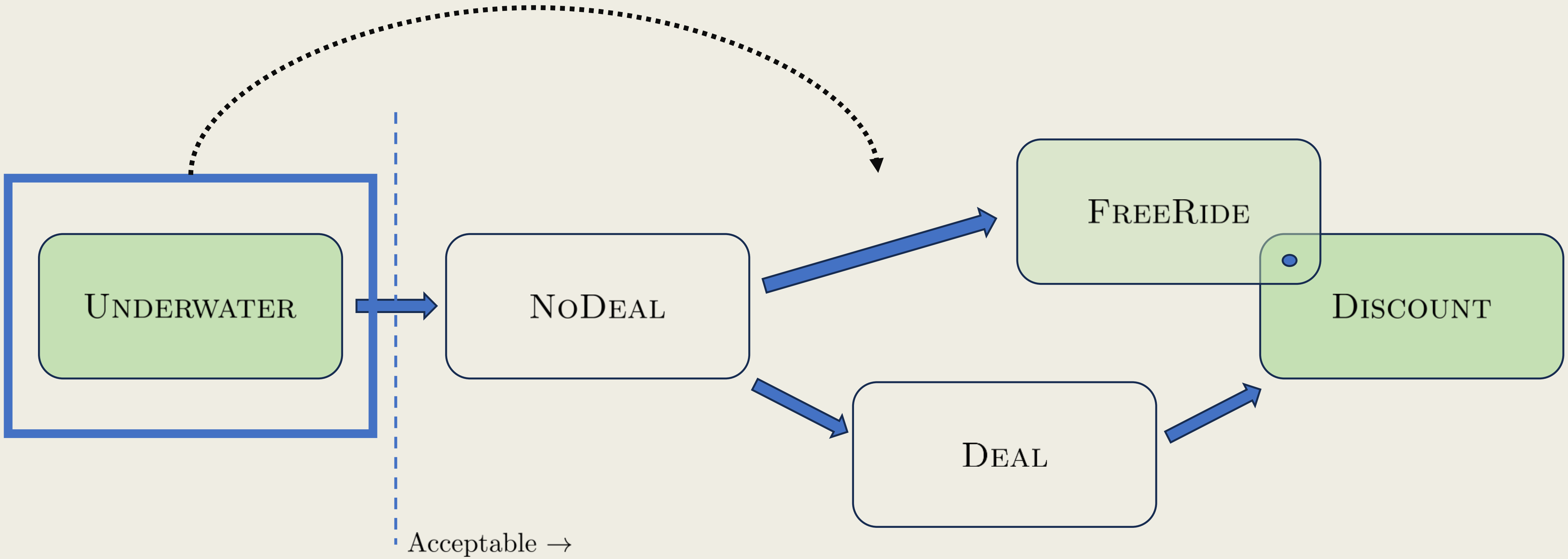
[Herlihy'18] gives an atomic protocol so long that:

- the swap digraph is strongly connected
- each party has the preference structure:

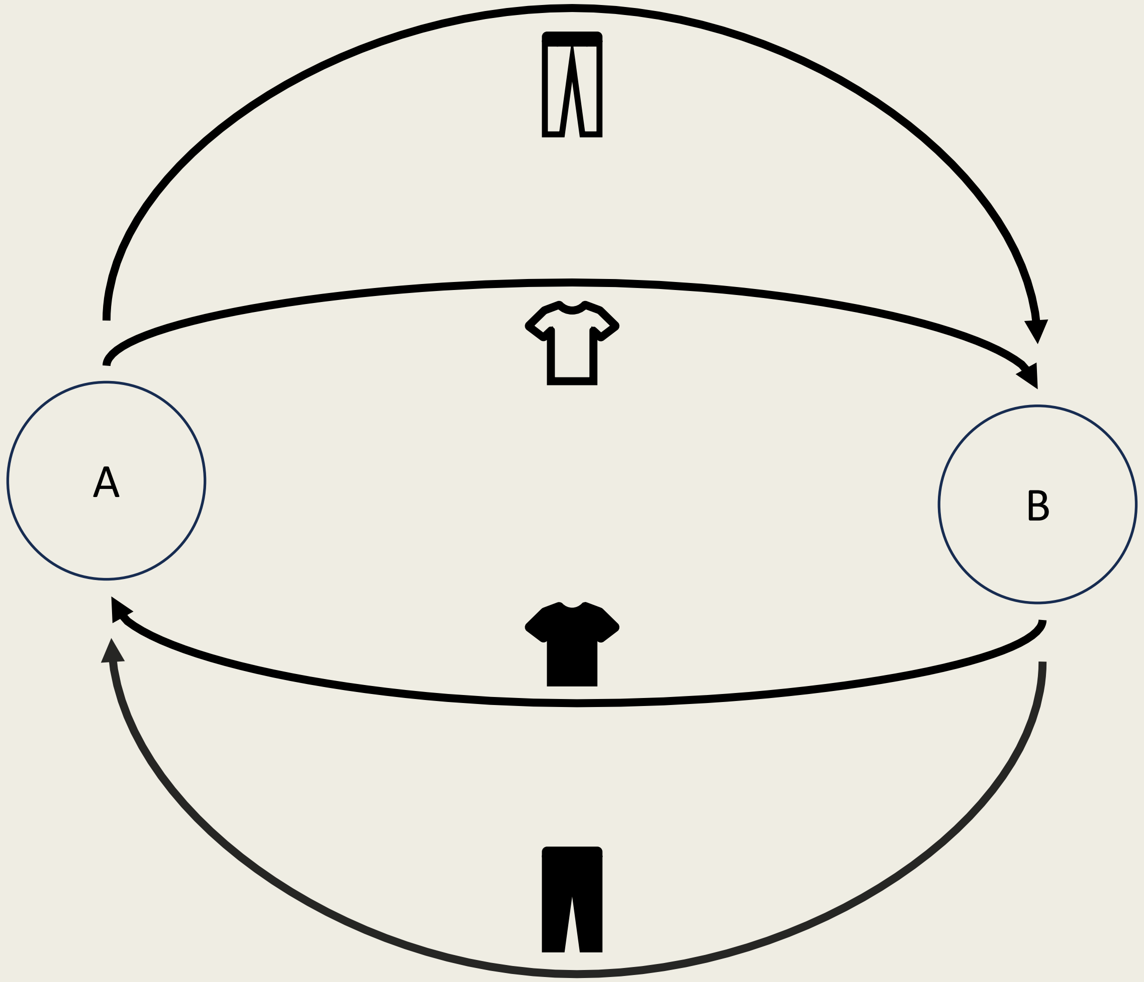


Can We Do Better?

The Underwater Class



Preferences



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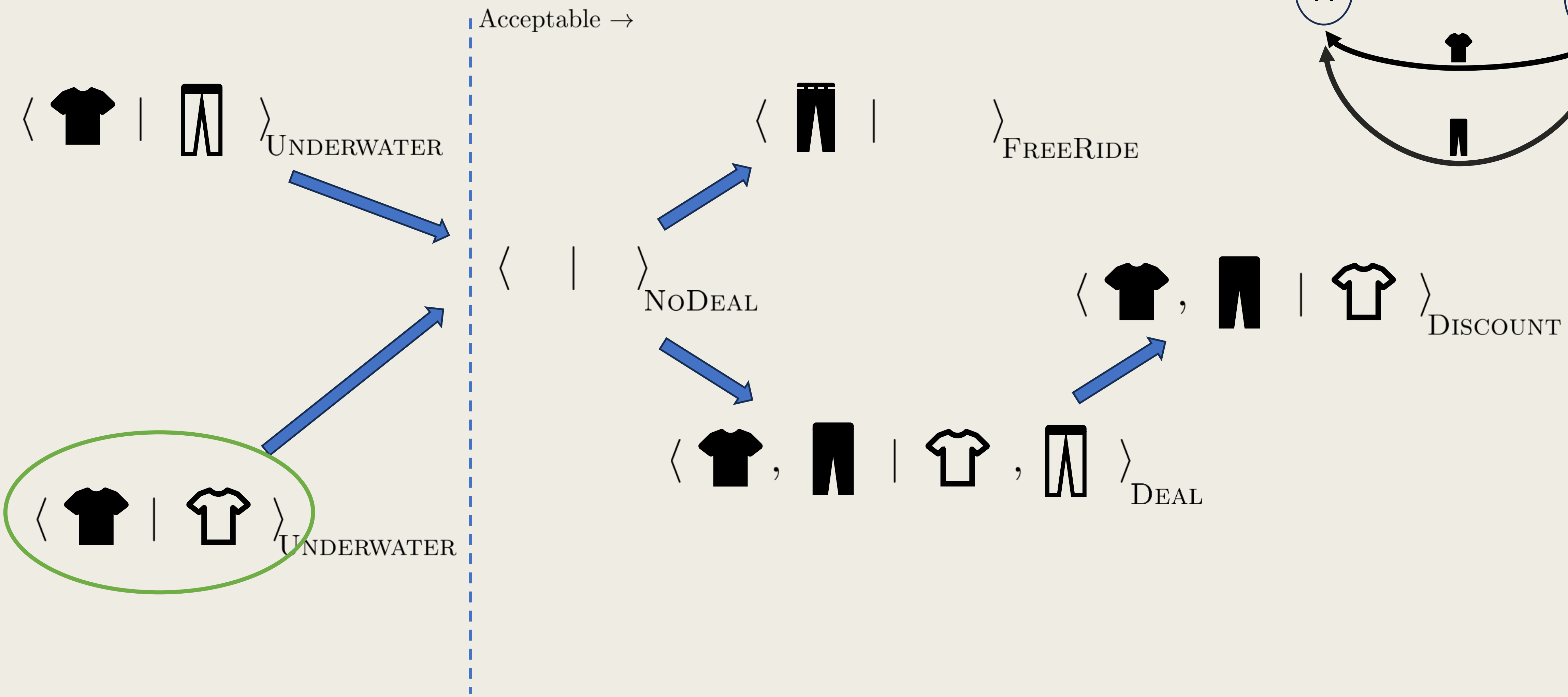
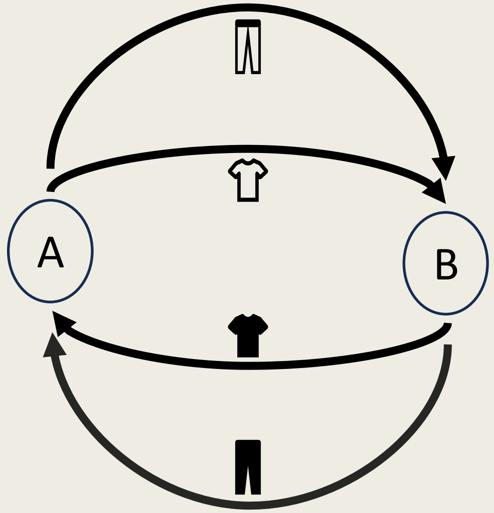
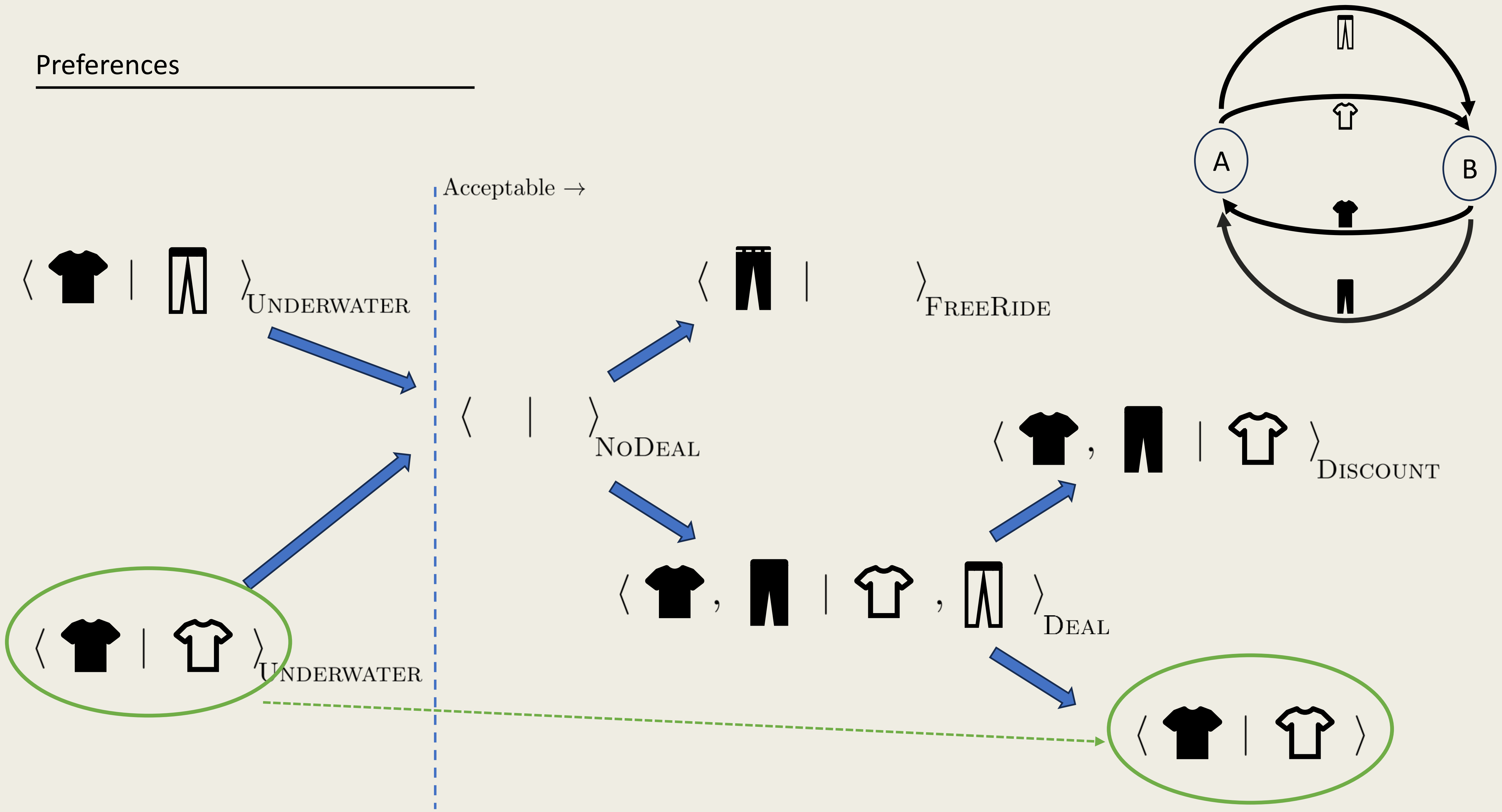
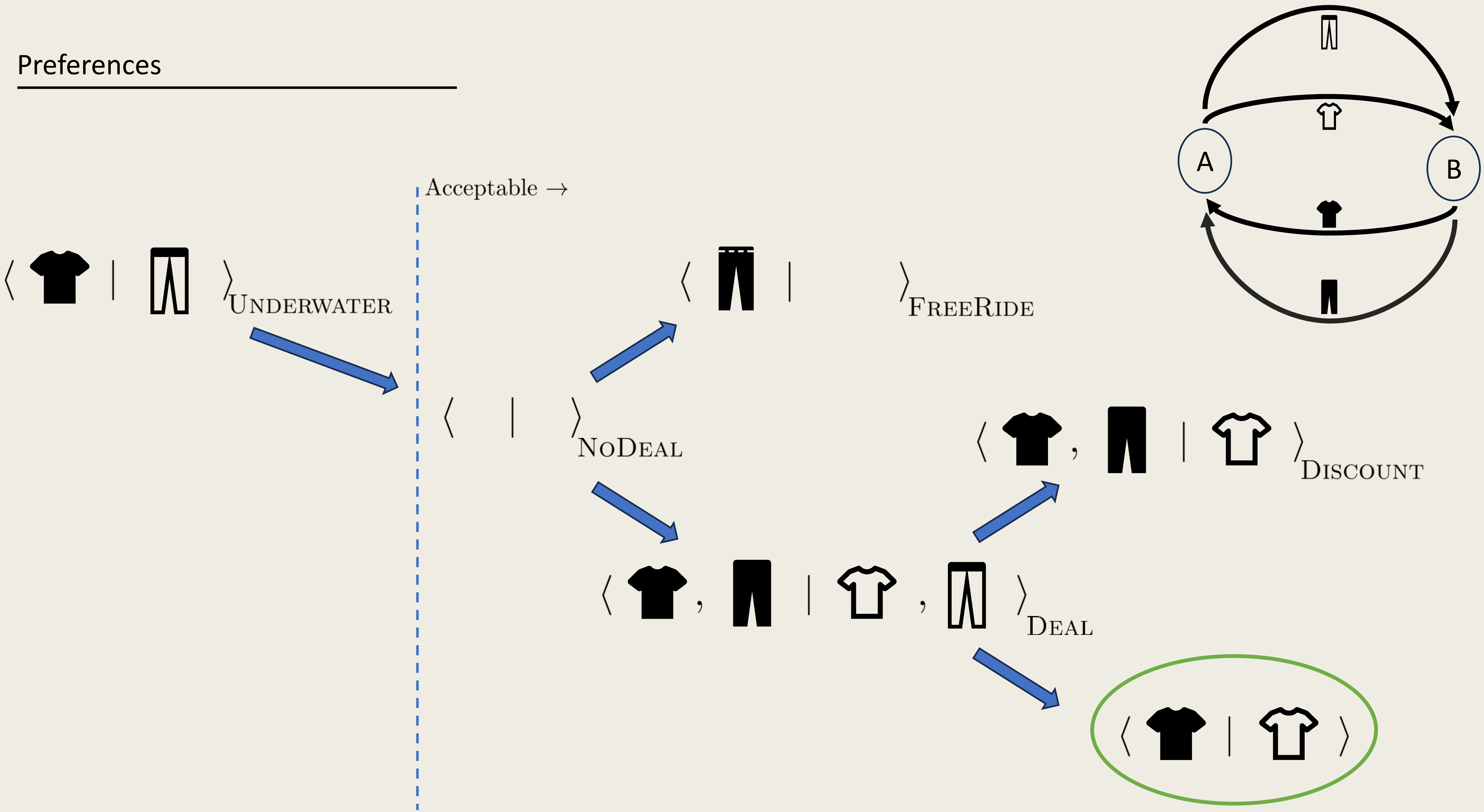




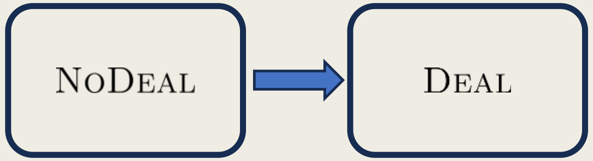
Preferences



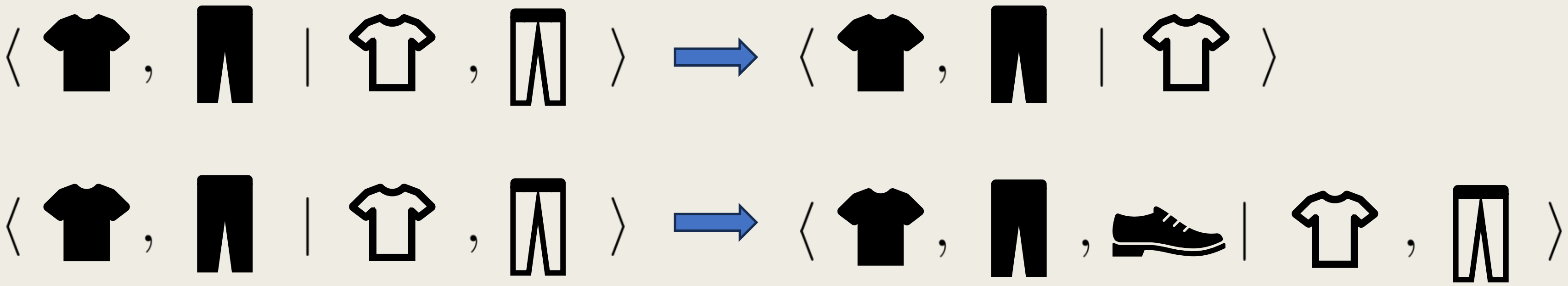
Preferences



User-defined Preferences

- 

- *Inclusive Monotonicity*:



General Atomic Protocol?

- *Liveness*: if every party follows $\mathbb{P}$, then every party finishes DEAL *or better*
- *Safety*: if a party follows $\mathbb{P}$, then it finishes in an acceptable outcome
- *Strong Nash Equilibria*: No coalition improves its payoff by deviating from $\mathbb{P}$

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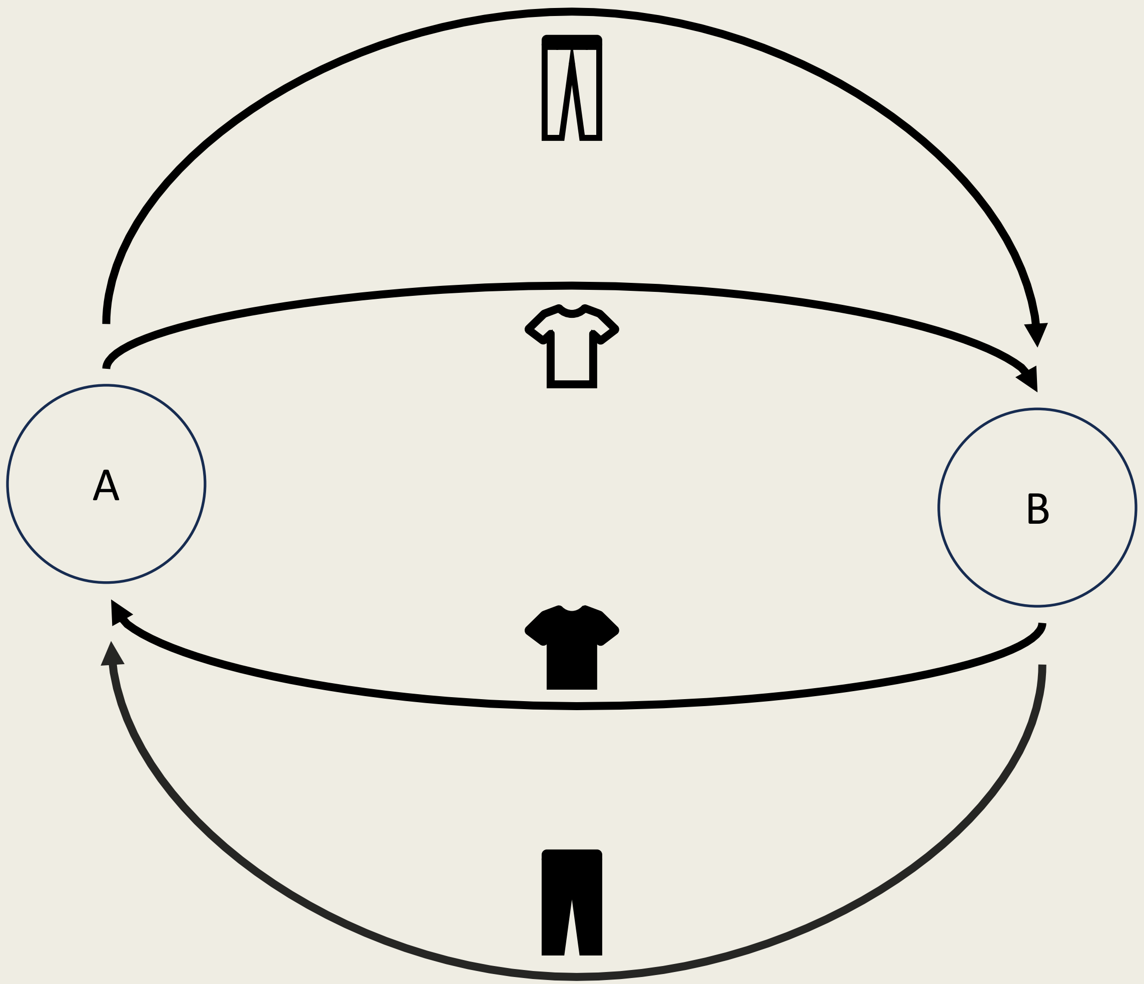
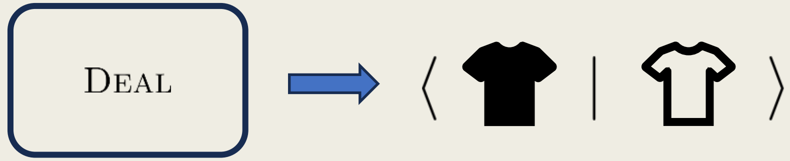
No, there is no atomic protocol (scheme) that works for every swap system.

No General Atomic Protocol



Preference of A:



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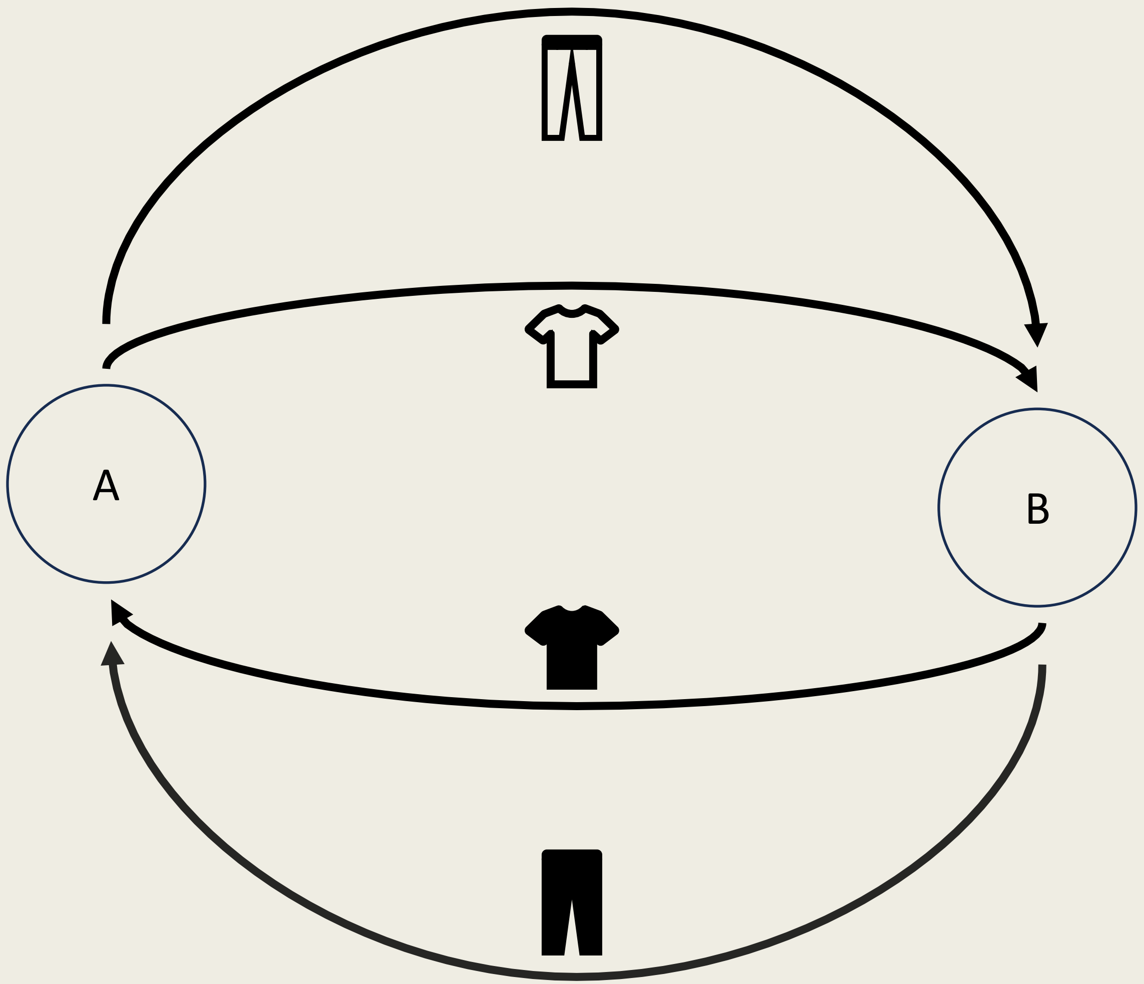

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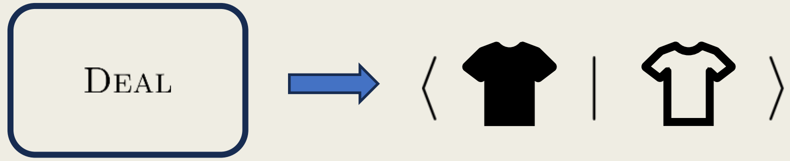
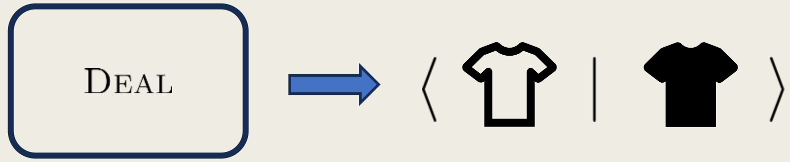


Preference of B:



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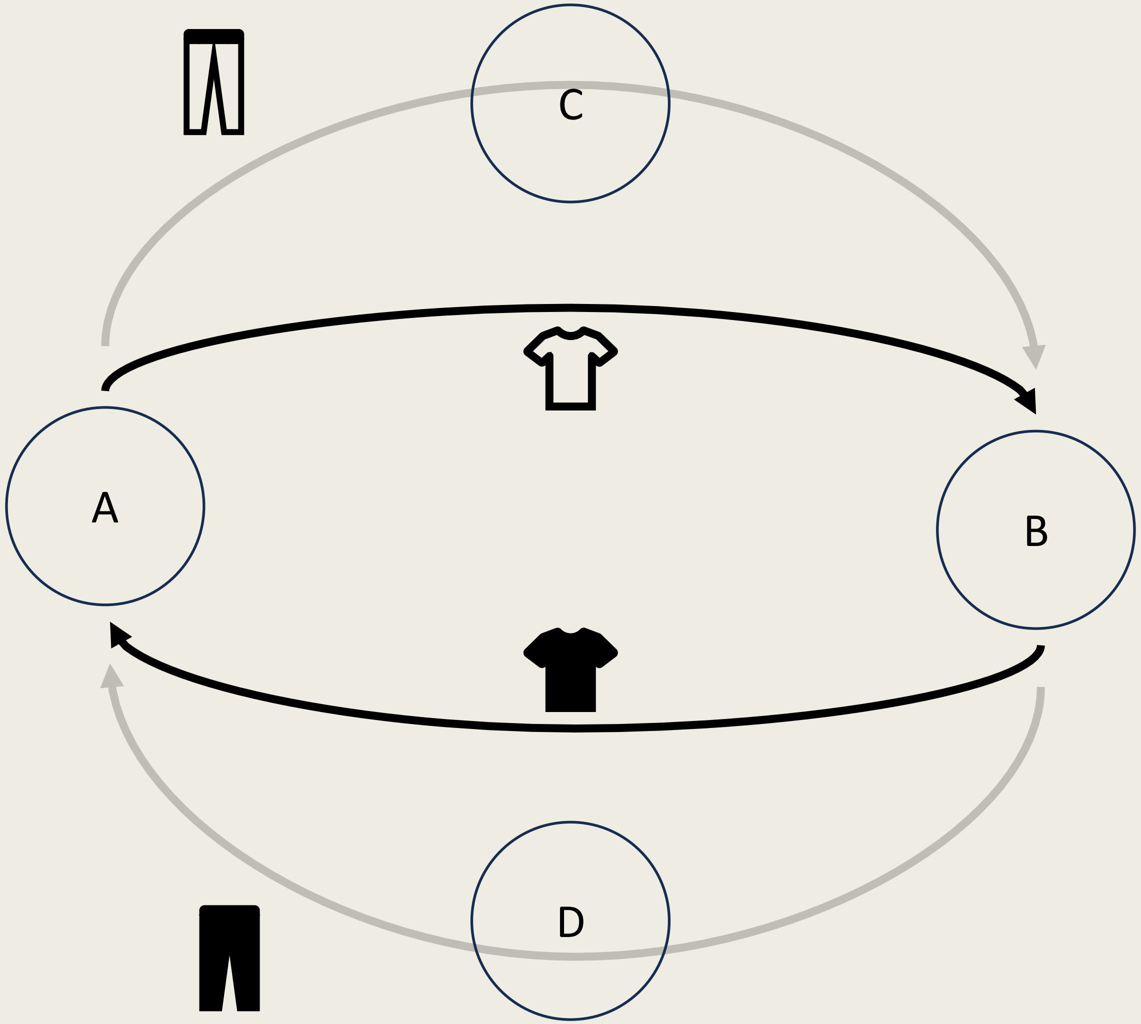

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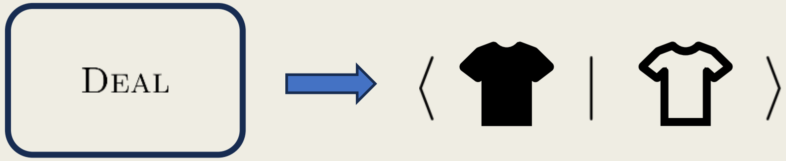

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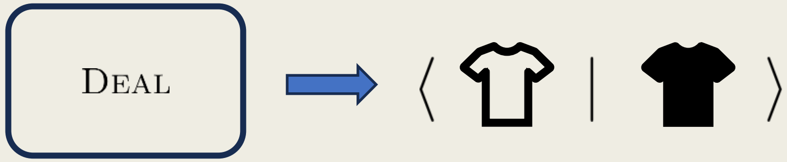

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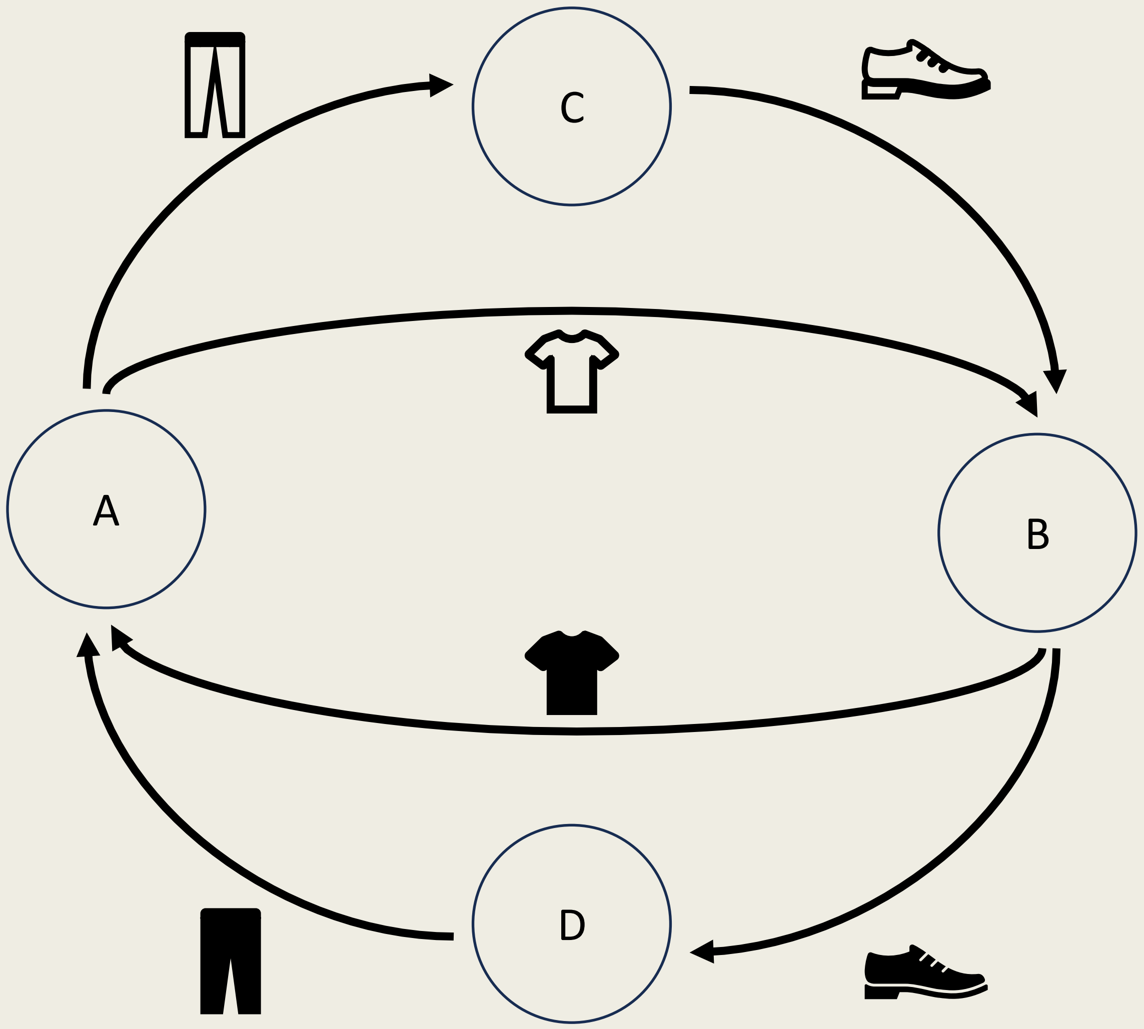

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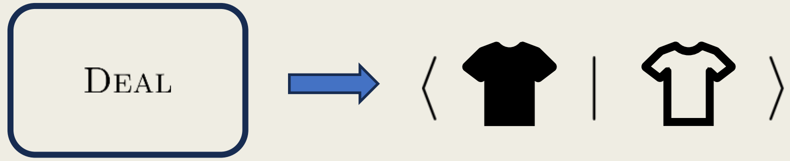
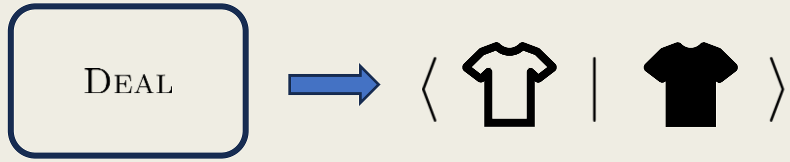
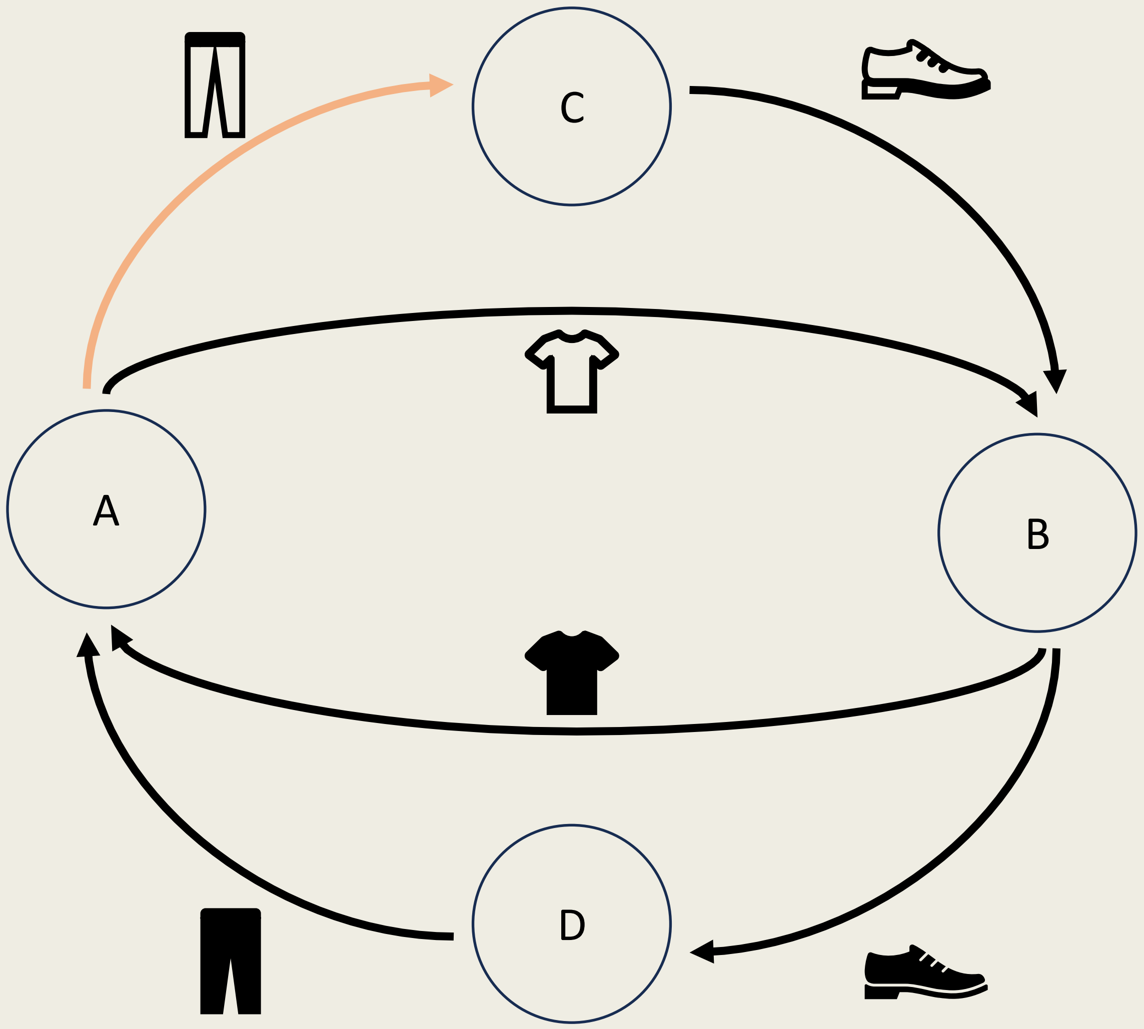


Preference of B:



No General Atomic Protocol – Case 1



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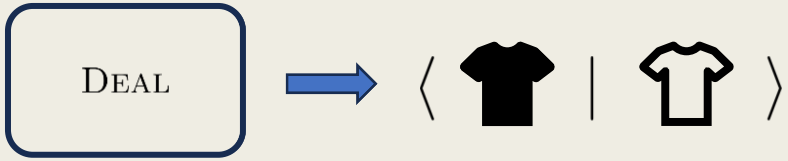
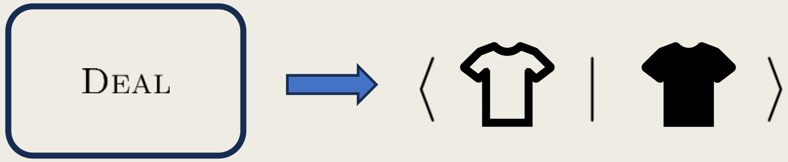


Preference of B:



Case 1

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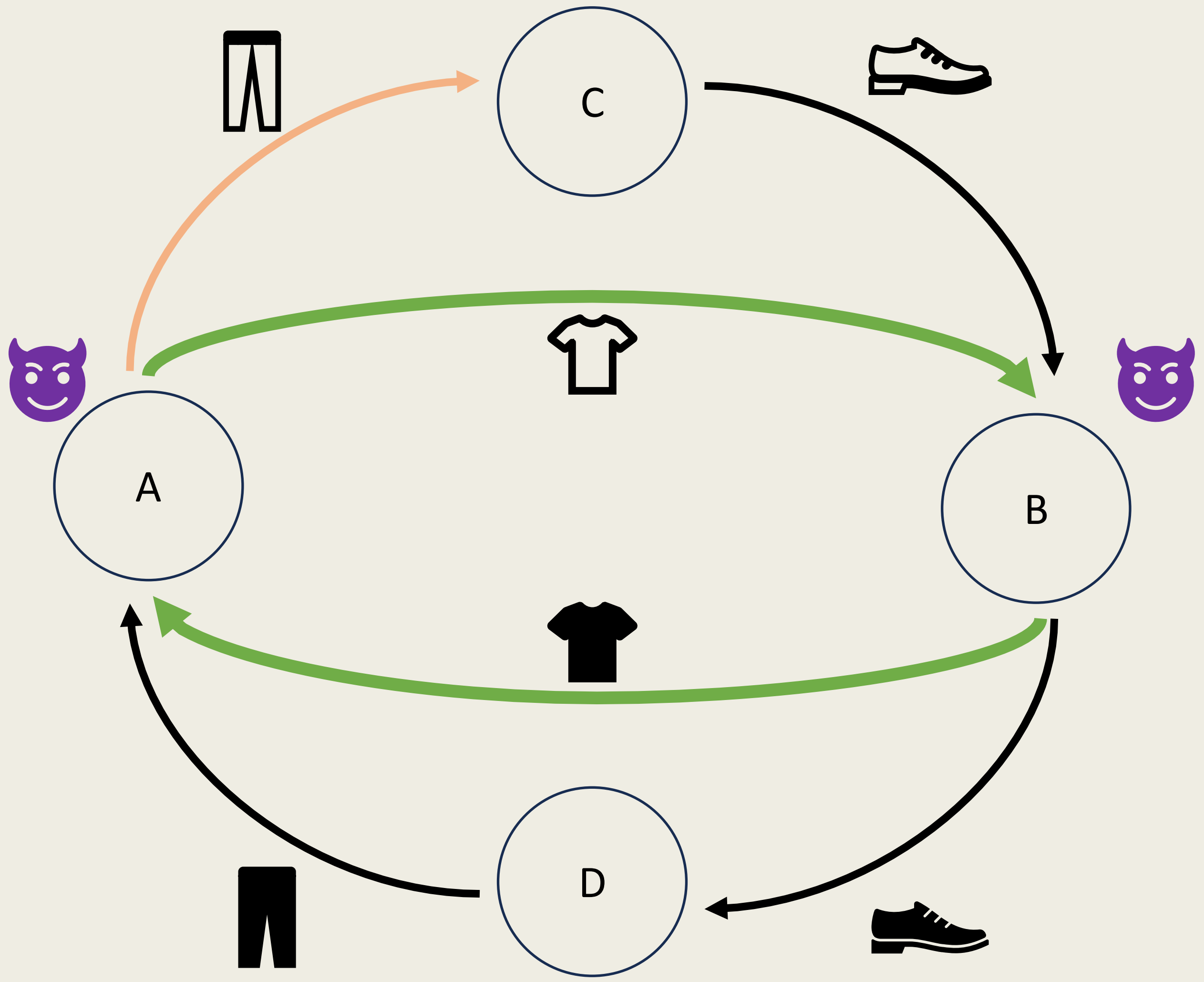

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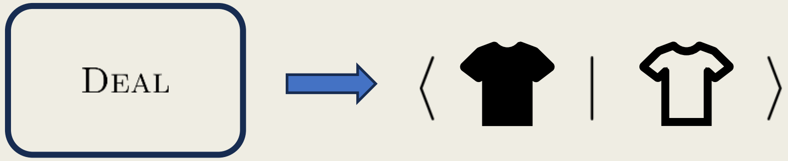
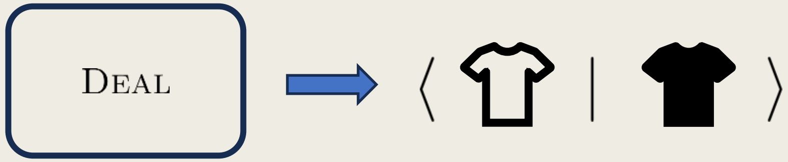
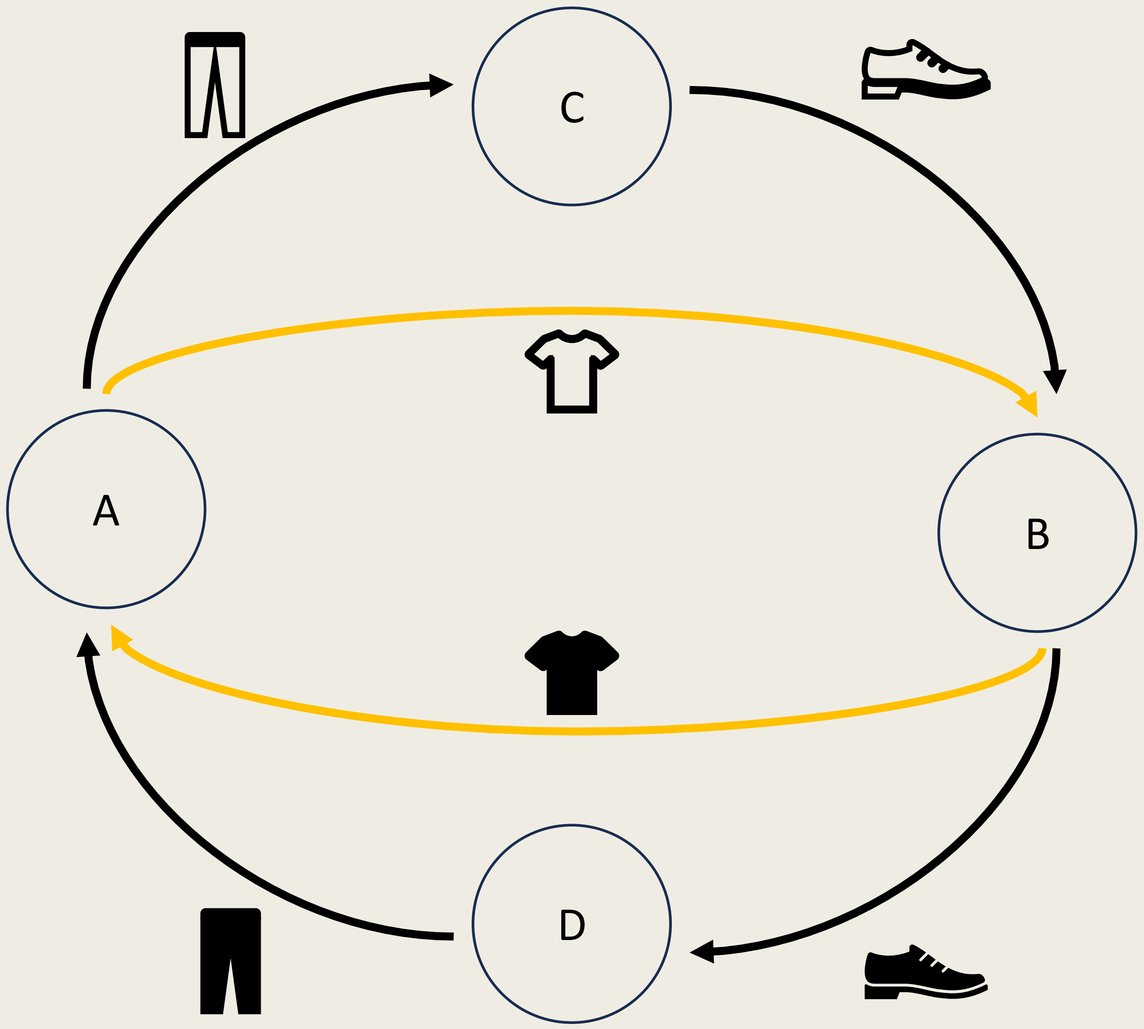


Preference of B:



Case 1: Not strong Nash equilibria

No General Atomic Protocol – Case 2



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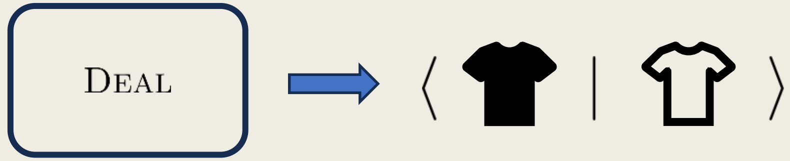
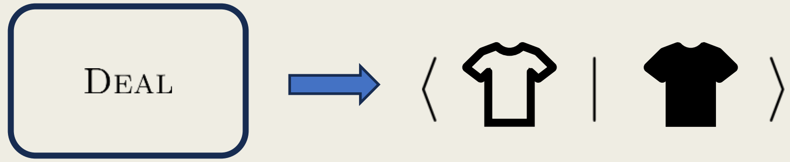
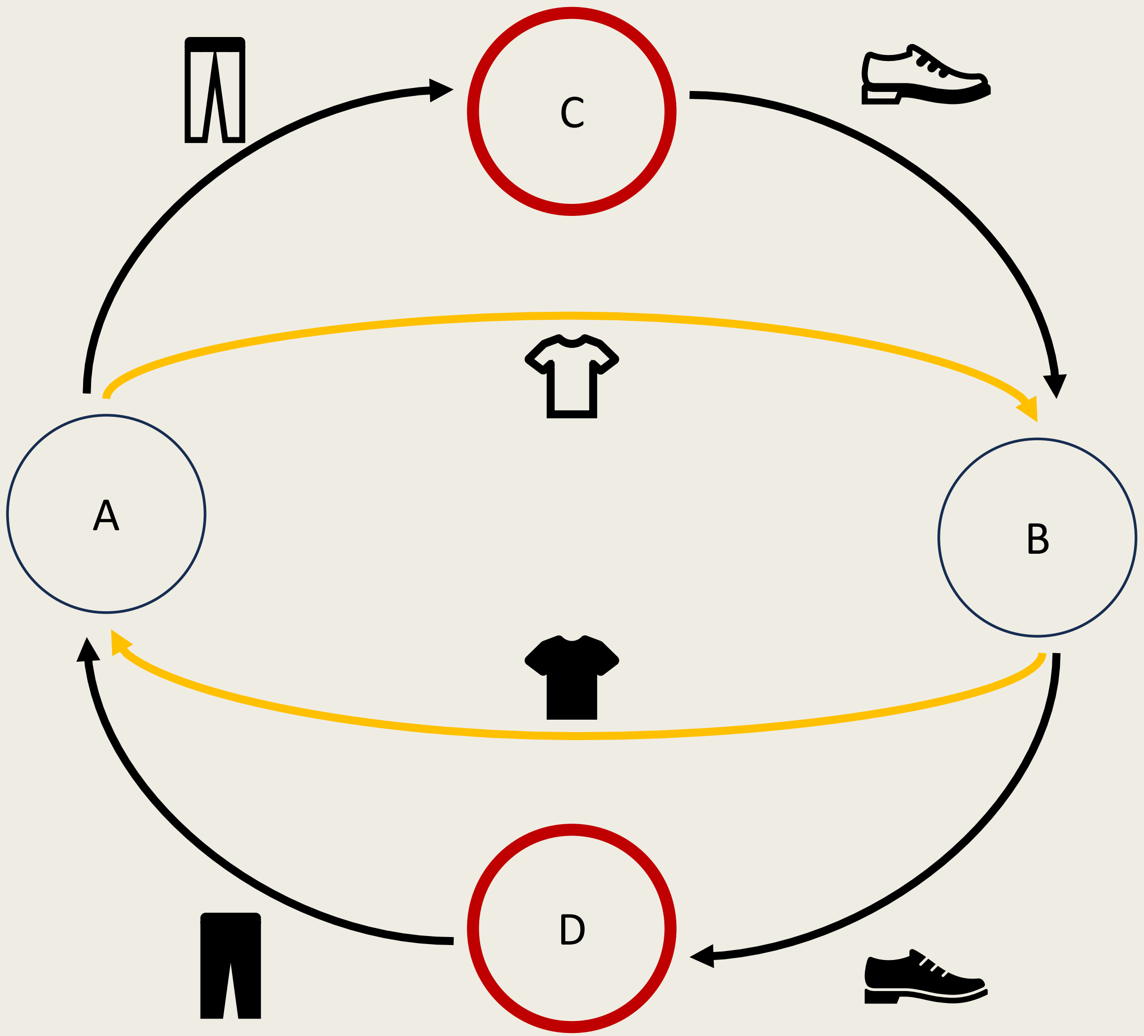


Preference of B:



Case 2

No General Atomic Protocol – Case 2



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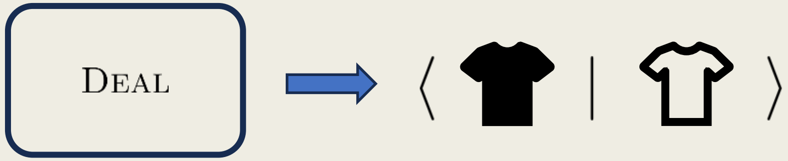
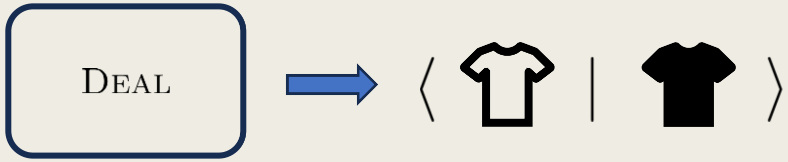


Preference of B:



Case 2: Not live

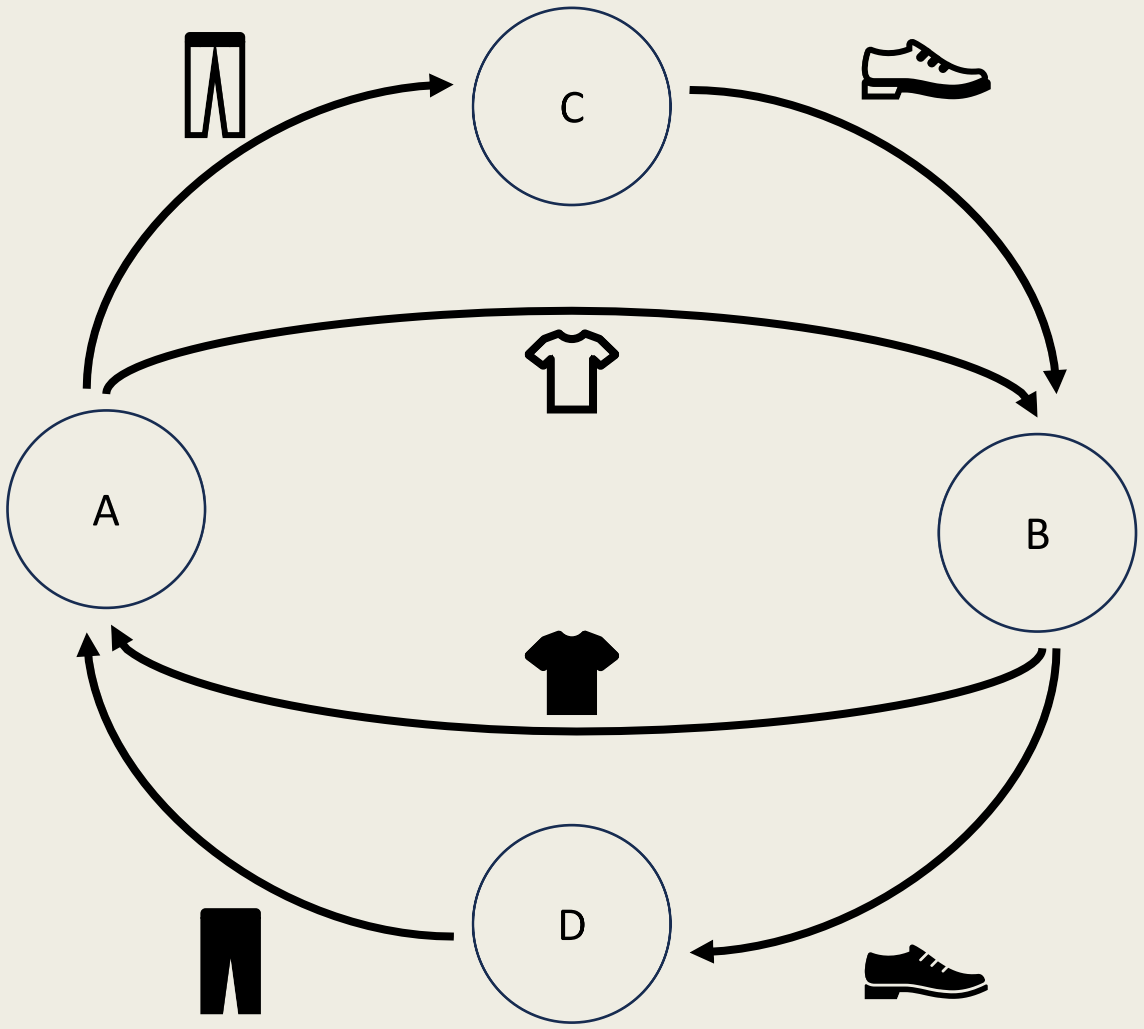
Sometimes, There Is a Protocol

Theorem

Theorem. $S = (D, P)$ has an atomic protocol **iff** there exists a spanning subgraph G of D such that:

- G is piece-wise strongly connected and has no isolated vertices
- G dominates D
- no subgraph H of D strictly dominates G

Example



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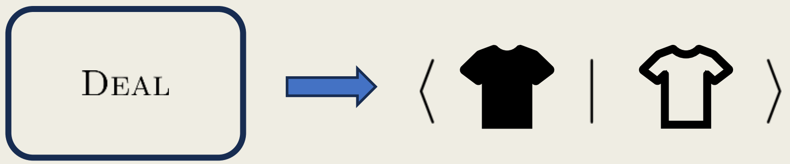
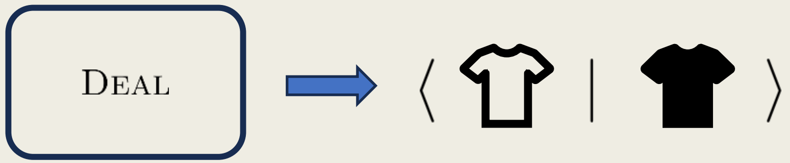


Preference of B:



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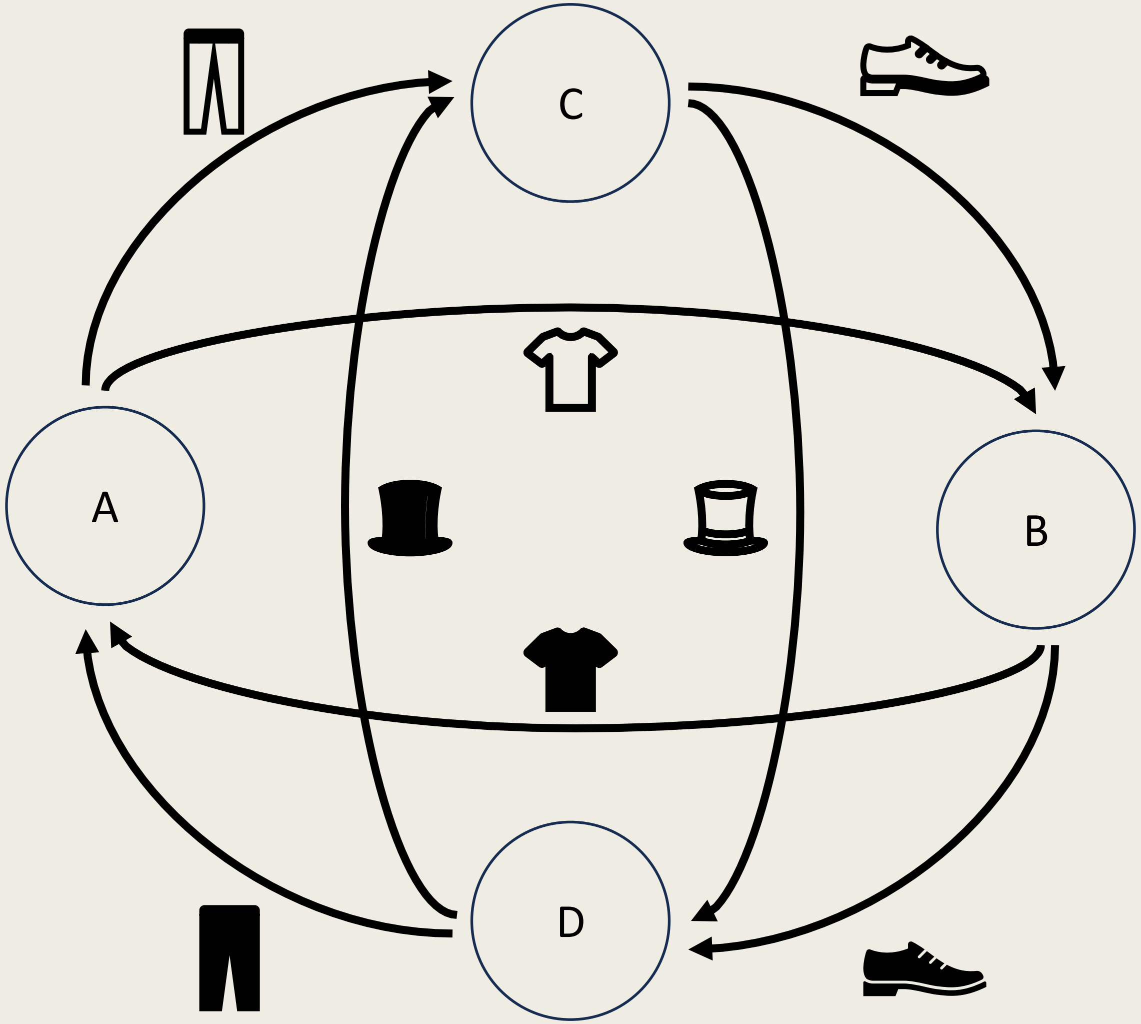

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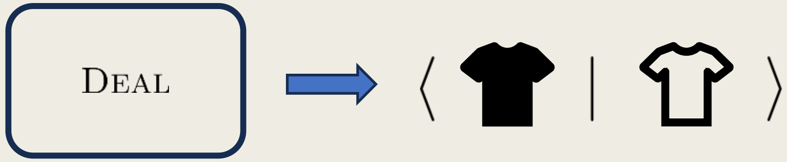
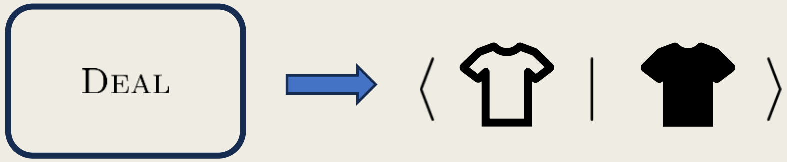


Preference of B:



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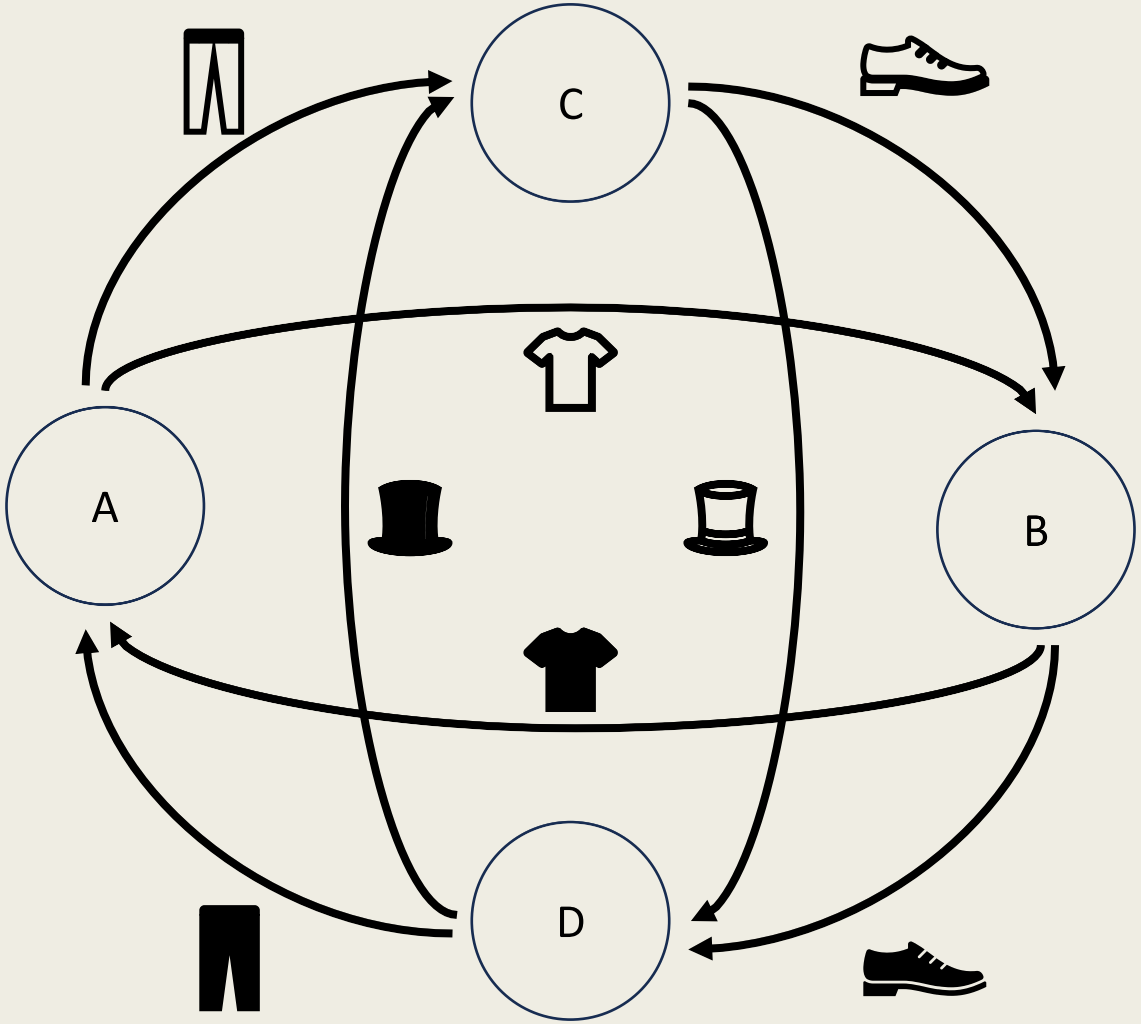

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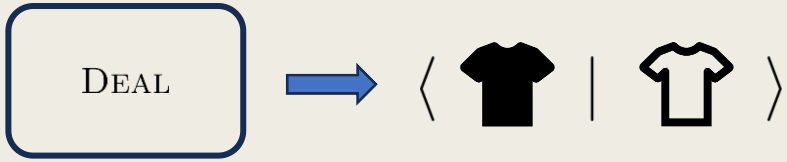
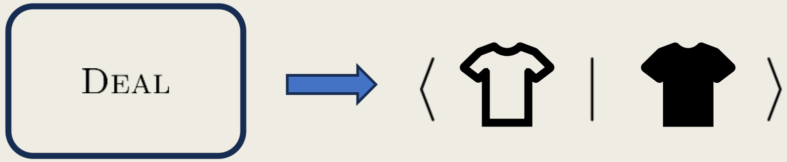
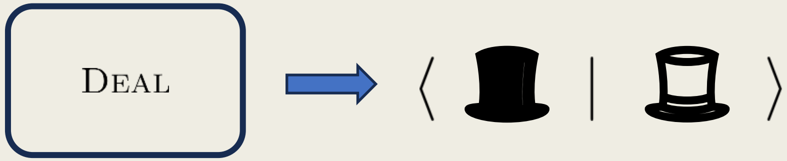


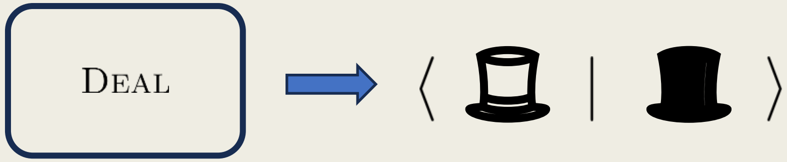
Preference of B:



Preference of C:

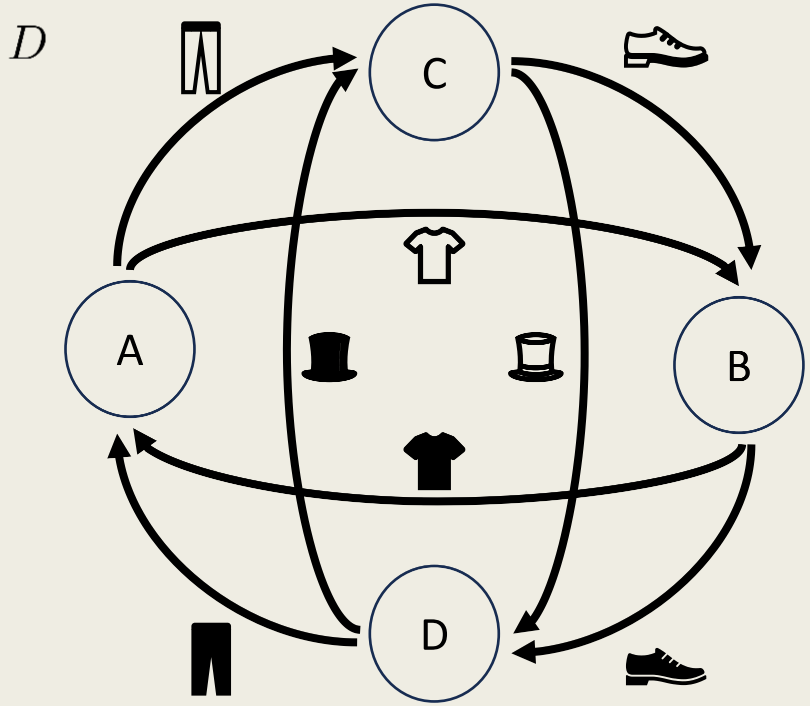


Preference of D:



Condition 1

G is piece-wise strongly connected and has no isolated vertices



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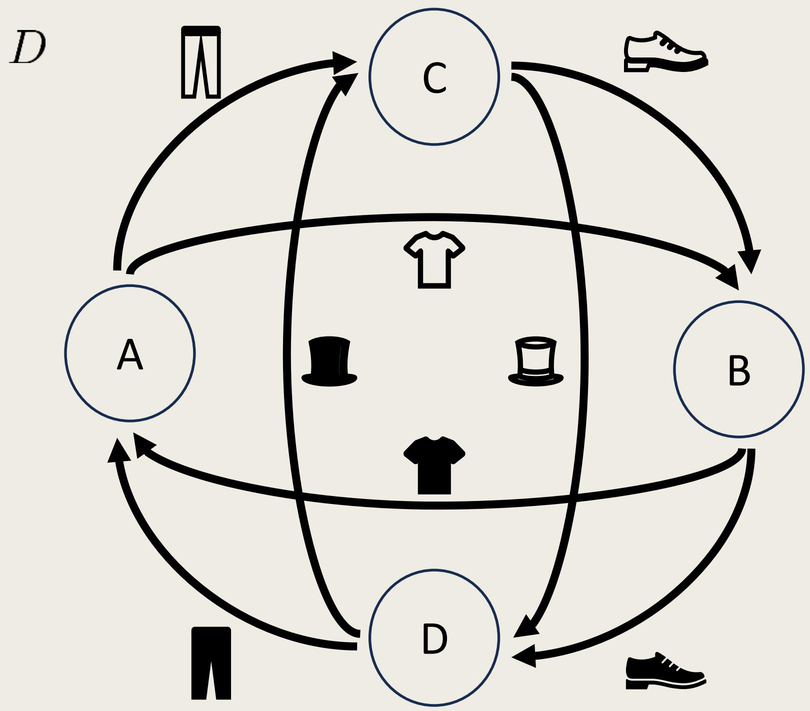
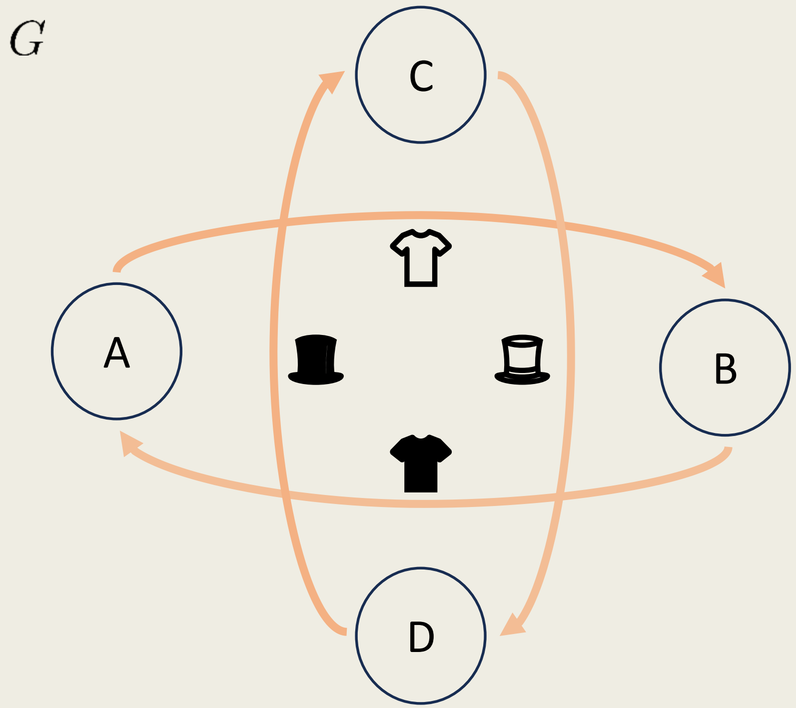
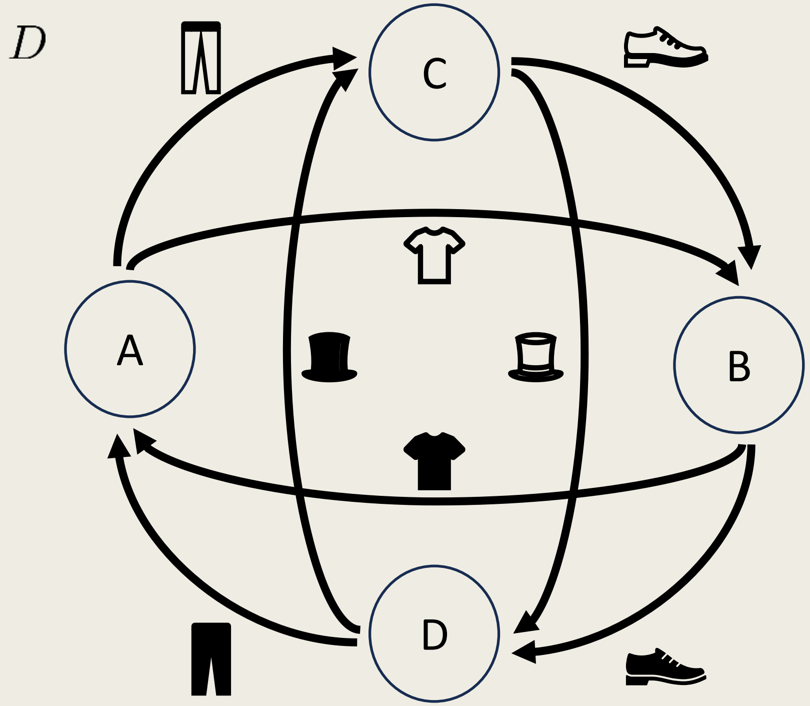
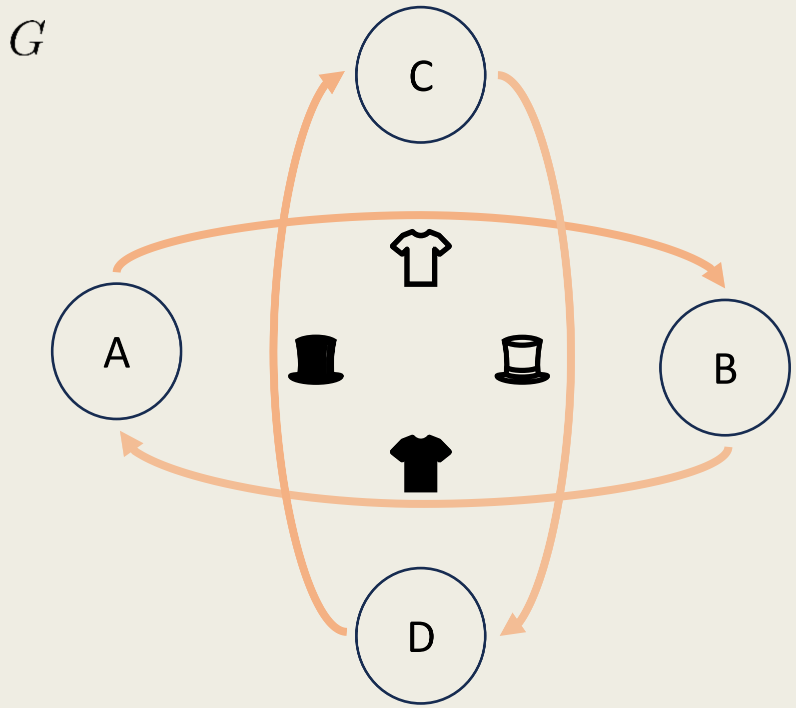




Condition 2

G dominates D: each party in G ends at least as good as they do in D





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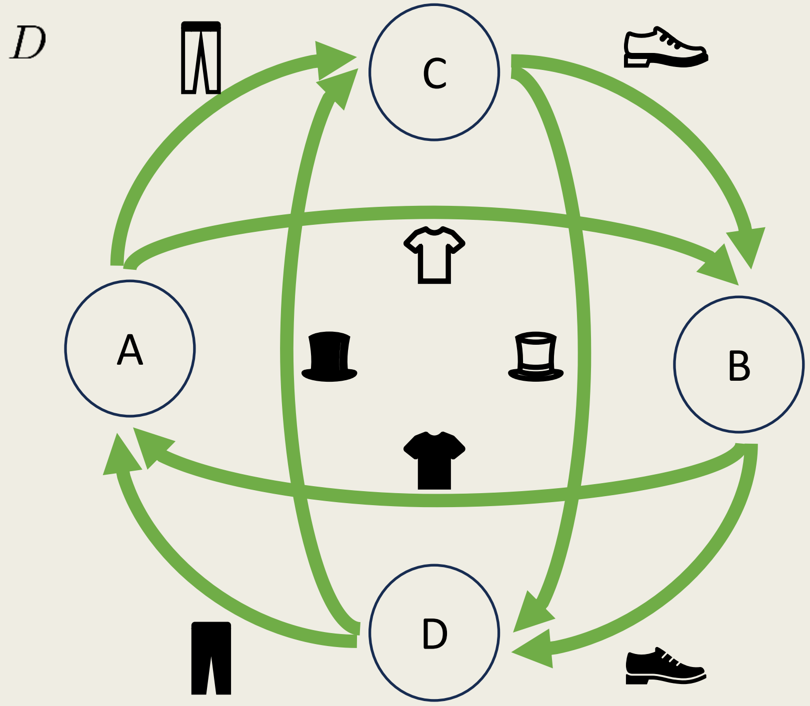

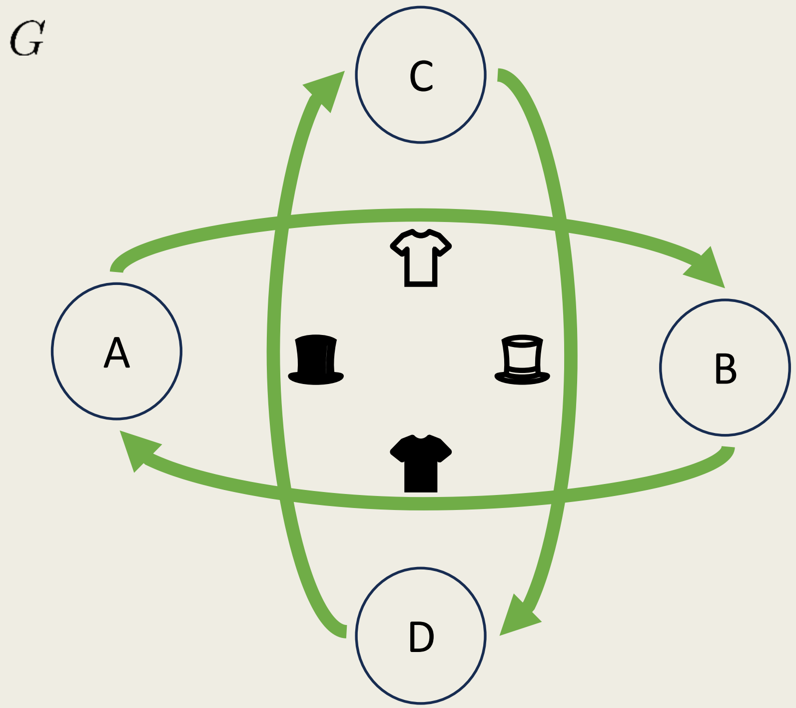

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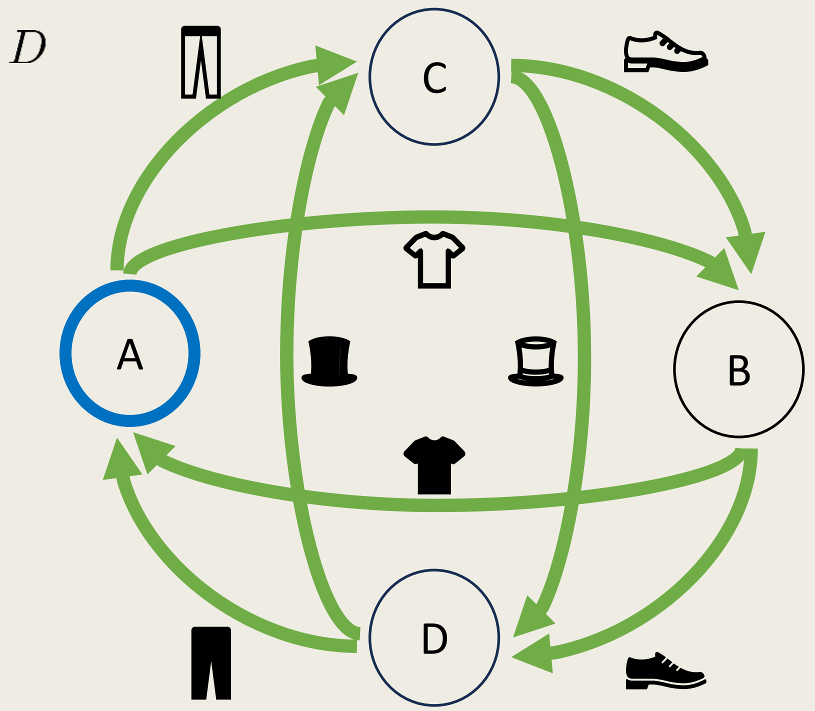

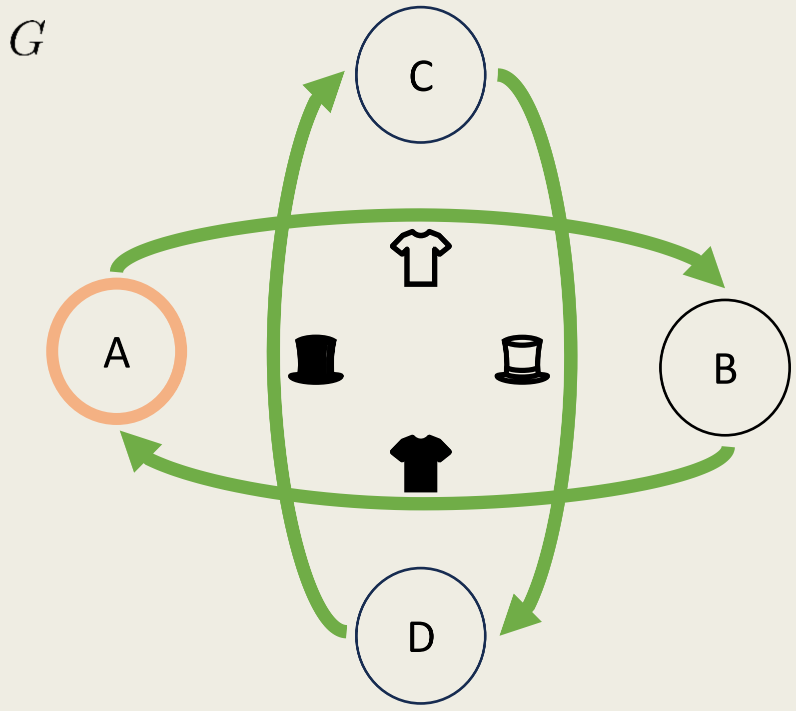

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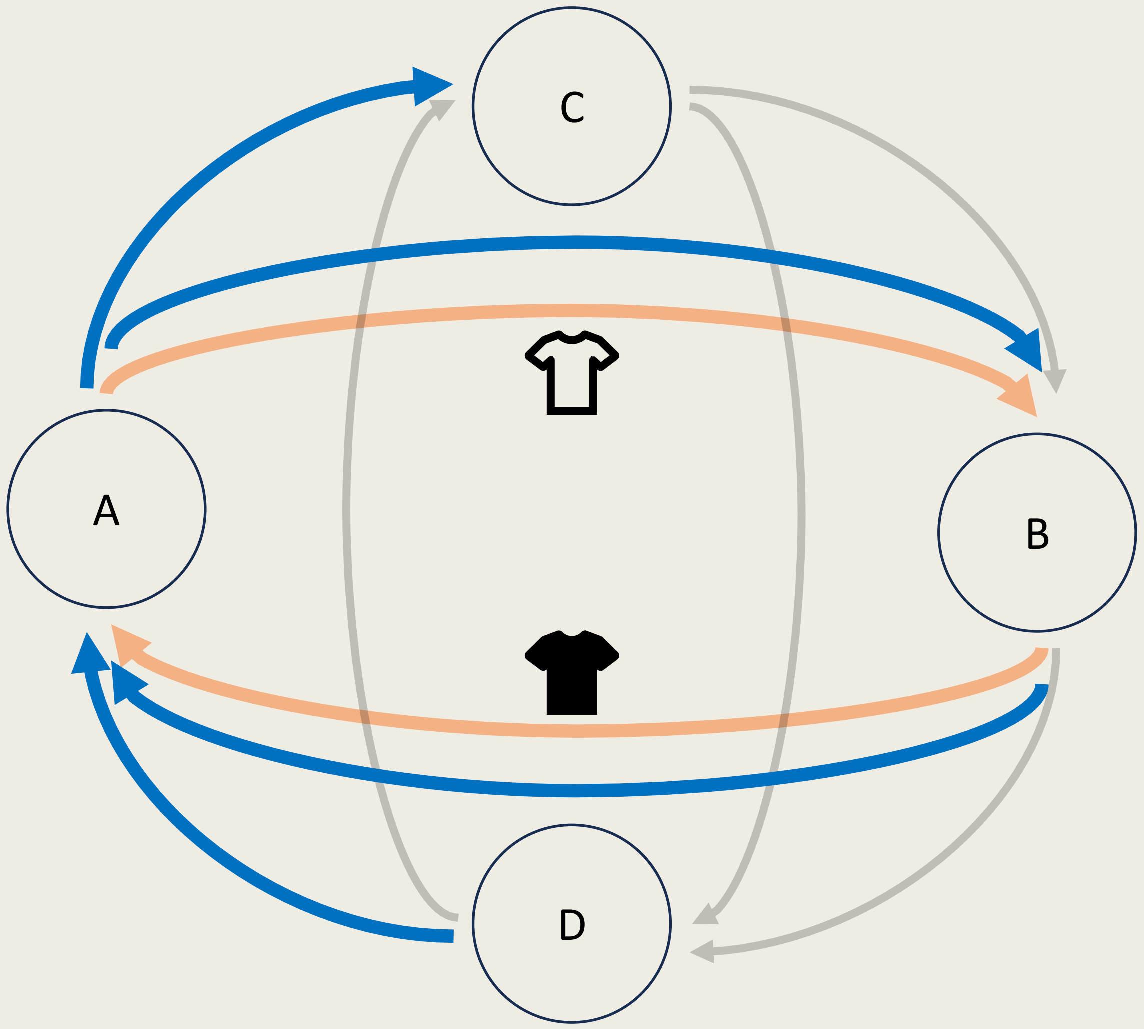
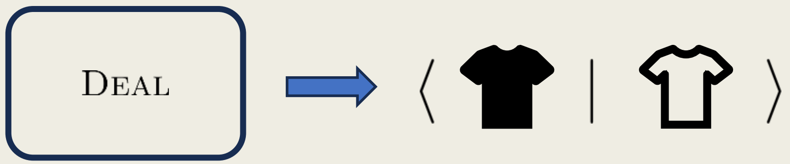


Preference of A:



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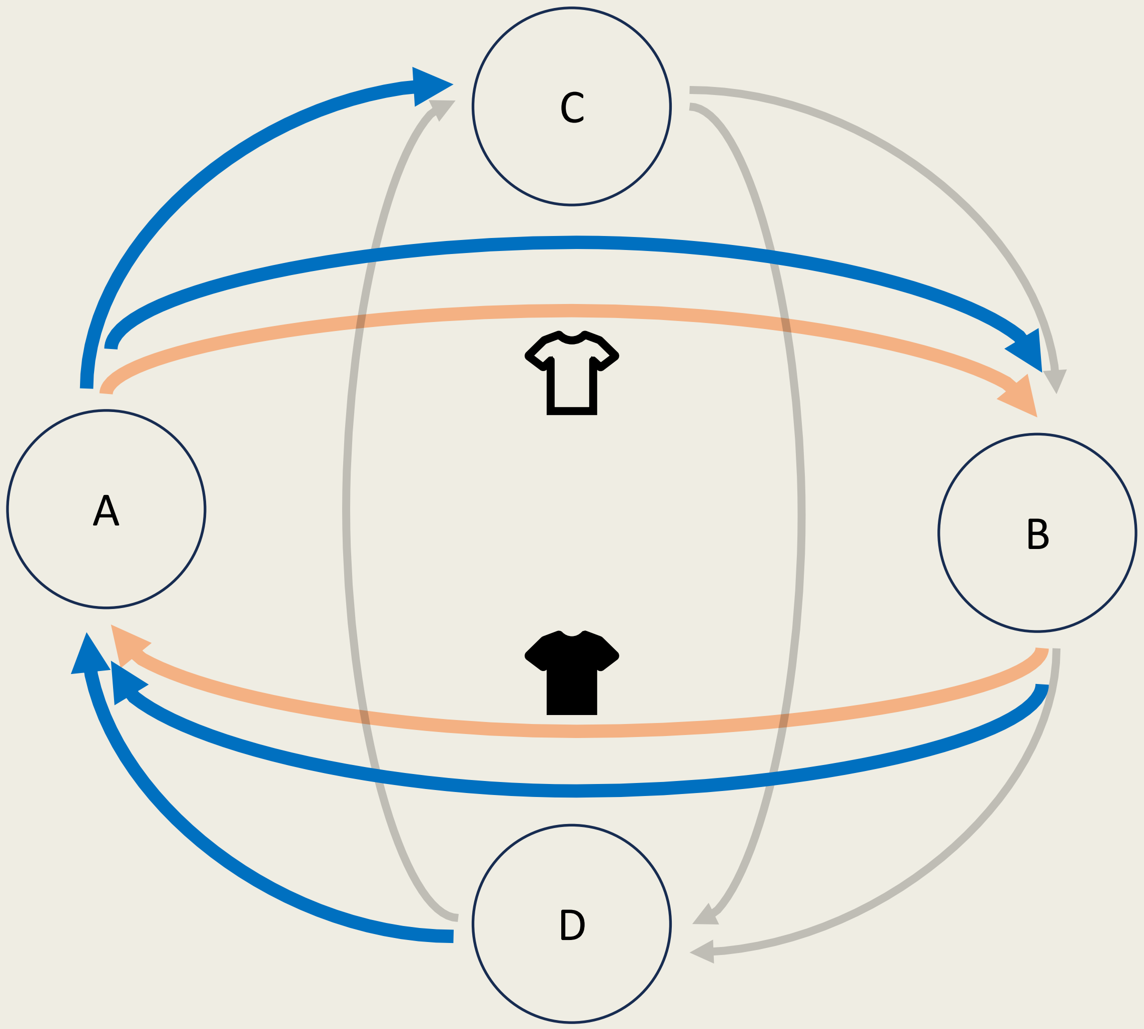

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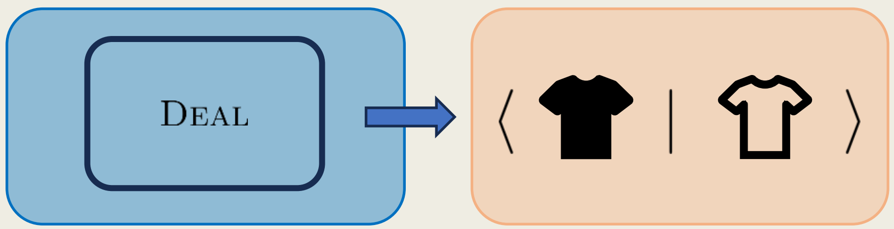

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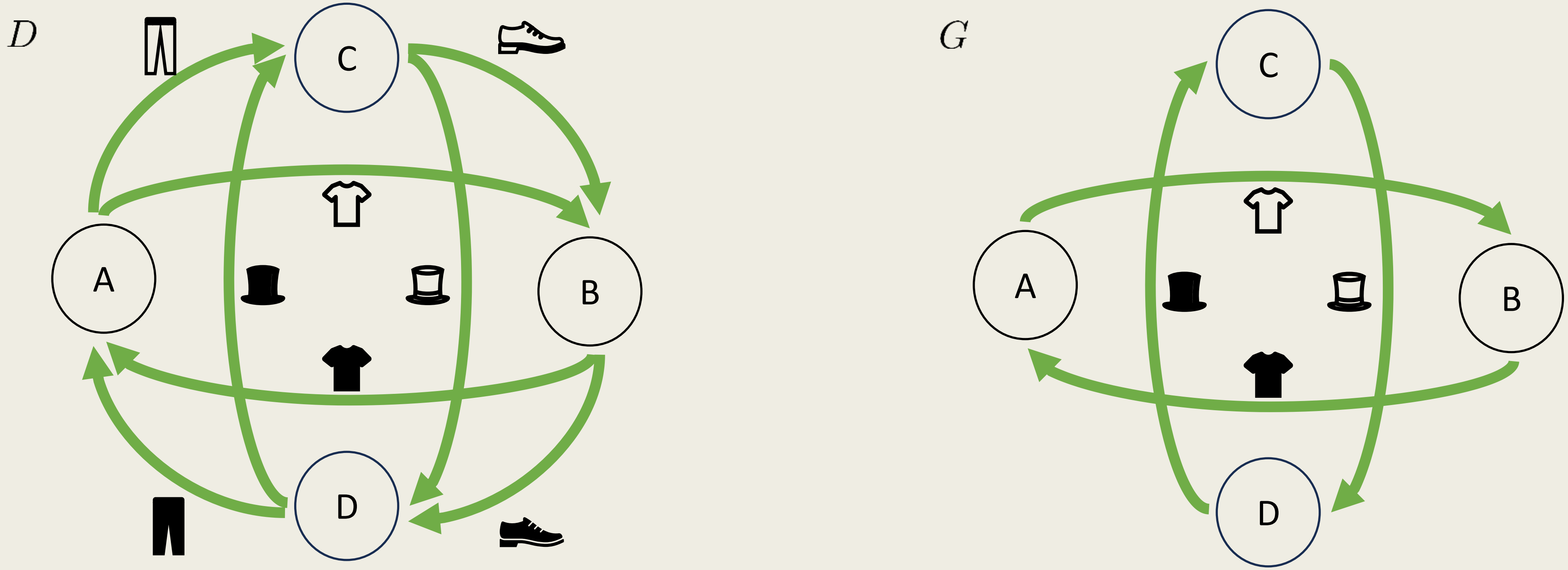

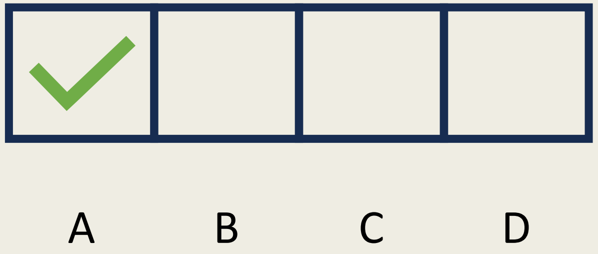

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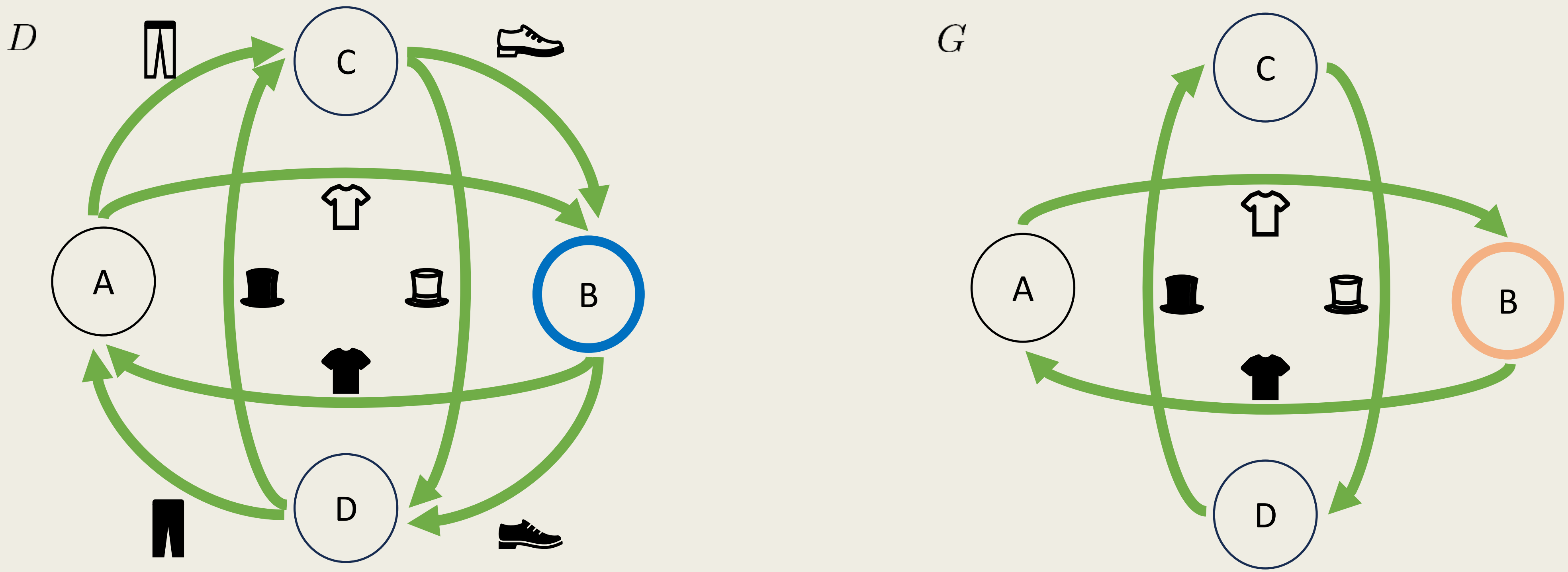

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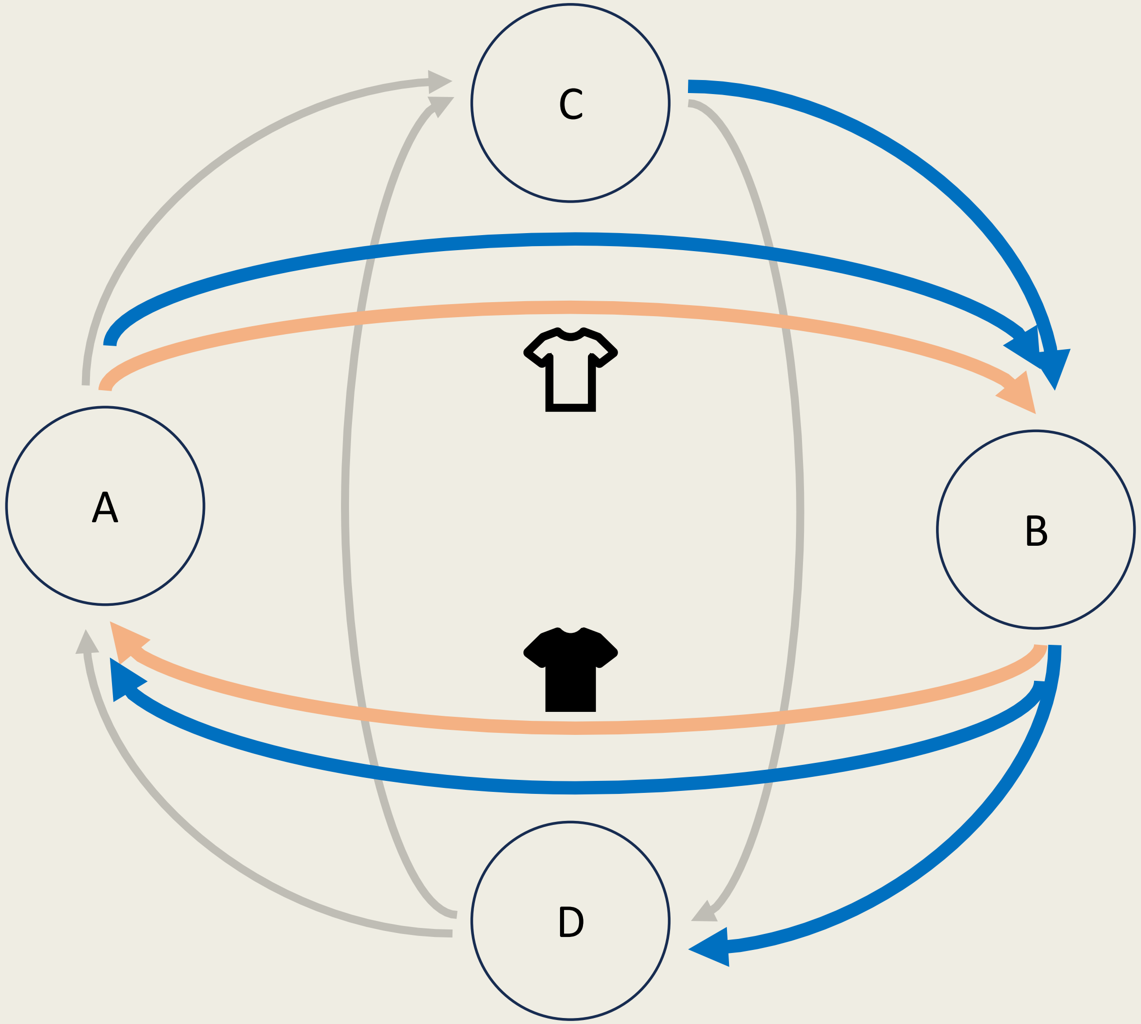
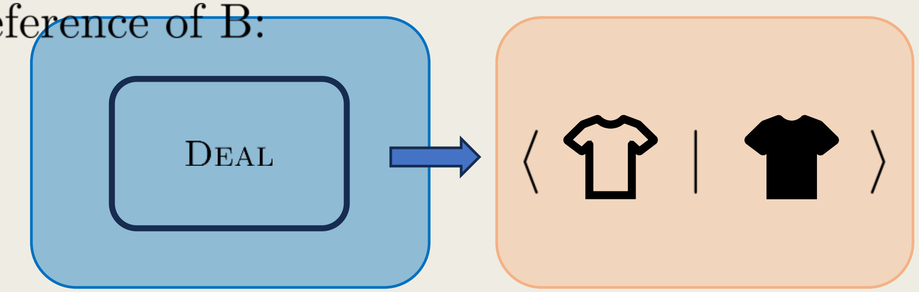


Preference of B:



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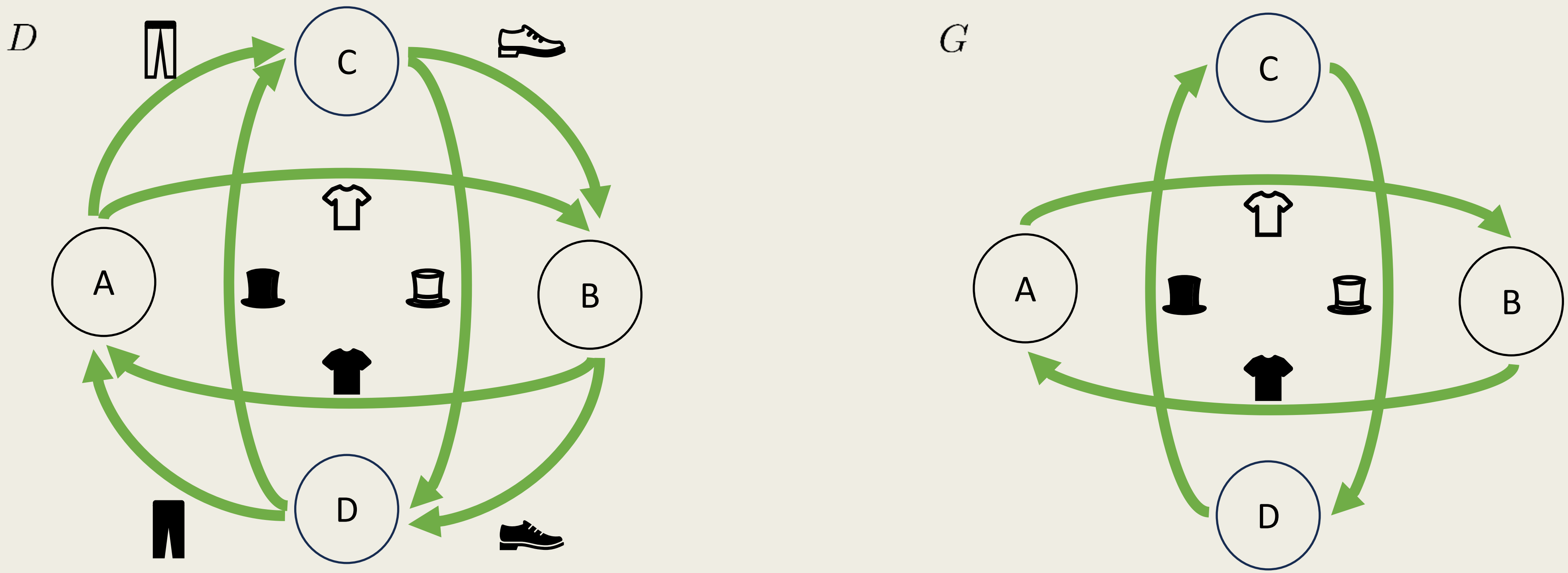

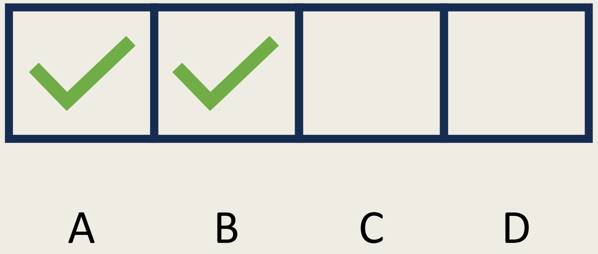

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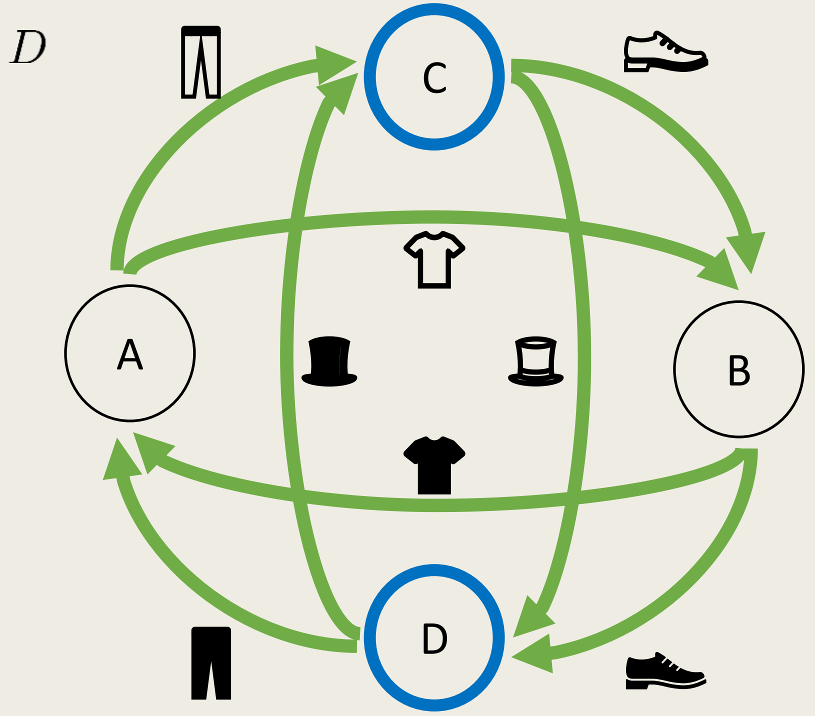

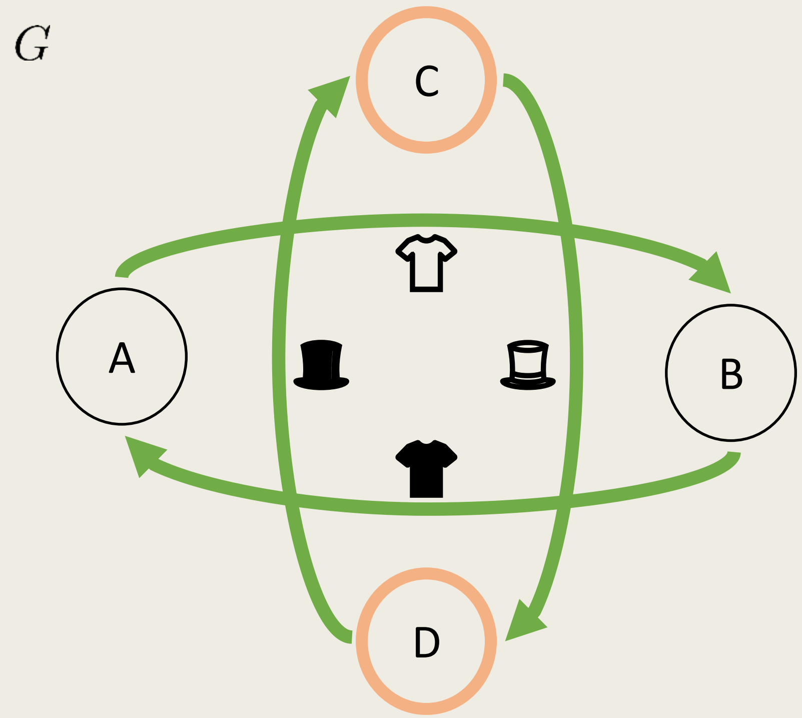

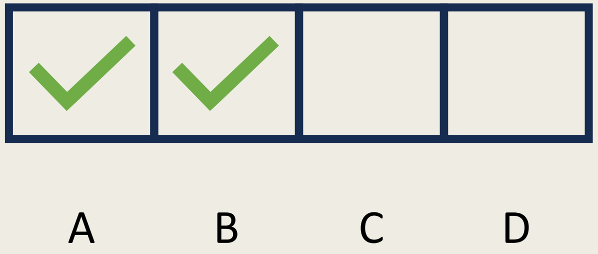

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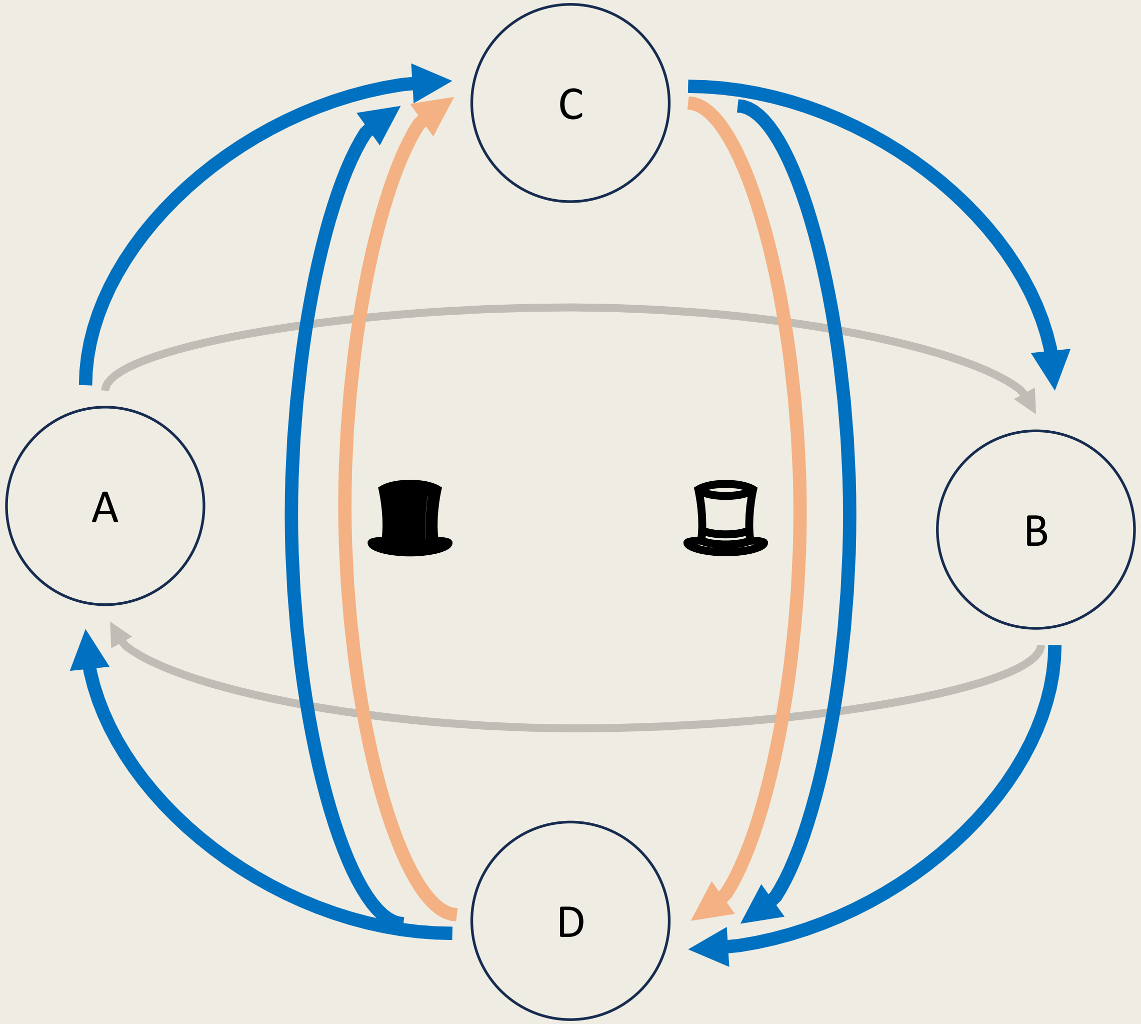

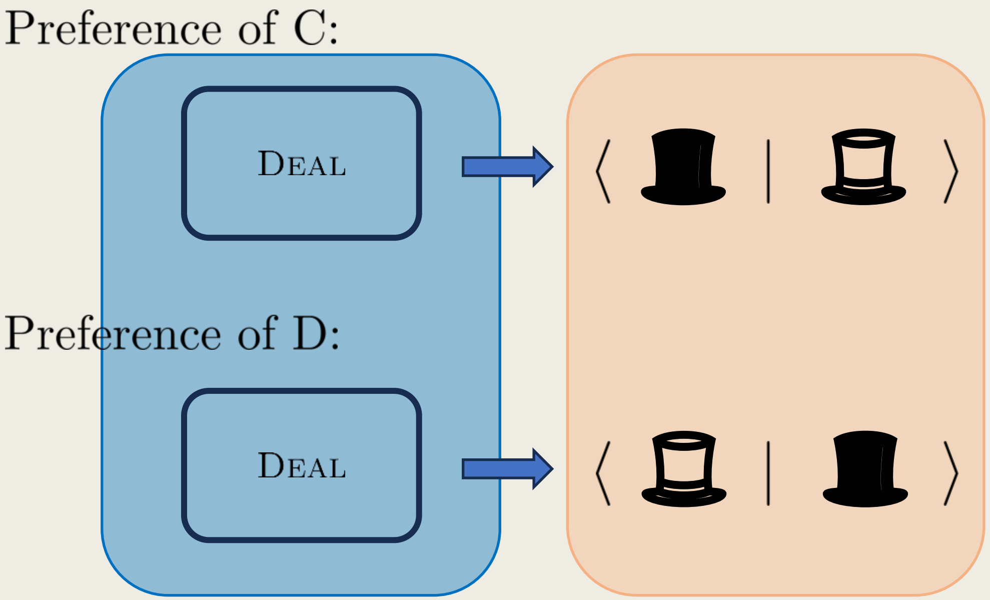

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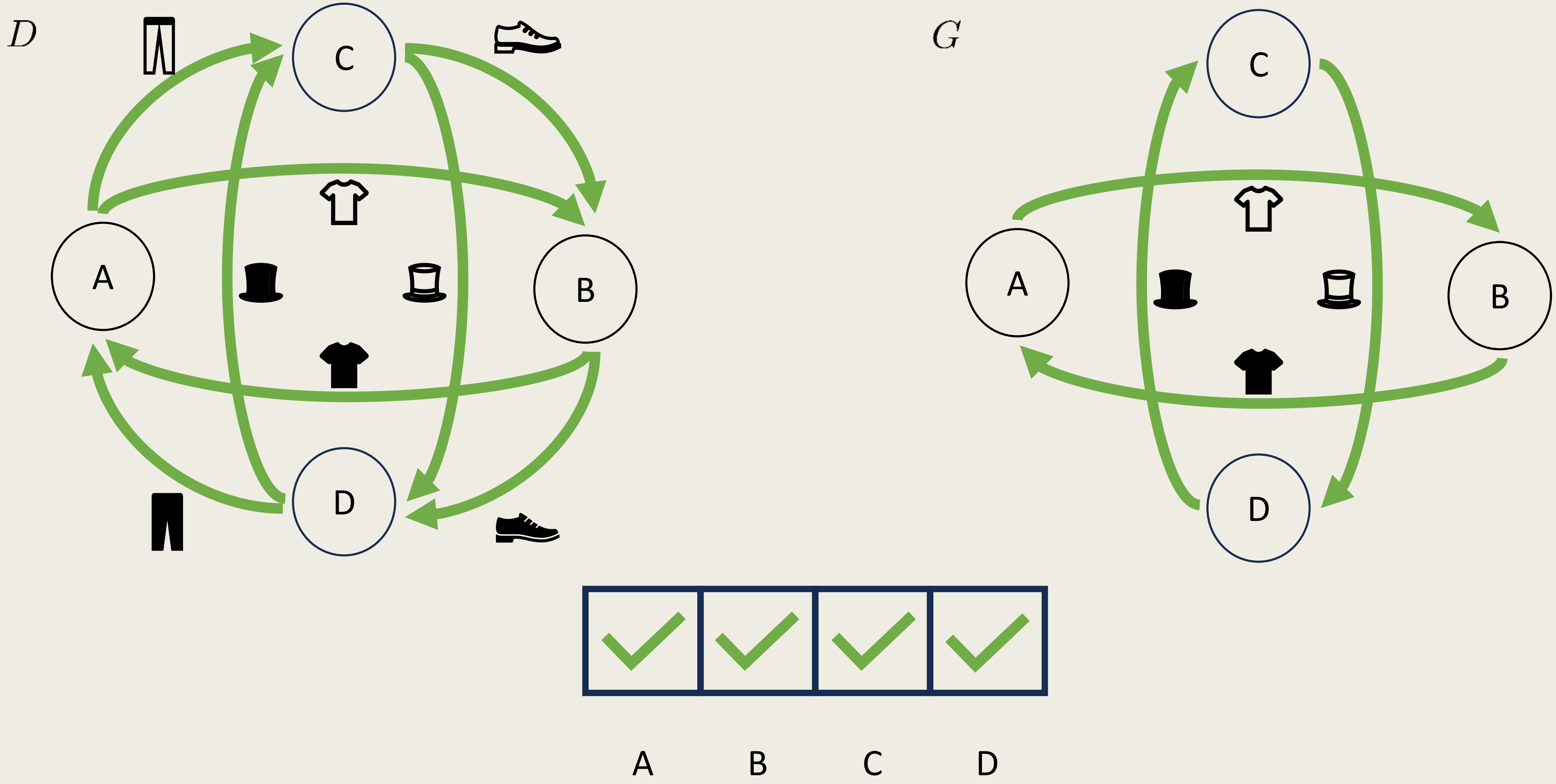

Condition 3

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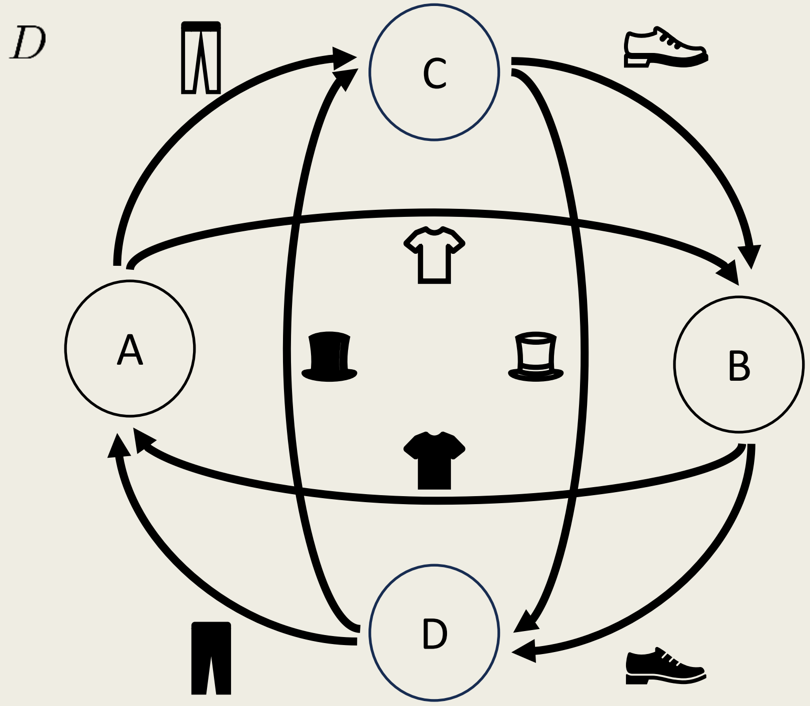

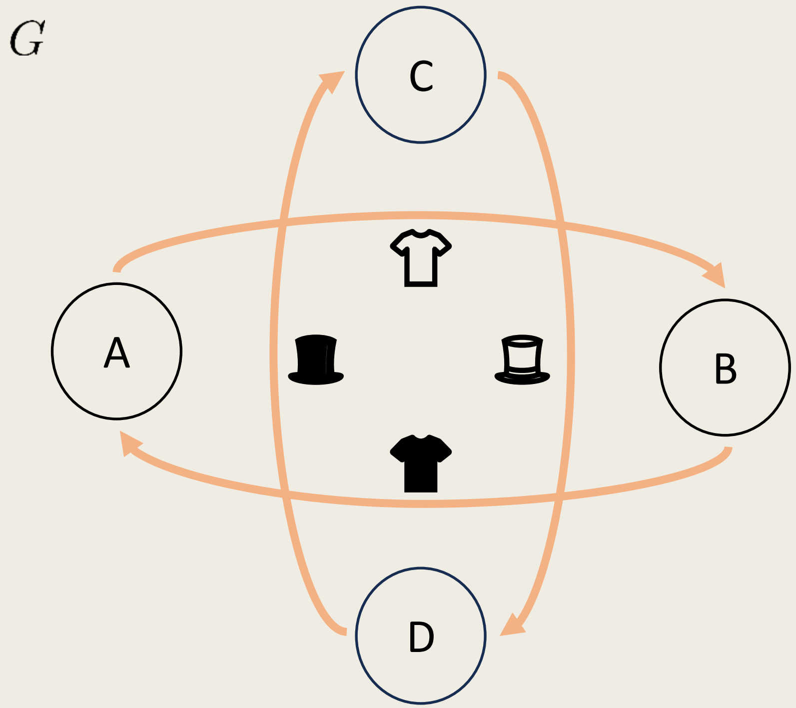

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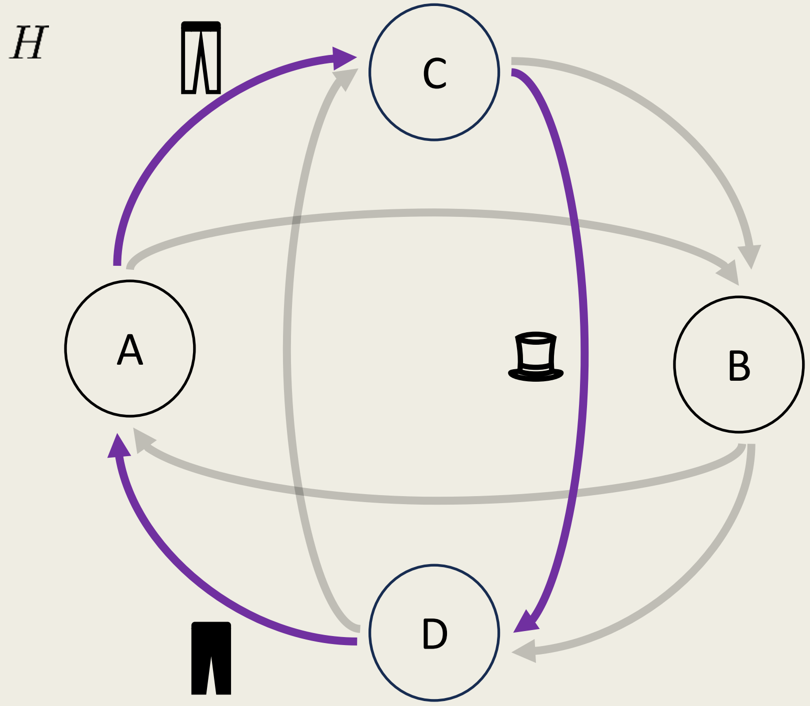

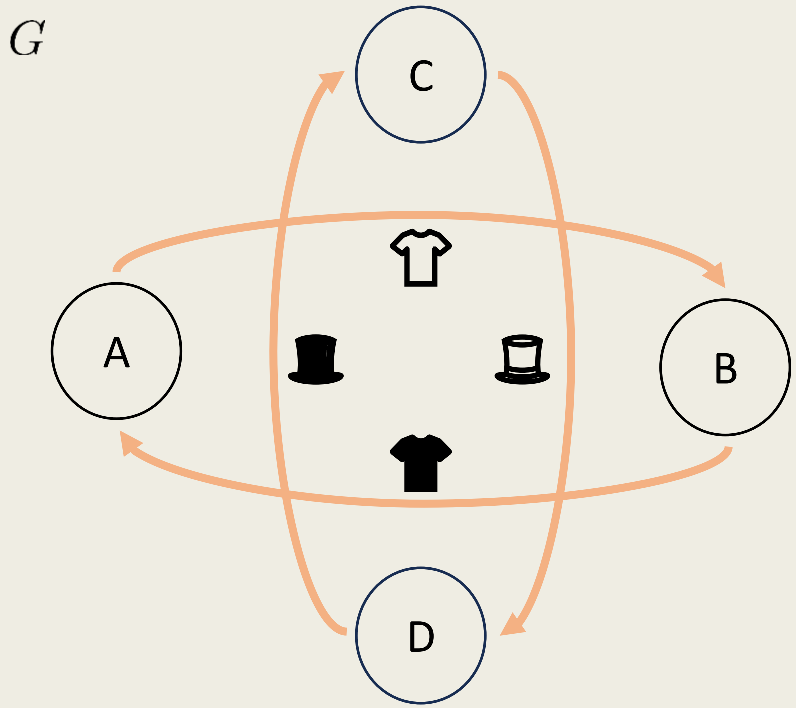

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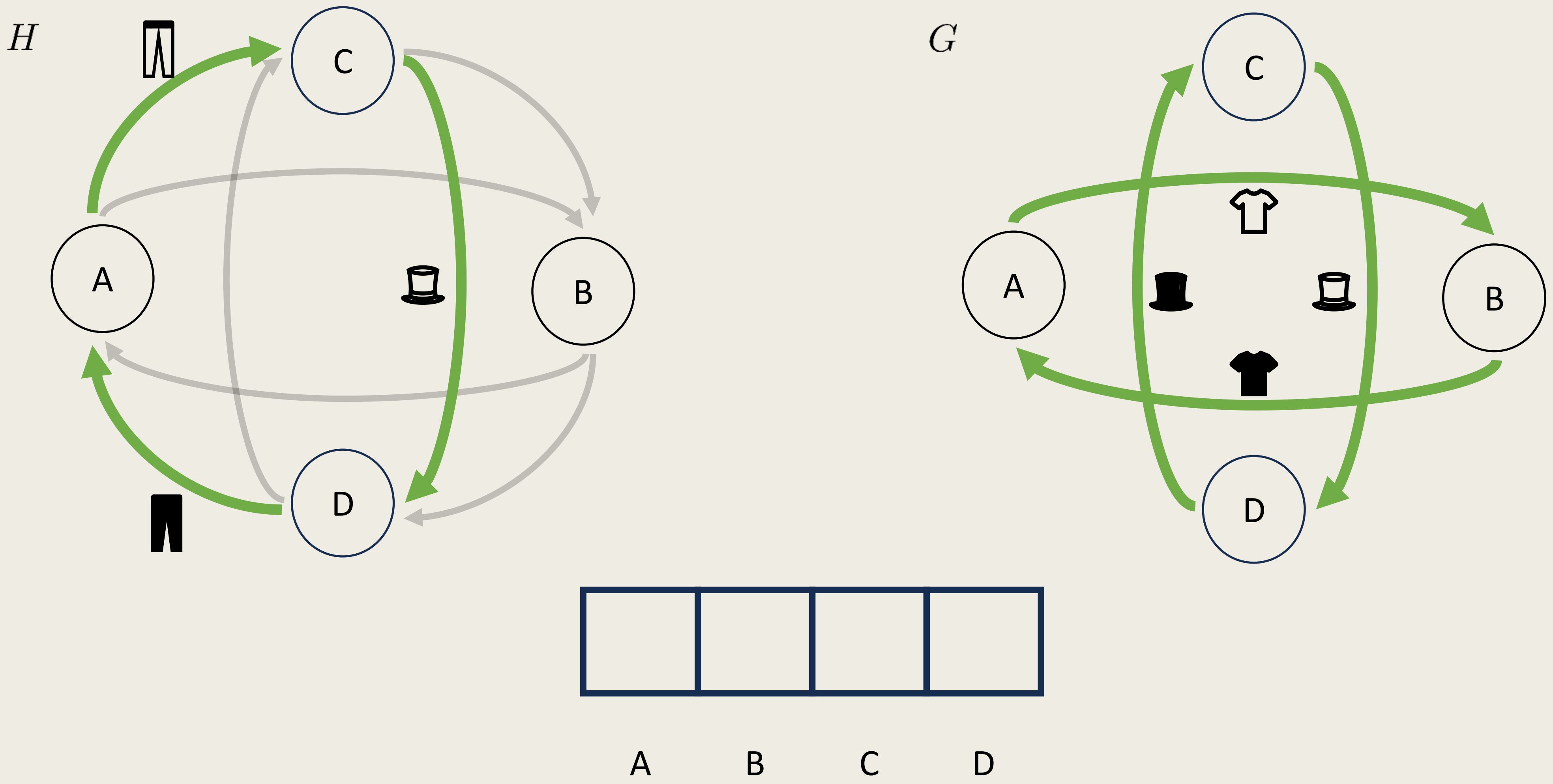

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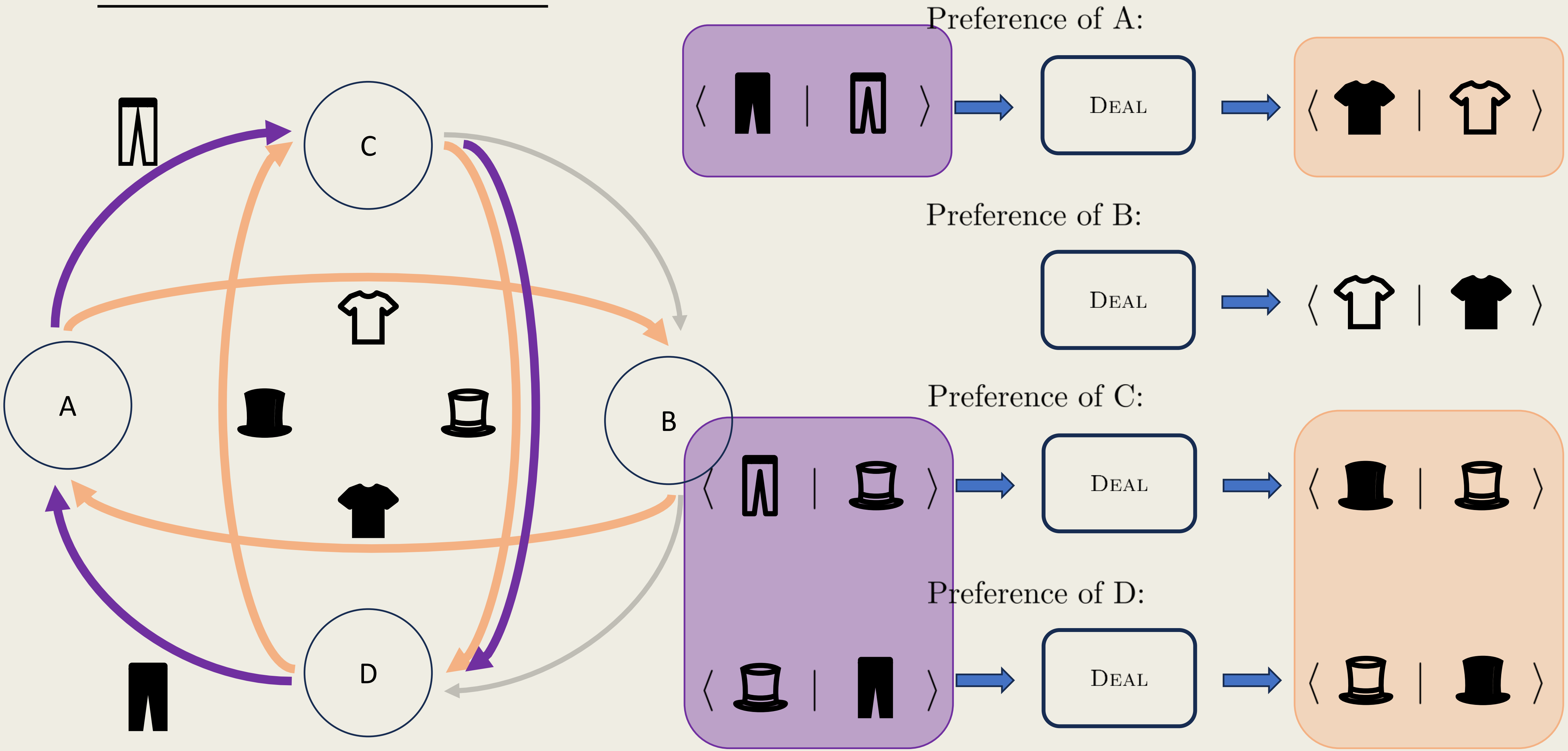
C
A
B
D
Preference of A:
DEAL
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DEAL
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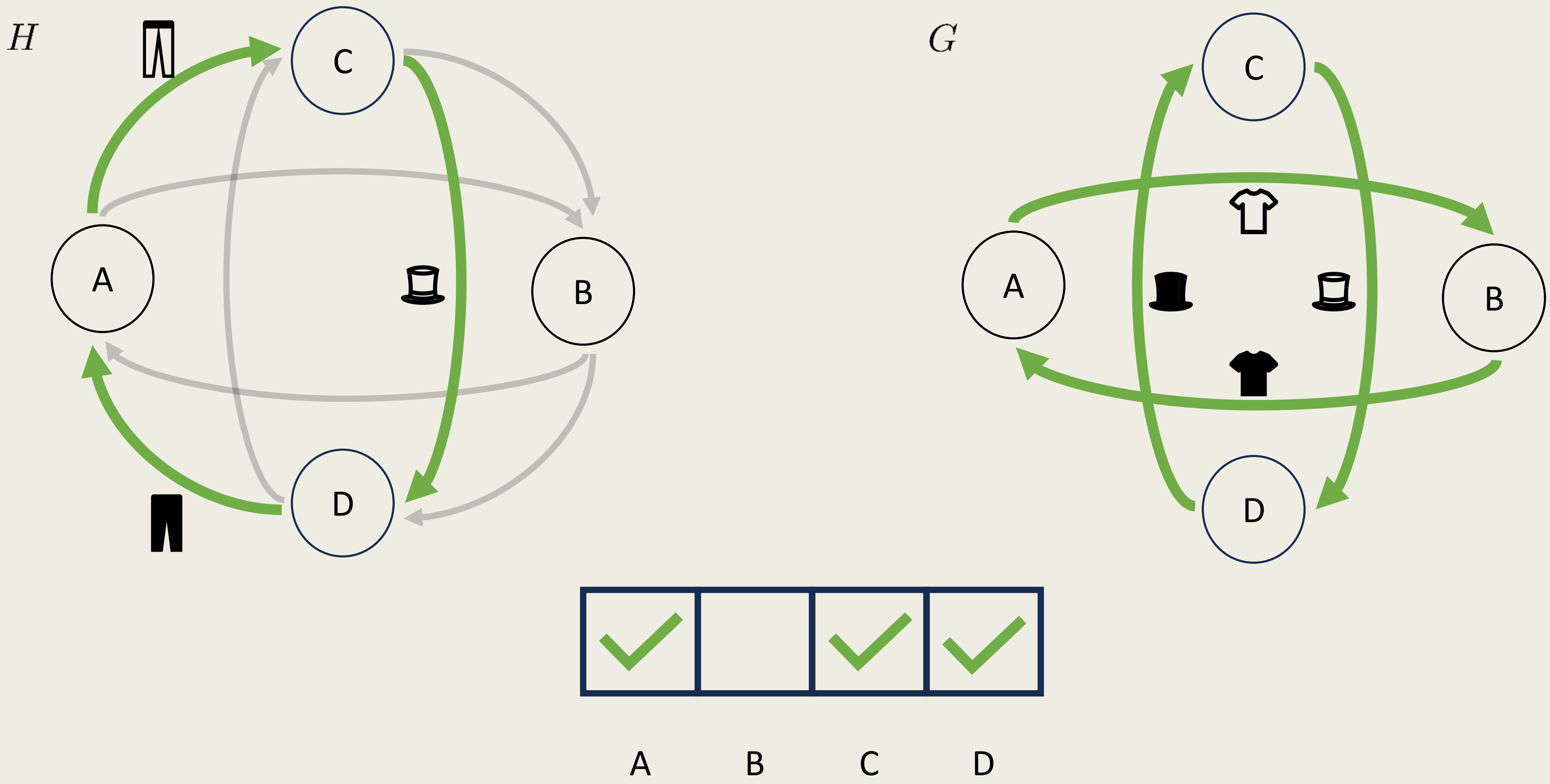

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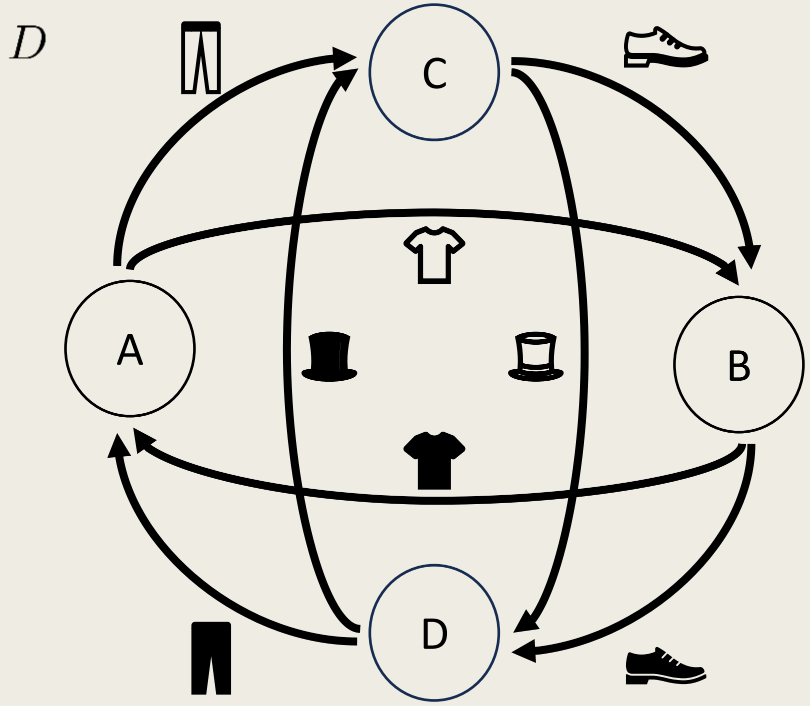
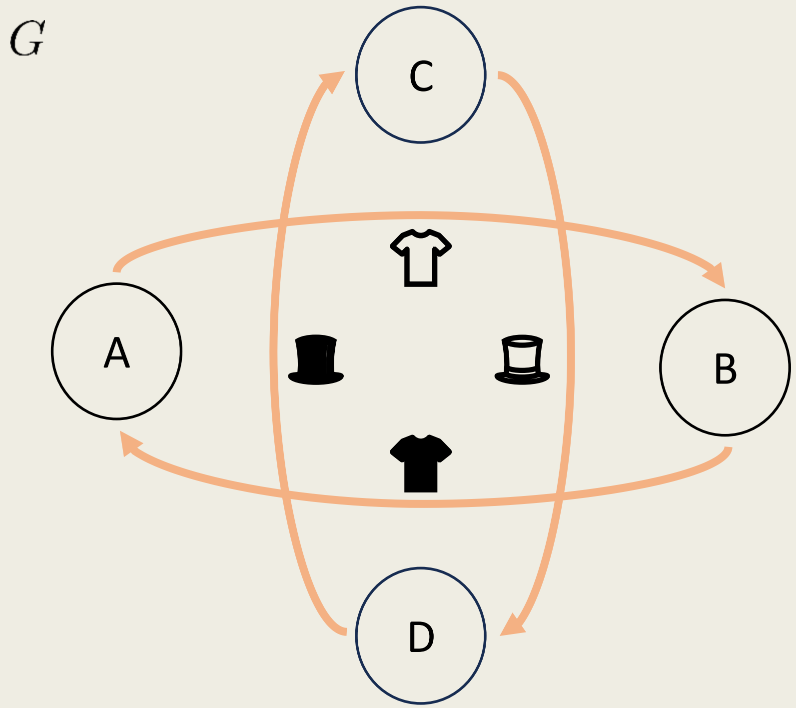
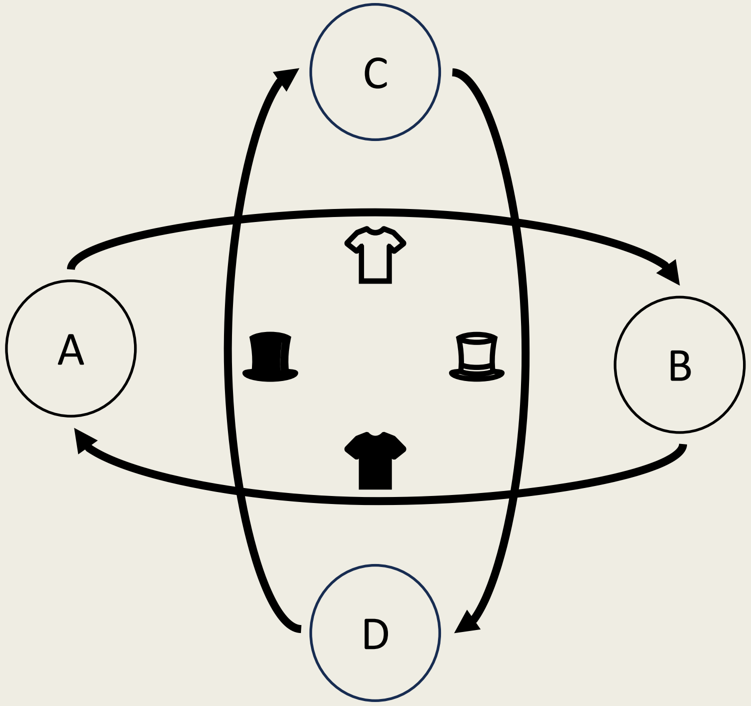


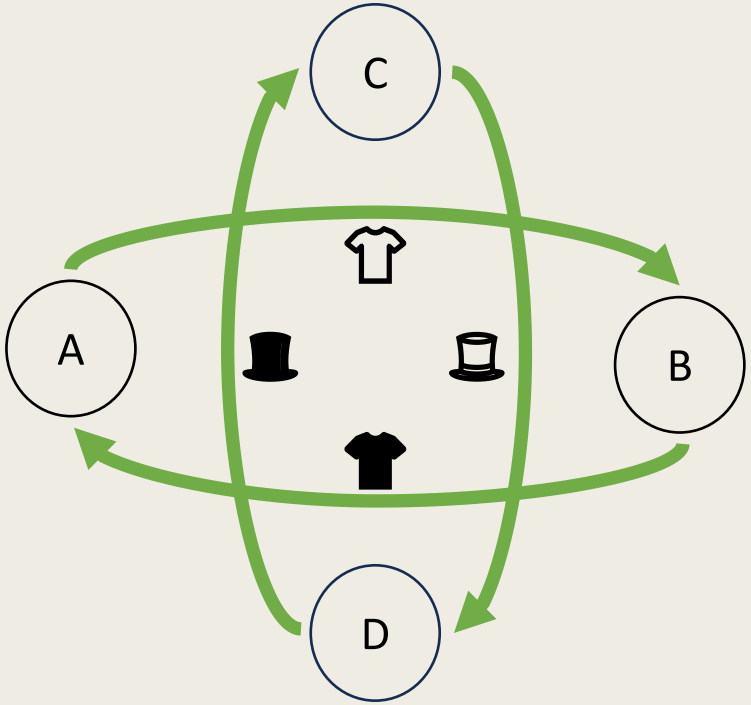


Protocol

Applying Herlihy's Protocol



Applying Herlihy's Protocol



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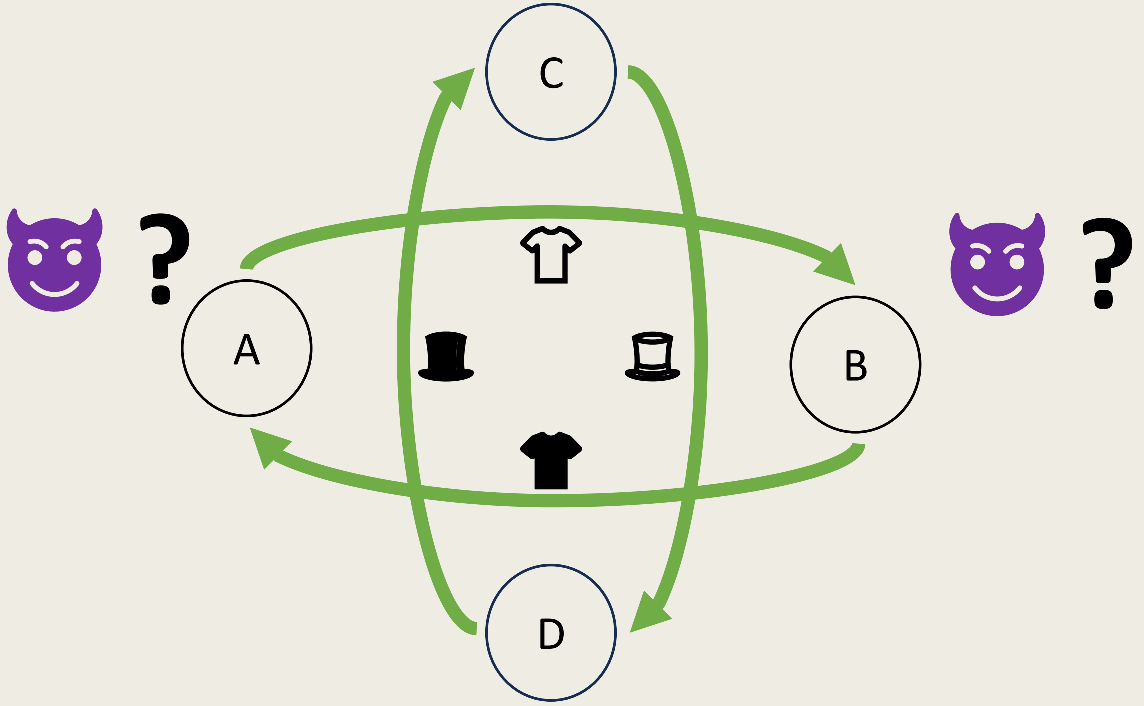

Applying Herlihy's Protocol

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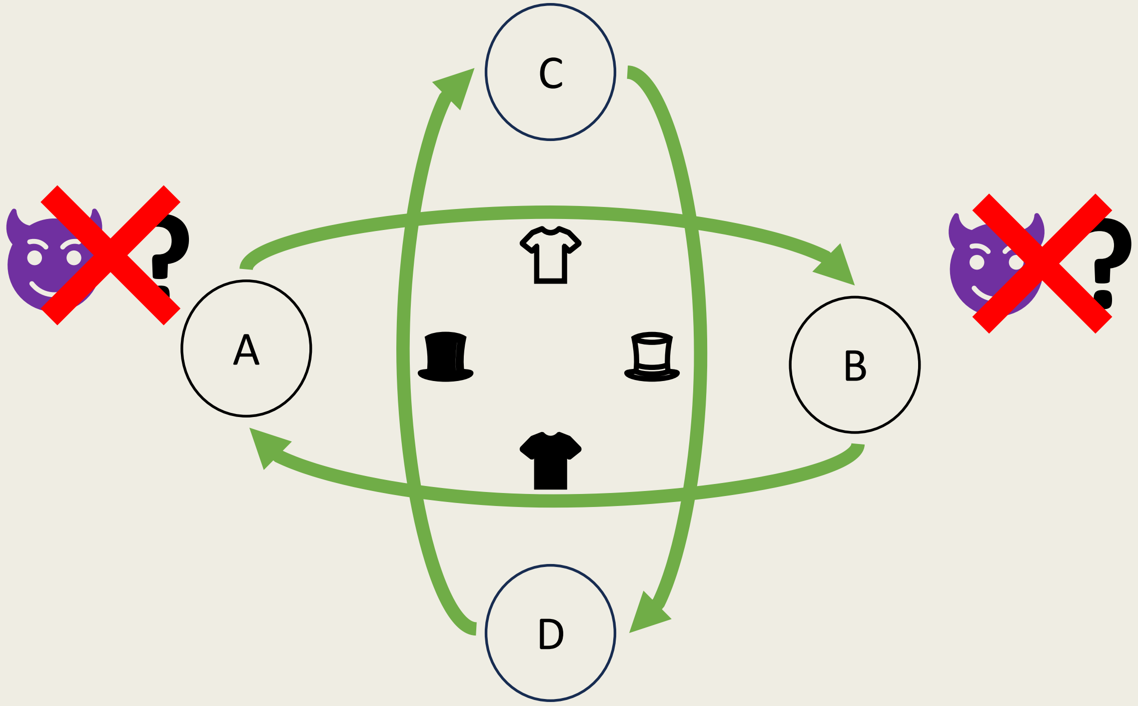

Complexity

SwapAtomic

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- **input**: swap system $S = (D, P)$
- **output**: YES if S has an atomic swap protocol, otherwise NO

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Theorem. SwapAtomic is Σ_2^{P}-complete.

Σ_2^{P}-completeness

Theorem. $S = (D, P)$ has an atomic protocol **iff** there exists a spanning subgraph G of D such that:

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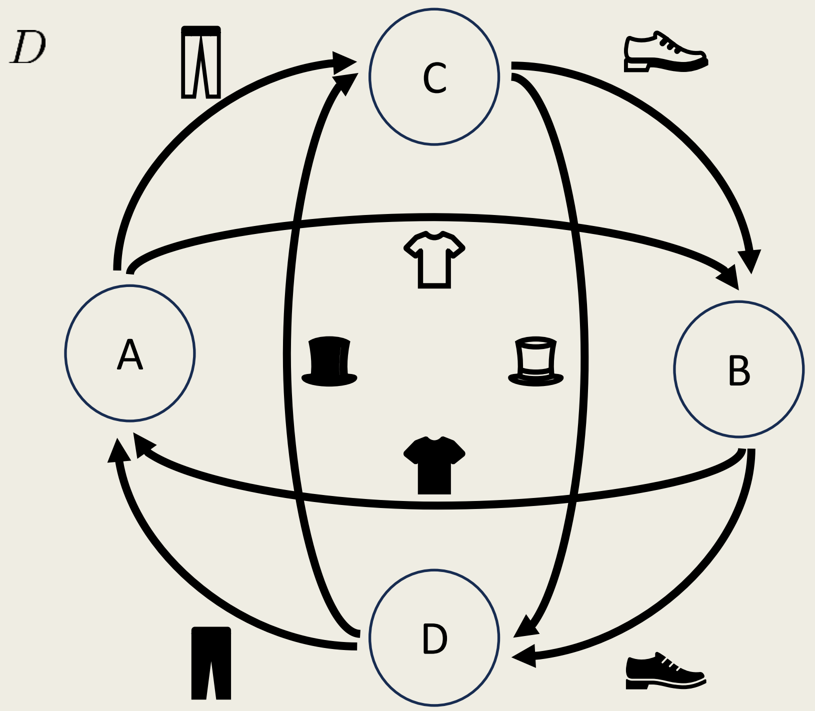

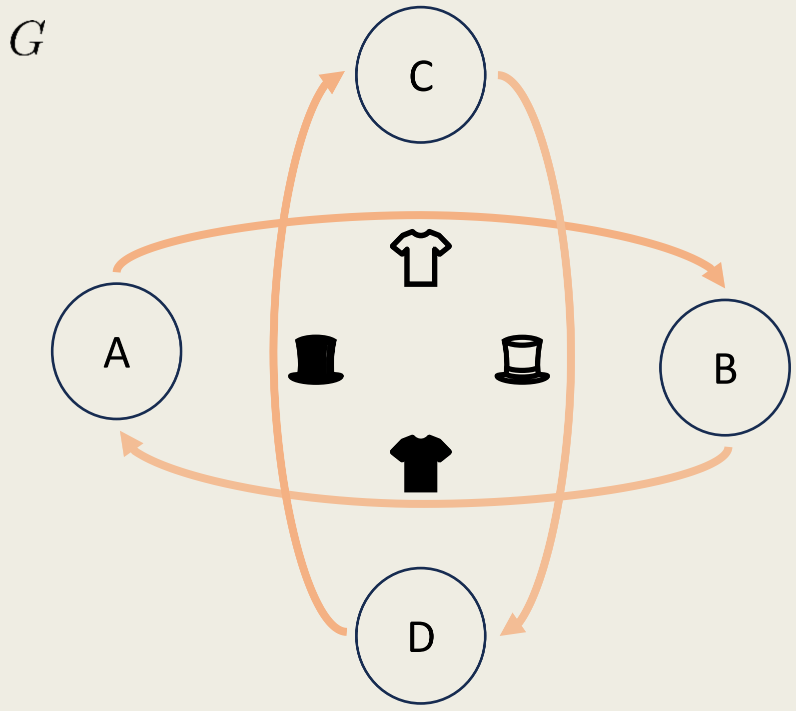

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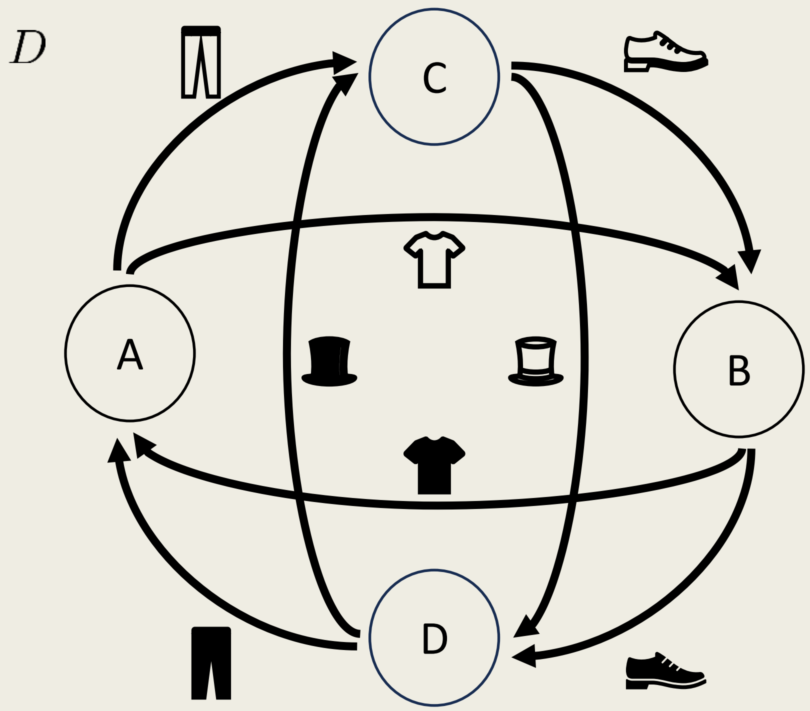

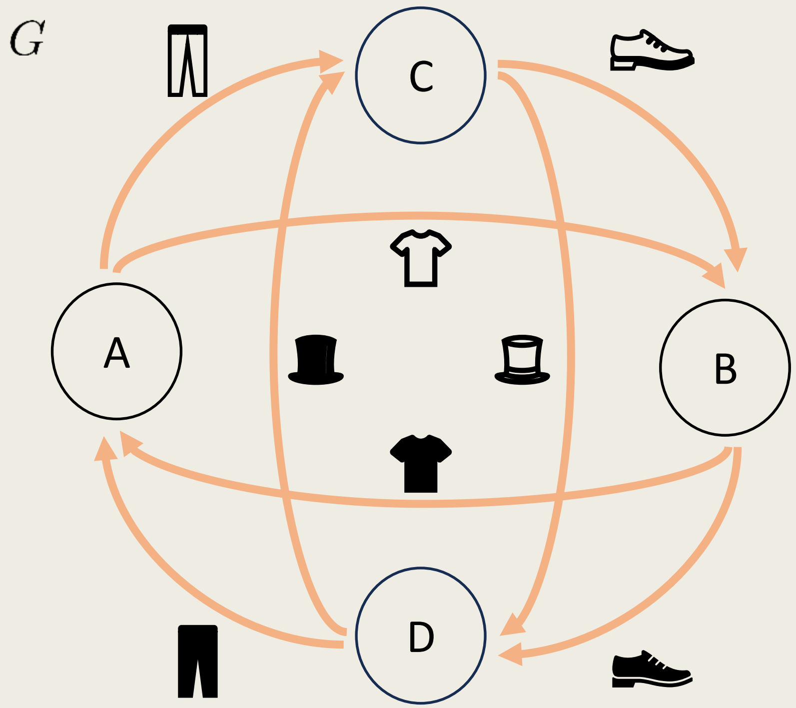

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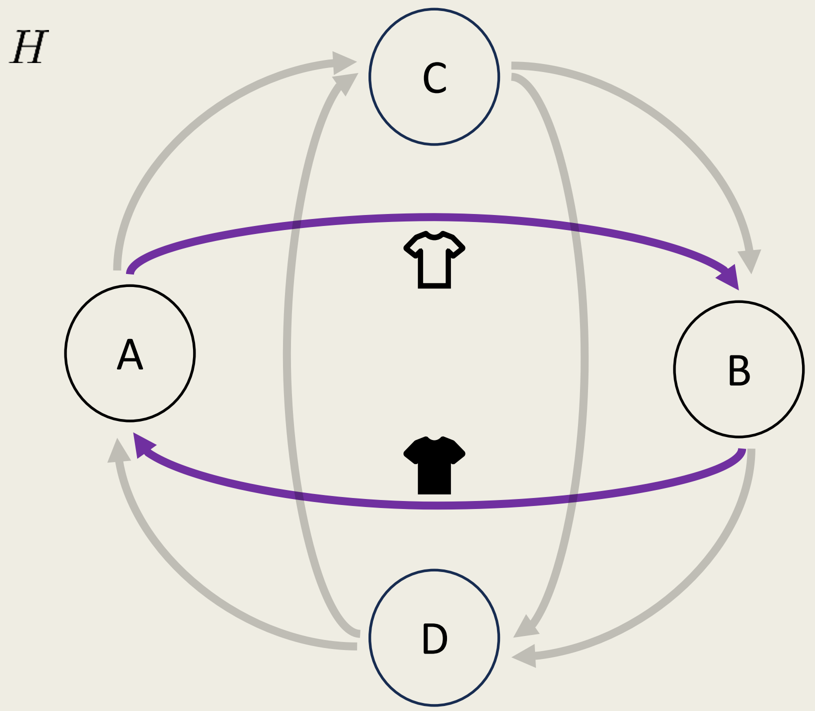

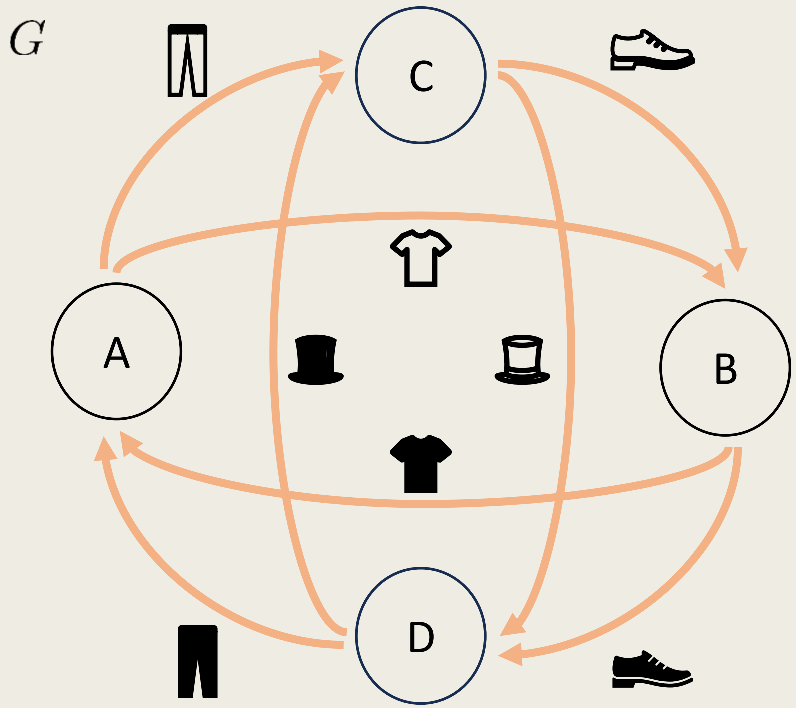

Summary

- Relax structure of preference posets
- Characterize when swap systems have an atomic protocol
- If there is an atomic protocol, we give one
- Complexity of deciding whether a swap system has an atomic protocol

Thank You