A Quantale of Information

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Information Flow
w depends on y and z
Conjunctive Dependency

- w depends on y and z
- w depends on y and z

Sources

Sink
Conjunctive Dependency

w depends on y and z
Fine-Grained Dependency

w depends on y and the first 5 minutes of z
Conditional Dependency
Various semantic models of information flow use **equivalence relations** to model information.
A missing abstraction: disjunctive information flow

w depends on y or z
Disjunctive Information Flow

if x then \( w = y \) else \( w = z \)

\[ w \text{ depends on } \begin{cases} x \text{ and } y \\ or \\ x \text{ and } z \end{cases} \]
Disjunctive Information Flow
What is it Good For?
THE CHINESE WALL SECURITY POLICY

Dr. David F.C. Brewer and Dr. Michael J. Nash

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ABSTRACT

Everyone who has seen the movie Wall Street will have seen a commercial security policy in action. The recent work of Clark and Wilson and the NIST initiative (the Workshop on Integrity Policy for Computer Information Systems) has drawn attention to the existence of a wide range of commercial security policies which are both significantly different from each other and quite alien to current "military" thinking as implemented in products for the security market place.

This paper presents a basic mathematical theory which implements one such policy, the Chinese Wall, and shows that it cannot be correctly represented by a Bell-LaPadula model.

The Chinese Wall policy combines commercial discretion with legally enforceable mandatory controls. It is required in the operation of many financial service organizations and is, therefore, perhaps as significant to the financial world as Bell-LaPadula's policies are to the military.

INTRODUCTION

Until recently, military security policy thinking has

However, the analyst is free to advise corporations which are not in competition with each other, and also to draw on general market information. Many other instances of Chinese Walls are found in the financial world.

Unlike Bell and LaPadula, access to data is not constrained by attributes of the data in question but by what data the subject already holds access rights to. Essentially, datasets are grouped into "conflict of interest classes" and by mandatory ruling all subjects are allowed access to at most one dataset belonging to each such conflict of interest class; the actual choice of dataset is totally unrestrictive provided that this mandatory rule is satisfied. We assert that such policies cannot be correctly modelled by Bell-LaPadula.

It should be noted that in the United Kingdom the Chinese Wall requirements of the UK Stock Exchange [5] have the authority of law [7] and thus represent a mandatory security policy whether implemented by manual or automated means.

Furthermore, correct implementation of this policy is important to English Financial Institutions since it provides a legitimate defence against certain penal classes of offences under their law.
Contributions

A semantic model for disjunctive information flow generalising the lattice of information

• Disjunctive policies (ethical wall)

Model enjoys properties useful for reasoning about programs

• Disjunctive completion of IF lattices

• Compositional reasoning principles
The Lattice of Information
Equivalence Relations as Partitions

Assume (e.g.) a data domain $D = \{0, \ldots, 3\}$

We assume that the observer knows the data domain itself.

Zero partitions = zero knowledge.
Equivalence Relations as Partitions

Assume (e.g.) a data domain $D = \{0, \ldots, 3\}$

output $(d \div 2)$
Equivalence Relations as Partitions

Assume (e.g.) a data domain $D = \{0, \ldots, 3\}$

output $(d == 0)$
The Lattice of Partitions

output (d div 2)

output (d == 0)

Least upper bound
The lattice of Information

A sublattice of LoI(D)

Parity \sqcup isZero

Parity

isZero

All

Represents full knowledge of the value

knowledge of parity and whether it is zero

knowledge of whether it is zero or not

Represents no knowledge of the value
Information Flow Properties

A computation modelled as a function
Information Flow Properties

What can be observed by a given observer
Information Flow Properties

"P maps R-equivalent things to S-equivalent things"
Expressing disjunctive policies?

LoI can express a *specific* instance of a disjunctive flow:

"VW data flows to Consultant if they speak German, otherwise Volvo data flows to consultant"

But such conditions may be complex, unknown, or irrelevant
# Lattice of Information

<table>
<thead>
<tr>
<th>Elements</th>
<th>Equivalence relations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conjunction of information</td>
<td><img src="image" alt="Diagram" /></td>
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<tr>
<td>Disjunctive of information</td>
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![Lattice Diagram](image.png)
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<td>$\times$</td>
</tr>
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</table>

\[
\begin{array}{c c c}
0 & 2 & 2 \\
1 & 3 & 3 \\
\end{array} \sqcup \begin{array}{c c c}
0 & 2 & 2 \\
1 & 3 & 3 \\
\end{array} = \begin{array}{c c c}
0 & 2 & 2 \\
1 & 3 & 3 \\
\end{array}
\]
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| Conjunction of information | $\begin{array}{c}
0 \ 2 \\
1 \ 3
\end{array}$ $\sqcup$ $\begin{array}{c}
0 \ 2 \\
1 \ 3
\end{array}$ = $\begin{array}{c}
0 \ 2 \\
1 \ 3
\end{array}$ |

### Diagram

```
<table>
<thead>
<tr>
<th>0</th>
<th>2</th>
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<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
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2 \\
1 \\
3
\end{array}$ = $\begin{array}{c}
0 \\
2 \\
1 \\
3
\end{array}$ | $P \otimes Q = \{ p \sqcup q \mid p \in P, q \in Q \}$ |
| Disjunctive of information    |                        |                         |
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<td>Tensor operator $\otimes$</td>
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<td>Lattice least upper bound $\sqcup$ = set union</td>
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**Table:**

- Equivalence relations $\sqcup$:
  - Elements: 0, 2, 1, 3

- Tensor operator $\otimes$:
  - $P \otimes Q = \{ p \sqcup q \mid p \in P, q \in Q \}$
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<td>Sets of Equivalence relations (*)&amp;</td>
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<tr>
<td><strong>Conjunction of information</strong></td>
<td>Lattice least upper bound $\sqcup$ 0 2 1 3 = 0 2 1 3</td>
<td>Tensor operator $\bigotimes$</td>
</tr>
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<td>Lattice least upper bound = set union (*)</td>
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(*$
  \left\{ p \sqcup q \mid p \in P, q \in Q \right\} \right.$) tiling

(*$
  \left\{ p \sqcup q \mid p \in P, q \in Q \right\} \right.$) tiling
Tiling closure

Equivalence classes = observations

Tiling closure of a set of partitions = all relations that can be built by mixing and matching observations
Conclusion

A Quantale of Information: a strict generalisation of the Lattice of Information

More in the paper, including:

• Capture the essence of ethical wall policies in a precise sense
• Nice compositional properties which make reasoning easier:

\[
\begin{align*}
  & f_1 : P \Rightarrow Q, \quad Q \sqsubseteq Q', \quad f_2 : Q' \Rightarrow R \\
  \implies & f_1; f_2 : P \Rightarrow R
\end{align*}
\]